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Tolerance analysis methods for the application of ISO AND ASME GD&T to mechanical component: 2D and 3D case studies

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Abstract: This study examines the application of two- and three-dimensional tolerance analysis methods in accordance with ISO 2768:2017 and ASME Y14.5-2018 standards to assess dimensional and geometric compliance in mechanical component assemblies. By analyzing a series of case studies—including a simple shaft-hole coupling and a complex pulley assembly—the study compares deterministic and statistical methods, such as Worst-Case Scenario (WCS) and Root Sum Square (RSS), to explore their effectiveness in real-world design. The analysis demonstrates how 3D tolerance methods can capture complex geometrical interactions and cumulative deviations more accurately than traditional 2D approaches, particularly for assemblies with intricate mating features or spatial dependencies. Key differences between ISO and ASME standards, such as the implications of the envelope principle (ASME Rule #1) and the independence principle in ISO, are highlighted to help practitioners choose the appropriate standard for specific applications.

Keywords: ASME Y14.5:2018; ISO GD&T; tolerances; mechanical drafting; assemblies

1. Introduction

The computation method of unsteady transonic flow based on N-S equations should be best accurate, but to three-dimensional complex problems, it can be achieved.

As widely known, engineering drawings without dimensional, geometrical tolerances and datums are difficult for manufacturers to interpret and may result in non-functional manufacture of components. Insufficient and unclear part tolerancing in engineering drawings not only increases the cost of manufacture and inspection, but it also makes it difficult to validate complaints of faults and increases legal liability[1]. Accordingly, when approaching reliability design from the standpoint prevailing in modern "quality engineering," the designer is required to specify dimensional and geometric tolerances[2]. In addition to defining two levels of judgment (acceptable and unacceptable) during the part's production phase, this approach requires strict control of the parts geometry[3]. Since industrial production is impacted by the normal uncertainty of a part and, by extension, of an assembly, it is well known that it is impossible to manufacture exactly identical parts[4]. Furthermore, during its operation in real environment, the mechanical part could be subjected to geometry modifications because of, for instance, wear and mechanical/thermal deformations. The features of the environment that a product is meant for must be considered by the designer, however this is rarely covered in enough detail.

Dimensional discrepancies are not only determined by random components possibly occurring during manufacturing of parts; systematic mistakes, such as wear-induced flaws during a machining process, also play a significant role. Consequently, individual parts must fulfil certain dimensional constraints to be assembled into sub-assemblies and function correctly. In turn, the sub-assemblies must be able to interface and must therefore fall within a certain tolerance range. This design concept is widely known and named "dimensional specification". In addition, the "geometrical specification", i. e., the definition of geometrical characteristic on or between geometrical features, is used to support the design of mechanical parts and assembly[5]. For this reason, a tremendous effort is being made to create a novel and cogent geometric tolerance management scheme to better define the relationship between functional requirements, geometrical specifications, and relative control procedures. These relationships can be summed up as the principles of geometric dimensioning and tolerancing (ISO GPS and ASME GD&T) and if they are applied correctly and cogently, they will enable the shortcomings of the current methodologies to be overcome and intra- and inter-company communication to be revolutionized. Not by chance, GD&T is widely acknowledged as industry practice as a result of the introduction and popularization of new generations of tolerancing standards, such as ASME Y14.5-2018[6] and ISO 8015: 2011[7]. As widely recognized, GD&T system describes the nominal (theoretically perfect) geometry of controlled features, as

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well as the allowable variation in their size, other geometrical characteristics (form, orientation, and location), and variation between features[8]. It does this by using a symbolic language on engineering drawings. Following this terminology, tolerances and dimensions are chosen based on the function and mating relationship of a part and are open to interpretation. It enables efficient and effective description of part shape and permissible variation by design engineers, production staff, and quality inspectors[9]. When dealing with GD&T methods, three main categories can be defined[10]:

(1) Parts tolerancing. All assembly components have features that must be tolerated, starting with the product data. A subset of each part most significant features is chosen to serve as datums for the remaining features. Next, a set of tolerance types is selected for each feature to restrict variance on both datum features and the feature's nominal geometry.

(2) Tolerance analysis. In this case "the component tolerances are all known or specified, and the resulting assembly tolerance is calculated"[11]. All tolerances are given numerical values, which are decided via optimization (minimization of a cost function according to production limitations) or modification (i. e., refinement of original empirical values).

(3) Tolerance allocation. On the other hand, assembly tolerance is known because of design constraints, while parts tolerances are unknown when it comes to tolerance allocation. The available assembly tolerance must be carefully distributed or allocated among the components. Design criteria are validated whenever necessary throughout an allocation process by computing geometrical entities including gaps, angles, and dimensions involving various assembly components (tolerance stack up).

Regarding the core topic of this work, tolerance analysis methodologies, they may be categorized as one-dimensional (1D), two-dimensional (2D), or three-dimensional (3D) depending on their dimensions. According to the ISO standards, and as also stated by Literature 12, the ever tighter and more complicated needs of tolerance analysis in diverse sectors cannot be met by the conventional 1/2D tolerance analysis methods. More precisely, 1/2D approaches cannot consider the three-dimensional fluctuations of a feature that result from geometric tolerances. Accordingly, it was necessary for researchers and engineers to develop a new technique that can examine the representation and propagation of such geometric tolerances in three dimensions. Deterministic and statistical approaches are the two primary techniques used in engineering design to manage tolerance analysis, regardless of dimensionality. Deterministic methods, mainly used to deal with 2D cases, are often known as worst case methods or methods of extremes; statistical approaches primarily rely on Root Sum Square (RSS).

A thorough analysis of deterministic techniques aimed at forecasting functional characteristic variations from component tolerances is put out by Scholtz[13]. Methods covered by this author are, among others, arithmetic tolerancing (also known as worst-case analysis)[14], simple statistical tolerancing[15] and RSS methods[16].

The goal of worst-case analysis is to determine the dimensions and tolerances so that the chance of non-assembly is exactly equal to zero in every conceivable combination that results in a functioning assembly[17, 18]. It looks at the functional characteristic and considers the worst conceivable combinations of individual tolerances. Thus, worst-case tolerancing may result in extremely tight part tolerances, which in turn may result in elevated manufacturing expenses. The mean center of the process distribution is thought to be represented by the nominal value. Thus, the linear summation of the component tolerances yields the assembly tolerance, especially if the tolerance is symmetrical to the nominal dimension as well as distribution of the dimension values of the measured component is considered symmetrical. Component tolerances defined by worst case are tight and costly to produce. Statistical tolerance analysis, such the widely used RSS (Root Sum of Squares), makes it possible to ascertain whether or not the functional condition will be satisfied. Complex statistical techniques for tolerance analysis allow the defect probability of an existing design to be computed given the functional requirements and size tolerances. Different assumptions about the component dimensions' statistical distributions may be made depending on their tolerances and capability levels. Specifically, RSS approaches rely on modelling component variations using a statistical Normal or Gaussian distribution. The concept is predicated on the notion that manufacturing variances follow a Normal or traditional bell-shaped distribution, symmetrically positioned at the middle of the tolerance limits. Tolerances are assumed to equate to six standard deviations.

Using this approach, it is possible to consider random errors in tolerance stack: by applying a Gaussian distribution model, tolerance analysis can estimate the likelihood of dimensional deviations based on standard deviations around the nominal dimension. Wear-related deviations, however, might suit a Weibull distribution, although uniform distributions might be applicable when variations within a range are equally frequent. Therefore, the designer should be aware of the manufacturing process which stands behind a given part to be assembled to implement a different distribution of random errors.

Despite these techniques are widely known, it is still interesting to carry out a series of case studies to show the main concepts underpinning the use of ISO and ASME GD&T, and to highlight the main differences.

Accordingly, the primary objectives of this work are to apply 2D and 3D tolerance allocation methods on a set of case studies to help practitioners in gaining awareness of the methods most commonly used in mechanical design and to better understand the main differences between an ASME-based design and an ISO-based design. Therefore, the paper investigates which 2D and/or 3D analytical technique may be used to ensure that mechanical components meet both dimensional and geometric tolerances. Accordingly, case studies are used in this type of feasibility research to demonstrate how well-suited basic methods are for tolerating even the most intricate assembly. In particular, the method relies on the application of WCS and modified RSS methods, as explained in the following Sections.

2. Materials and methods

Starting from a simple kinematic pair (i.e., a dovetail guide) used to establish some theoretical principles, a 2D tolerancing analysis of a shaft-hole coupling case, extracted from the ASME Y14.5-2018 standard, is examined. Finally, 3D tolerance methods are applied to assess the analysis of tolerances for an assembly (pulley) to demonstrate the applicability of these methods even on more complex machines.

2.1. Stack analysis

Consider the simple assembly of a dovetail guide (see Fig. 1) which, in its parts, shows the presence of both dimensional and geometric tolerances to define a simple tolerance analysis strategy. To simplify the discussion, only an angularity tolerance is applied on the contact planes between the two parts of the guide.

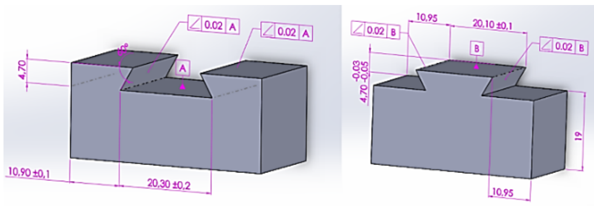


Fig. 1. Elements of dovetail guide under inspection.

To facilitate a more systematic study of the element, the features of the parts are qualified with alpha-numeric codes, indicating the "hole" component with the number 1 and the "shaft" component with the number 2. Note that the modifier " * " is intended to indicate the difference from the ideal condition if the element exploits the entire tolerance indication. These indexations will be useful in defining the path to be taken to analyse the tolerances and in particular the RSS value.

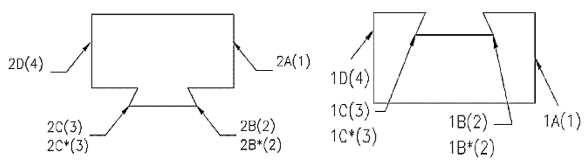


Fig. 2. Functional path to tolerance analysis addressed for the male part (left) and the female part (right).

Before proceeding to the calculation of the parameters, let consider the geometry in the case of the tolerance under consideration. Knowing the definition of angularity tolerance, it is possible to analyse the horizontal space that will be included within the two fictitious planes, and which thus defines a degree of freedom to the horizontal development of the analysed feature. From Fig. 3 it can be agreed that:

$$AN = \frac{AB}{\sin 60°} = 0.0231 \text{ mm} \tag{1}$$

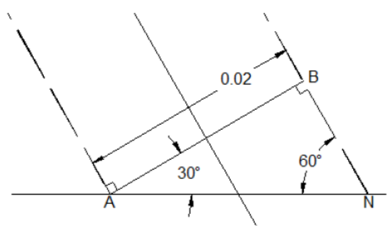


Fig. 3. Geometric representation of analyzed feature.

Once the models have been formed, it is possible to move on to assembly by bringing both elements into the assembly environment; couplings are imposed to relate the two parts and then a tolerance analysis is conducted.

In detail, a stack-up analysis will provide indications of the impact of the geometric tolerance of inclination as well as the dimensional tolerances present. It is possible to derive the dimension Z, initially unknown, which represents the distance from surface A of body 1 to edge B of body 2 through the definition of a "stack path" which, using the references in Fig.3, is as follows:

$$\begin{aligned} &\overline{1A2B^*} + \overline{2B^*2B} + \overline{2B2C} + \overline{2C2C^*} + \\ &\overline{2C^*1C^*} + \overline{1C^*1C} + \overline{1C1B} + \overline{1B1B^*} = 0 \end{aligned} \tag{2}$$

which becomes:

$$Z(\text{WCS}) \pm 0.0115 + (20.1 \pm 0.1) \pm 0.0115 \pm 0.0 \pm 0.0115 - (20.3 \pm 0.2) - (10.9 \pm 0.1) = 0 \tag{3}$$

where Z(WCS) is the dimension of the unknown feature in the Worst Case Scenario (WCS), i.e. the condition where all of the tolerances are set at their limits in order to make a measurement the largest or smallest possible by design.

By solving Eq.(3) it is possible to state the WCS, which returns a nominal value with a relative tolerance of

$$Z(\text{WCS}) = 11.1 \pm 0.4345 \text{ mm} \tag{4}$$

By further applying the Root Sum Square (RSS) Approach it is possible to define the tolerance value for the statistical method:

$$Z(\text{RSS}) = 11.1 \pm 0.2458 \text{ mm} \tag{5}$$

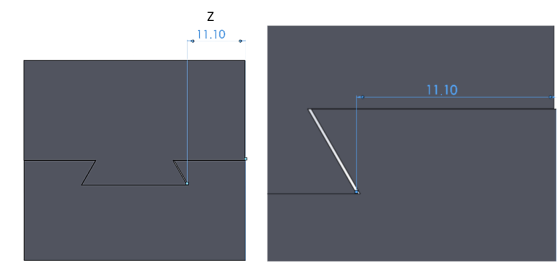


Fig. 4. Analysis of the assembly composed of two elements and detail of section affected by play between two faces.

2.2. 2D tolerancing analysis of a shaft-hole coupling case

To demonstrate the reliability of the tolerance analysis methods, the shaft-pulley coupling presented as an example in the ASME Y14.5M-2018 standard is analysed (see Fig. 5). The values for dimensions and tolerances were chosen from those found within the standard (see Fig. 6). The choice of reference elements is made on the basis of functional assembly and coupling conditions. With regard to the pulley, the inner bore was selected as the primary reference element (identified with A), based on the amount of contact it has with the diameter of the shaft on which it is mounted. The outer edge, in contact with the shaft shoulder, was chosen as the secondary reference (identified with B). Since the installation of the pulley depends on the tightening of the washer and screw, a tertiary reference element is not deemed necessary. The same analysis was conducted on the shaft, where the shoulder, due to the greater amount of contact it has with the support, was taken as the primary reference. Secondary contact is made between the shaft guide and its seat on the bearing, so the former was chosen as the secondary reference. In this case, a tertiary reference is not mandatory as the rota-

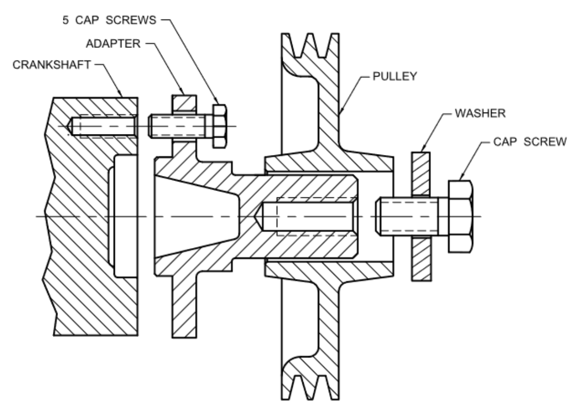


Fig. 5. Adopted figure from ASME Y14.5M-2018 standard: Shaft-pulley coupling.

tion is constrained by the five through holes and the high references on the parts do not have to be checked for rotation. This choice of references minimises tolerance build-up within the component.

Three different scenarios are examined. In the first scenario, the distance, perpendicular to the axis of the two components, between the maximum (minimum) side of the shaft and the maximum (minimum) edge of the pulley bore were chosen as control measurements.

This measurement is called external/internal dimension. The sequence of the simplified assembly was defined to establish both a tolerance chain between the two measurement functions and the order in which the parts are positioned within the assembly.

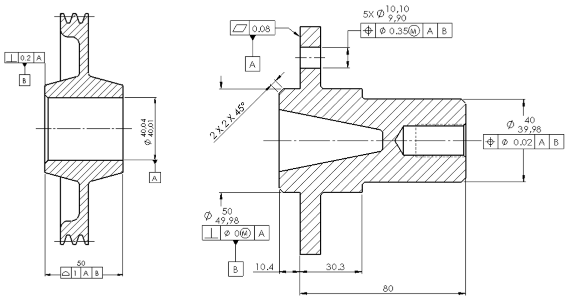


Fig. 6. Example of tolerance values set for shaft-pulley coupling parts.

First the shaft was selected and then the pulley. The concentricity constraint between seat and cavity was considered. In Table 1 the nominal value of the measurement, the minimum and maximum worst condition, and the minimum and maximum sum of the square root (RSS) are shown. It is worth noticing that the results can change depending on the two analysis parameters that are selected. The first is that of the orientation tolerances, which includes the geometric orientation tolerances plus or minus the angular location tolerances in the worst-case evaluation. The second is that of the moving pins and fasteners which use the distances between holes and fasteners to increase the minimum and maximum worst-case results. Each part can move by a value equivalent to the radial distance between hole and fastener.

Accordingly, let ΔZ be the accumulated tolerance for a given control dimension (e.g., Radial clearance) and let δ_i be the tolerance associated to the ith dimension Z_i contributing to control dimension, with $i \in (1, 2, \cdots, n)$. The WCS value is defined by the following simple equation, which is a generalized form of Eq.(3):

$$\Delta Z(\mathrm{WCS}) = \sum_{i=1}^{n} \delta_i \tag{6}$$

By assuming a Gaussian distribution for component variations, it is also possible to define the accumulated tolerance for RSS:

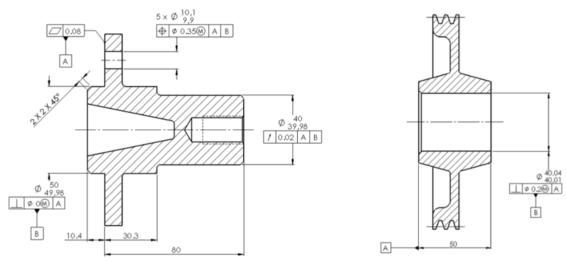
$$\Delta Z(\mathrm{RSS}) = \sqrt{\sum_{i=1}^{n} \delta_i^2} \tag{7}$$

The evaluated RSS dimension in this study is calculated assuming that, instead of causing the variation of each component dimension to be symmetrically distributed about the mean or nominal dimension, setup errors or drifts from time-varying variables, such as tool wear, have displaced the variation. Accordingly:

$$Z(\mathrm{RSS}) = \sum_{i=1}^{n} \alpha_i \delta_i + \sqrt{\sum_{i=1}^{n} \left(1-\alpha_i^2\right) \delta_i^2} \tag{8}$$

where $\alpha_i \in (0\text{-}0.1)$ is the mean shift factor associated with the manufacturing process for dimension Z_i. Table 1 shows the obtained results.

In the second scenario, the locating tolerance on the shaft was changed to the circular oscillation tolerance, keeping the numerical value unchanged. On the pulley, on the other hand, the same dimensions and tolerances used in the first study were left (see Fig. 7(a)). Under these conditions it is possible to evaluate the tolerance stack using both the deterministic and the statistical approaches, by using Eqs.(5) and (7).



(a) (b)

Fig. 7. (a) Modified tolerance values for the shaft in the second scenario; (b) Modified tolerance values for the pulley in the third scenario.

Under these conditions it is possible to evaluate the tolerance stack using both the deterministic and the statistical approaches, by using Eqs.(6) and (8).

In the third scenario, the primary and secondary reference elements were changed for the pulley (see Fig. 7(b)), under the consideration that this condition is not typically used in industrial scenarios. The pulley width was chosen as the primary reference plane, while the bore was selected as the secondary reference to which a perpendicularity tolerance was added. Results for external/

Table 1 Results obtained for the shaft-pulley coupling in the first scenario.

Radial clearance	Nominal value(mm)	ΔZ(WCS) minimum value (mm)	ΔZ(WCS) maximum value (mm)	ΔZ(RSS) minimum value(mm)	ΔZ(RSS) maximum value(mm)
Shaft	0	0⩽ ΔZ ⩽0.03	0⩽ ΔZ ⩽0.03	0.006⩽ ΔZ ⩽0.002	0.017⩽ ΔZ ⩽0.021
Pulley	0	−0.06⩽ ΔZ ⩽0	0⩽ ΔZ ⩽0.06	0.004-0.051	−0.03⩽ ΔZ ⩽0.054
External /Internal dimension	Nominal value(mm)	ΔZ(WCS) minimum value (mm)	ΔZ(WCS) maximum value (mm)	ΔZ(RSS) minimum value(mm)	ΔZ(RSS) maximum value(mm)
Shaft	0	39.99	40.02	39.994	40.017
Pulley	40	39.98	40.04	39.986	40.035

Table 2 Results obtained for the shaft-pulley coupling in the second scenario.

External dimension	Nominal value(mm)	ΔZ(WCS) minimum value(mm)	ΔZ(WCS) maximum value (mm)	ΔZ(RSS) minimum value(mm)	ΔZ(RSS) maximum value(mm)
Shaft	40	39.98	40.04	39.987	40.034
Pulley	40	39.903	40.117	39.93	40.098

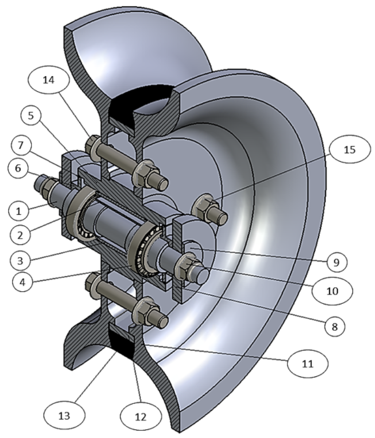
internal dimensions of shaft and hole are depicted in Table 3.

2.3. 3D tolerancing analysis of a shaft-hole coupling case

To validate the 3D tolerance analysis process, consider the assembly of a pulley. The assembly consists of fifteen mutually interfaced elements produced on different lines and subsequently assembled. Fig.8 shows the assembly and its parts list (bill of materials, BOM).

Table 3 Results obtained for the shaft-pulley coupling in the third scenario.

External dimension	Nominal value(mm)	ΔZ(WCS) minimum value (mm)	ΔZ(WCS) maximum value (mm)	ΔZ(RSS) minimum value(mm)	ΔZ(RSS) maximum value(mm)
Shaft	40	39.99	40	39.992	40.013
Pulley	40	39.98	40	39.989	40.013



BOM		
N.	Part	Quantity
1	Shaft	1
2	Spacer inner rings	1
3	Spacer outer rings	1
4	Tapered roller bearings	2
5	Central sleeve	1
6	Sealing cap	2
7	Gasket	2
8	Rim	2
9	Washer ISO 7043 30x60	2
10	Nut ISO 7043 M20	
11	Half-Pulley	2
12	Pulley spacer	2
13	Ring	1
14	Screw ISO 4162 M16x100x38	4
15	Nut ISO 4161 M16	4

Fig. 8. Pulley 3D assembly.

Assuming the scenario in which the components of the assembly are produced by different companies and subsequently assembled in the final system by the company that markets them, the design of the individual parts should include the allocation of the tolerances required for correct assembly. For a more comprehensive analysis of the effects of using a given tolerance system, the critical assembly measurements performed on the assembly are shown below, analysing the variability of dimensions as the finish class changes to demonstrate the tolerance values retrievable by using deterministic and statistical methods from GD&T standard. In detail, a comparison between the retrievable results applying the ISO 2768:2017 and by applying ASME Y14.5-2018 are compared. In both cases, the tolerance analysis is applied by examining two main dimensions: the maximum external diameter of the pulley and the maximum axial size of the assembly (see Fig. 9). Eqs. (5) and (7) are used, respectively for WCS and RSS parameters. Fig. A1 (see Appendix) shows the entire dimensioning and tolerancing of the non-commercial pulley assembly components.

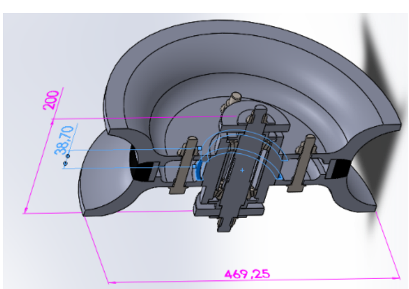


Fig. 9. Dimensions controlled using tolerance analysis.

2.3.1. ISO 2768: 2017 based tolerance analysis

The analysis starts from the study of the axial size of the pulley whose nominal dimension is 200 mm and whose variations caused by the imposed tolerance chain are visible and comparable in the table below (Table 4). It must be noticed that the tolerance analysis is performed by applying the tolerance classes according to ISO 2768:2017 standard. Subsequently, studies can also be set up to investigate the radial size, shown in Table 5, as well as the space between the two faces of the pulley in Table 6.

Table 4 Pulley axial size.

Tolerance class	Medium	Fine
Nominal dimension	200	200
Min Dim. WCS	199.4	199.95
Max Dim. WCS	201.212	200.974
WCS Variation	1.812	1.024
RSS min	199.668	199.974
RSS max	200.456	200.3
RSS Variation	0.788	0.326

Table 5 Pulley radial size.

Tolerance class	Medium	Fine
Nominal dimension	469.25	469.25
Min Dim. WCS	468.05	468.75
Max Dim. WCS	470.45	469.75
WCS Variation	2.4	1
RSS min	468.65	468.995
RSS max	470.019	469.563
RSS Variation	1.369	0.568

All parts comprising the assembly were considered to be affected by dimensional and geometrical limits inherent to the tolerance

Table 6 Inner distance between inner faces of pulley.

Tolerance class	Medium	Fine
Nominal dimension	38.969	39.696
Min Dim. WCS	38.696	38.696
Max Dim. WCS	39.604	39.311
WCS Variation	0.908	0.615
RSS min	38.755	38.755
RSS max	39.222	39.092
RSS Variation	0.467	0.337

class analysed, with the exception of part 4 (tapered roller bearing), the choice of which is often constrained by catalogue consultation and subject, by its very nature, to tolerances generally of the 'fine' class. It can be noticed that is a recurrence in the measurements of a significant reduction in variations from the medium to the fine tolerance class and that these become more pronounced as the nominal size increases.

2.3.2. ASME Y14.5-2018 based tolerance analysis

The ASME Y14.5-2018 standard renders the indication of general tolerances unnecessary, as it uses the envelope principle as a default (while this is prescribed using the modifier Ⓔ in the ISO standard), which allows shape errors to be limited. Such a principle is named Rule#1 which states that "the form of a regular feature of size is controlled by its limits of size". A regular feature's real surface can never be larger than the envelope specified by MMC. Therefore, "if the feature measures at MMC, the form of the feature must be perfect, which in the real world is impossible to achieve". Table 7 shows the results of tolerance analysis for the two control dimensions of the pulley assembly obtained by using the Rule #1.

Table 7 Results obtained using Rule#1 included in ASME Y14.5-2018.

Rule #1	Axial size (mm)	Radial size (mm)	Inner distance (mm)
Nominal dimension	200	469.25	38.696
Min Dim. WCS	199.9	468.15	38.646
Max Dim. WCS	201.056	470.35	39.297
WCS Variation	1.156	2.2	0.651
RSS min	199.949	468.67	38.646
RSS max	200.345	469.961	38.952
RSS Variation	0.396	1.291	0.306

3. Results and Conclusions

The main purpose of the present study was to emphasize the need of communicative and operational standards in mechanical drafting for the practice of mechanical design that, better than others, can create a direct, unambiguous link between the design and manufacture of a component or assembly. It was therefore essential to focus on those that still represent the most influential and globally recognized standardization societies (ASME and ISO) in the search for 'ideal' codes for the definition of a 'perfect' geometry desired by all designers.

The work emphasizes the distinct effects of implementing ASME Y14.5 -2018 and ISO 2768:2017 for tolerance evaluations, particularly when employing statistical (Root Sum Square, RSS) and deterministic (Worst-Case Scenario, WCS) approaches. In detail, the following key findings can be drafted:

- The Medium and Fine variation classes offered by ISO 2768 produce noticeably differing tolerance spreads. For example, the pulley's axial and radial dimensions show a reduction of at least 50% when going from the "Medium" tolerance class to the "Fine" tolerance class. But when size is at maximum material condition (MMC), Rule #1 of the ASME Y14.5 -2018 standard applies an envelope principle that restricts form fluctuation.
- Part form at MMC is impacted by ASME Y14.5 -2018 Rule #1, which states that even small manufacturing errors can drastically change fit and functionality. Similar restrictions might be imposed by ISO's Ⓔ modifier, but it needs to be explicitly stated to allow for design freedom.
- As evidenced by the broader WCS variance in ISO as opposed to ASME, ISO's independence principle treats each tolerance separately unless otherwise noted, resulting in higher cumulative variation.

Furthermore, it should be noted that number of potential problems could occur that could lead to deviations from the intended tolerances, such as:

- Manufacturing Variability: Dimensional and geometric variances can be caused by variables such as machine calibration, temperature variations, and tool wear. Part consistency may be impacted if machine precision deteriorates over time. To deal with this complex topic, engineers should implement model-based simulations (e.g., using Finite Element Method - FEM) for assessing deformations possibly induced by external forces and coupling of parts as well as considering contact mechanics, especially for mating surfaces such as gears or bearings.
- Material Inconsistencies: Unexpected dimensions and geometric changes in the finished product might result from variations in material qualities, particularly during operations like casting or forging. In this case, engineers should estimate the amount of material loss over time, based on parameters such as loads and hardness of parts.
- Assembly Sequence Errors: In components interfaced from several vendors, misalignment in assembly phases might result in tolerance stacking problems that may not precisely meet the required standards. For this reason, WCS is adopted to consider the most conservative estimates for tolerance.
- Interpretational Differences: Given ISO's latitude in tolerance stacking and ASME's more stringent Rule #1 application, engineers or manufacturers may have differing interpretations of ISO and ASME standards.

Consequently, to ensure better consistency and compliance with specified tolerances, engineers could integrate Statistical Process Control (SPC) in tolerance analysis[19] modifying design components to lessen the need for extremely tight tolerances, which can minimize manufacturing stress and increase overall reliability. Standardized inspection procedures could be useful to guarantee that all parts fit together fulfilling a desired set of tolerances for assemblies involving several suppliers.

Regarding the 2D analysis, the present work further stresses the fact that it relies on simplified assumptions, such as linear stack-ups, which may overlook interactions between complex shapes or features; therefore, its applicability should be considered effective for assemblies with few interdependent parts, as the cumulative effect of tolerances is likely manageable. In fact, the outcomes for various scenarios have demonstrated that, in certain circumstances, results vary from one another by two or three decimal places after zero. At other occasions, there was a greater difference—one or two orders of magnitude. When building a component, the designer has to be aware of this fact since selecting one or more tolerances will have an impact on the assembled product, perhaps compromising its functionality.

With reference to 3D analysis, it offers a more realistic assessment by modelling three-dimensional geometric interactions, allowing for the consideration of actual part geometry and assembly constraints. It works especially well for handling tolerances for complicated assemblies and parts, like those with complicated connections, asymmetrical geometries, or important spatial relationships. This enables a more thorough understanding of how fit, alignment, and performance are impacted by changes in all three axes. Tolerances for shape, orientation, position, and size are accommodated in approaches that more accurately reflect production variances in the real world. However, 3D analysis can be computationally intensive and time-consuming and therefore its use is recommended for high-stakes assemblies where failure to meet tolerance requirements could lead to significant performance issues or safety concerns.

Moreover, it is possible to appreciate strong differences in tolerance analysis using the two different standards. This is mainly due to the fact that there subsist some important differences between ISO and ASME GD&T. Firstly, ASME Y14.5-2018 is a standard that pertains solely to the dimensions and tolerances on a component design. It lacks the parameters needed for verifying the produced part's dimensions. ASME Y14.43, a different document, discusses this. Dimensioning and tolerancing guidelines for gauges and fixtures used to check manufactured components are provided by ASME Y14.43. On the other hand, three standards for coordinate measuring machines (CMMs) and eight standards for dimensional measurement equipment (gages) are included in ISO GPS standards. The ISO GPS standards provide a wide range of requirements for the design of various gauge types, acceptance testing, and coordinate measuring system calibration in addition to dimensioning and tolerancing. The main specifications and calibration standards for GPS measuring equipment (micrometres, callipers, and gauge blocks) are outlined in ISO 14978. The purpose of ISO 14253, a six-part standard, is to verify, using measuring equipment, whether a part being examined is conforming or non-conforming.

Furthermore, as explained in previous section where the pulley assembly is evaluated, the "Independency Principle" represents one of the core GPS principles. Unless a connection is shown on the design, the independence principle asserts that each dimensional or geometric tolerance on a drawing is satisfied separately. This guideline is not relevant in ASME Y14.5-2018, because the envelope principle—which controls the maximum variation in a part's form—indicates that size tolerance limitations govern a part's maximum variation. The choice of ISO or ASME GD&T standard should consider this issue since results (see Table 7) are strongly influenced by the need to either respect the Rule#1 or not in designing a part. As known, ISO and ASME tolerance systems converges when the Envelope principle is imposed in drawings by using the Ⓔ modifier. Focusing on tolerance analysis, then analysing different examples of assemblies, shows how the use of GD&T standards ensures a concise and effective communication of the physical characteristics of a component, even applying simple stack analyses like the ones proposed in this case studies work. Their use will therefore undoubtedly increase since it enhances the technical drawing's representation of reality by reducing annotations, makes the drawing simpler to comprehend, and cuts waste, all of which lower the overall cost of manufacturing.

Future research will be addressed to the study of the integration of GD&T standards with cutting-edge methods like additive manufacturing, Industry 4.0, and advanced machining processes, given the ongoing developments in manufacturing technologies. In particular, it can be beneficial to address, also using practical examples, how standards can be modified to meet the particular manufacturing layer-by-layer process construction in additive manufacturing, how to improve tolerance analysis using real-time data analytics and smart systems in Industry 4.0, and how to adapt current procedures to incorporate novel machining techniques.

References

1. Walter MS. Dimensional and geometrical tolerances in mechanical engineering-a historical review. *Machine Design* 2019;11(3):67-74.
2. Koch PN, Yang RJ, Gu L. Design for six sigma through robust optimization. *Structural and Multidisciplinary Optimization* 2004; 26: 235-248.
3. Martins Joaquim RRA, Ning A. *Engineering design optimization*. Cambridge: Cambridge University Press, 2021.
4. ElMaraghy W, ElMaraghy H, Tomiyama T, et al. Complexity in engineering design and manufacturing. *CIRP Annals* 2012;61(2):793-814.
5. Zhang M, Shi Z, Mathieu L, et al. Geometric product specification of gears: The GeoSpelling perspective. *Procedia CIRP* 2015;27:90-96.
6. ASME Y14.5-2018. Dimensioning and tolerancing.
7. ISO/TC 213. Dimensional and geometrical product specifications and verification.
8. Sahay C, Ghosh S. Interpretation of modifier Ⓜ "Circle M" ". In: *ASME International Mechanical Engineering Congress and Exposition.* 2020;84577:V009T09A005.
9. Colledani M, Tolio T, Fischer A, et al. Design and management of manufacturing systems for production quality. *CIRP Annals* 2014; 63 (2):773-796.
10. Hong YS, Chang TC. A comprehensive review of tolerancing research. *International Journal of Production Research* 2002; 40(11): 2425-2459.
11. Makelainen E, Ramseier Y, Salmensuu S, et al. Assembly process level tolerance analysis for electromechanical products. In: *Proceedings of the 2001 IEEE International Symposium on Assembly and Task Planning (ISATP2001). Assembly and Disassembly in the Twenty-first Century.* 2001.p. 405-410.
12. Chen H, Jin S, Li Z, et al. A comprehensive study of three-dimensional tolerance analysis methods. *Computer Aided Design* 2014; 53:1-13.
13. Scholz F. Tolerance stack analysis methods-a critical review. *Boeing Information & Support Services*, 2014.
14. Anselmi N, Rocca P, Massa A. Tolerance analysis of reconfigurable monopulse linear antenna arrays through interval arithmetic. *Journal of Electromagnetic Waves and Applications* 2023;37(13):1-16.
15. Talebi S, Koskela L, Tzortzopoulos P, et al. Deploying geometric dimensioning and tolerancing in construction. *Buildings* 2020;10(4):62.
16. Sahani AK, Jain PK, Sharma Satish C. Geometrical tolerance stack up techniques. In: *DAAAM International Scientific Book*, 2013. p. 857-872.
17. Roy U, Liu C, Woo TC. Review of dimensioning and tolerancing: Representation and processing. *Computer Aided Design* 1991; 23(7): 466-483.
18. Dantan JY, Eifler T. Integrated uncertainty management in parametric design and tolerancing. *Journal of Engineering Design* 2021; 32(10): 517-537.
19. Zhang Y, Yang M, Zhang Y. Concurrent design for process quality, statistical tolerance, and SPC. *Communications in Statistics-Theory and Methods* 2006; 35(10):1869-1882.

Appendix A

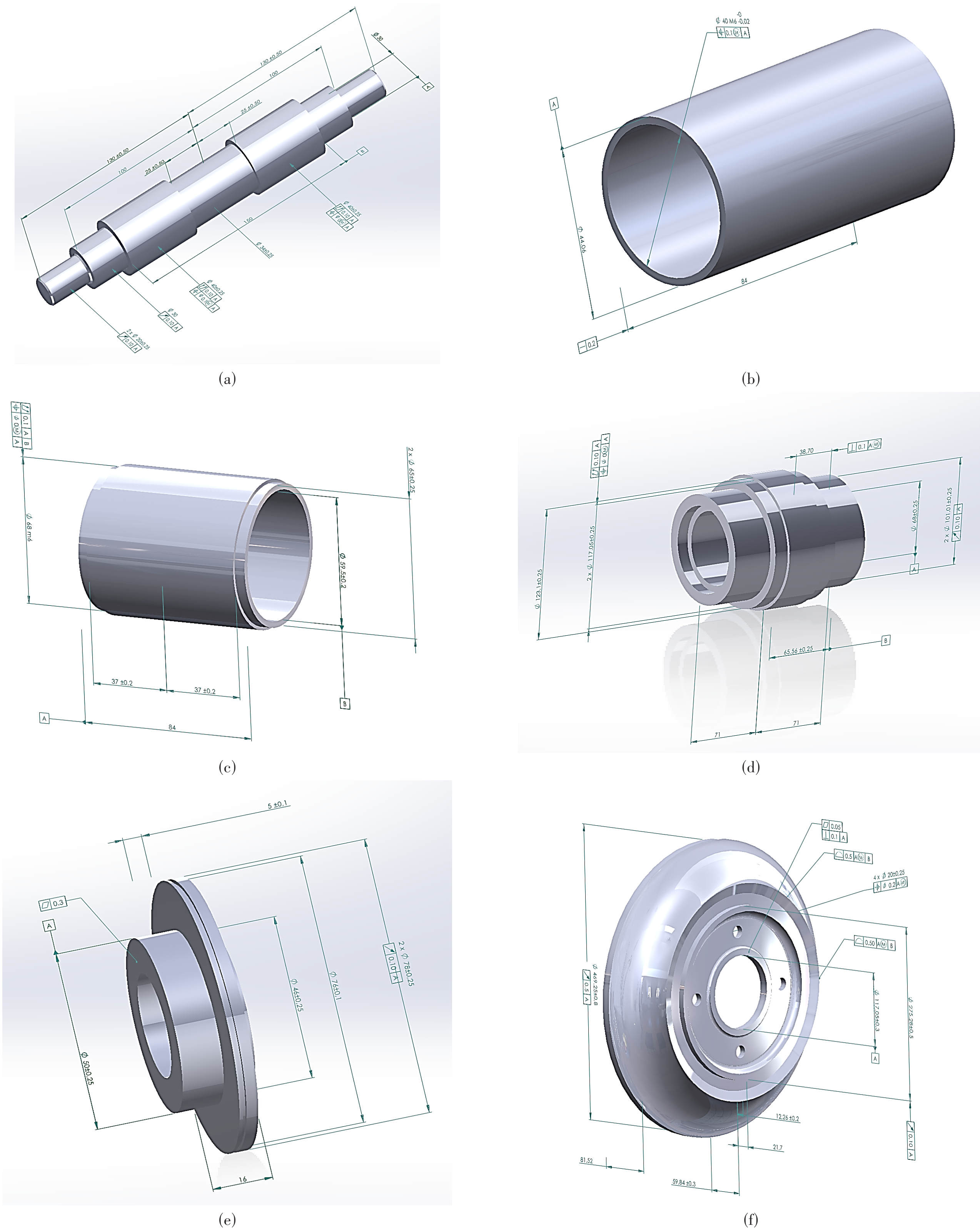


Fig. A1 Dimensioning and tolerancing of pulley assembly non-commercial parts; (a) Shaft (P1); (b) Spacer inner rings (P2); (c) Spacer outer rings (P3); (d) Central sleeve (P5); (e) Sealing cap (P6); (f) Half-pulley (P11).