

REVIEW ARTICLE

Advancements in Multipath Mitigation for GNSS Receivers: Review of Channel Estimation Techniques

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Multipath interference poses substantial challenges to global navigation satellite system (GNSS) receivers, leading to inaccuracies in the time of arrival (TOA) measurement of the line of sight (LOS) signal. Therefore, to mitigate the impact of multipath on receivers, the problem has been approached at several system development stages—signal design, reception, and processing. While efforts and advancements have been achieved over the years at each stage seeking navigation robustness, this article focuses on the signal processing stage by presenting a review of advanced multipath mitigation techniques using adaptive channel parameter estimation at the correlation level. The multipath mitigation literature often resorts to optimistic assumptions—high signal-to-noise ratio (SNR), static multipath channel, single fading channel distribution, and so on—while in real-world scenarios noise is prominent, the number of paths and their states vary at different rates, channels are nonstationary, along with other nonideal conditions. Moreover, it is important to directly compare different techniques to characterize their applicability and limitations. An analysis of adaptive algorithms is conducted for multipath mitigation applications.

Introduction

Multipath interference is a prevalent issue in global navigation satellite system (GNSS) receivers, wherein the transmitted signals reach the receiver through multiple paths due to reflections and diffractions [1,2]. These effects combine at the receiver antenna, effectively capturing a composite signal comprising the direct line of sight (LOS) and the delayed multipath components. The pseudorange primarily measures the time the signal propagates from the satellite to the receiver. The time of arrival (TOA) of the LOS signal becomes challenging to measure accurately due to the overlapping delayed signal components. The autocorrelation function (ACF) will appear distorted, negatively affecting the delay lock loop (DLL) ability to perform accurate LOS code delay measurements. The inaccuracy in the TOA measurement of the LOS signal leads to positioning errors.

In today's GNSS systems, multipath is a serious error source that compromises the viability of its use in critical applications such as autonomous driving [3,4]. Numerous mitigation techniques have been proposed to address the challenges multipath interference poses. Each technique offers specific advantages and is suitable for different use cases. While derivations of traditional techniques have been extensively studied and implemented in commercial receivers, recent advancements have introduced new methods that deserve attention and further development [4]. Alternatively, some studies take advantage of well-established old techniques and adapt them to mitigate multipath [5,6].

Integrating GNSS receivers in critical applications, mainly those subject to challenging environments, depends on effective multipath mitigation for improved accuracy and reliability of positioning solutions, ensuring safe and precise navigation. Given the pertinence of this research topic, it is essential to characterize the various multipath mitigation methods, particularly the most advanced ones, to understand their strengths, limitations, and applicability.

This article presents a comprehensive survey of multipath mitigation techniques (MMTs) in GNSS receivers. Our primary focus is to provide in-depth analysis and characterization of the most promising strategies based on channel parameter estimation. By comparing and evaluating these techniques, we aim to clarify their effectiveness, limitations, and potential for addressing multipath interference in real-world scenarios.

The remainder of the paper is organized as follows: Both the signal and channel models are presented in Signal and channel models to introduce the structure of the satellite signals and the properties of direct and indirect path signals. Multipath Mitigation at Correlation Stage contextualizes the problem of multipath mitigation by introducing different approaches explored in the literature. The approach based on channel parameter estimation is reviewed in Channel Parameter Estimation Review with a particular focus on adaptive estimation techniques. A comparison between all the techniques covered in the document is carried out in Summary and Discussion, where the limitations and applicability of each technique are addressed. Future research direction in multipath mitigation through channel estimation is presented

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in Future Directions. Conclusion and final remarks from this survey are provided in Conclusion.

Signal and channel models

One prevalent signal model in modern satellite navigation systems is the direct-sequence spread-spectrum signal. It reduces overall interference using a broader signal bandwidth than the information bandwidth. A code division multiple access scheme uses orthogonal codes to differentiate between satellites that transmit simultaneously. The transmitted signals consist of 2 main components: a ranging code, also known as the pseudo-random noise (PRN) spreading sequence, and a low-rate data link responsible for broadcasting essential information for positioning, including satellite orbital parameters and clock data. The complex baseband model of the signal transmitted by a GNSS satellite is given by [7]:

$$s_T(t) = \sqrt{P_T} \left(\gamma \sum_{u=-\infty}^{\infty} d_I(u) p_I(t - uT_{b_I}) + j\sqrt{1-\gamma^2} \sum_{l=-\infty}^{\infty} d_Q(l) p_Q(t - lT_{b_Q}) \right) \quad (1)$$

where P_T is the transmitted power, γ is the power balance, $d \in \{-1, 1\}$ is the navigation bits, T_b is the data bit period, and $p(t)$ is given by:

$$p(t) = \sum_{n=0}^{N_c-1} q(t - nT_{PRN}) \quad (2)$$

$$q(t) = \sum_{k=0}^{L_c-1} c(k) g_T(t - kT_c) \quad (3)$$

where N_c is the number of modulation code periods per data bit, T_{PRN} is the code period ($T_{PRN} = T_b/N_c$), L_c is the total number of chips, T_c is the chip period ($T_c = T_{PRN}/L_c$), $c(k) \in \{-1, 1\}$ is the chip values, and $g_T(t)$ is the code chip pulse shape. All parameters are equally representative of both in-phase and quadrature components, if applicable.

The received signal can be modeled for both LOS and multipath components. For a given satellite, the received complex baseband signal affected by $M - 1$ multipath signals is modeled as:

$$s_R(t) = \sum_{n=0}^{M-1} \alpha_n(t) \tilde{q}(t - \tau_n(t)) e^{j\phi_n(t)} + n(t) \quad (4)$$

where $\alpha_n(t)$, $\tau_n(t)$, and $\phi_n(t)$ are the amplitude, delay, and carrier phase of the n th received signal component, respectively, with $n(t)$ additive white Gaussian noise. $\tilde{q}$ is the received filtered version of the transmitted q . The time-varying Doppler shift due to the receiver-satellite dynamics is taken into consideration in the phase parameter:

$$\phi_n(t) = 2\pi\nu_n(t) + \phi_{n,0}(t) \quad (5)$$

where $\nu_n(t)$ is the time-varying Doppler shift and $\phi_{n,0}$ is the initial carrier phase of the n th received signal component.

During the tracking stage, the receiver keeps track of both the code phase and Doppler shift so the signal TOA can be measured and the navigation bits transmitted by the satellite

can be successfully extracted. This is achieved by 2 independent tracking loops—DLL and phase lock loop (PLL) [or frequency lock loop (FLL), used during pull-in]. While the PLL/FLL brings the signal to the baseband, the DLL despreads the signal, aiming to leave only the navigation bits in the data stream. After the despreading process—also known as autocorrelation—of the baseband signal represented in Eq. (4), the result is:

$$Y_k = \alpha_0 R_{\tilde{q}q}(\delta_k - \tau_0) e^{j\phi_0} + \sum_{i=1}^{M-1} \alpha_i R_{\tilde{q}q}(\delta_k - \tau_i) e^{j\phi_i} + N_k \quad (6)$$

where α_0 , τ_0 , and ϕ_0 are the amplitude, delay, and carrier phase of the LOS signal, respectively. The number of correlators is represented by $k \in [-N, N]$, forming a total of $2N + 1$ correlators, and δ_k is the correlator delay where $\delta_{k+1} > \delta_k$ and $\delta_0 = 0$. N_k represents the white Gaussian noise containing also spread spectrum interferences from other satellites, and $R_{\tilde{q}q}(t)$ is the ACF approximation of the given satellite signal defined by:

$$R_{\tilde{q}q}(\delta_k - \tau_i) = \frac{1}{T} \int_0^T \tilde{q}(t - \tau_i) q(t - \delta_k) dt \quad (7)$$

where T is the coherent integration time. Due to geometrical and physical reasons, the relationship between the LOS and multipath components in outdoor channels can be established as follows:

$$\tau_i(t) > \tau_0(t) \quad \forall i \in \{1, \dots, M-1\} \quad (8)$$

Effectively, the LOS and multipath signals are combined, resulting in a distorted ACF. Depending on the phase at which the multipath signals arrive at the antenna, the interference can be classified as constructive (in-phase with LOS) or destructive (out-of-phase with LOS).

Multipath Mitigation at Correlation Stage

Multipath mitigation can be performed at different stages of the receiver. This work focuses on multipath mitigation at the correlation stage, as highlighted in Fig. 1. More specifically, this work is dedicated to techniques based on channel parameter estimation. Before characterizing such methods, it is relevant to briefly mention the evolution of the techniques that brought increased robustness to the tracking stage of receivers against multipath interference.

There are 2 major categories of techniques for multipath mitigation—(a) geometry-based techniques and (b) channel estimation techniques.

Geometry-based techniques

Geometry-based techniques exploit the geometry of the ACF to reduce the sensitivity to multipath signals, thus improving the performance of the correlator or discriminator of the code-tracking loop. The wide correlator first used for GPS L1 C/A code tracking applied only 3 correlators—early, prompt, and late—with a spacing of one chip between early and late replicas. This simple correlator architecture enabled successful signal tracking but with a reported pseudorange measurement bias of up to a hundred meters when multipath is present. Despite the low complexity, the code delay measurement limitations were evident, and the accuracy was severely degraded in the presence of short-delayed replicas, as coherent multipath appears for relative delays lower than the chip period.

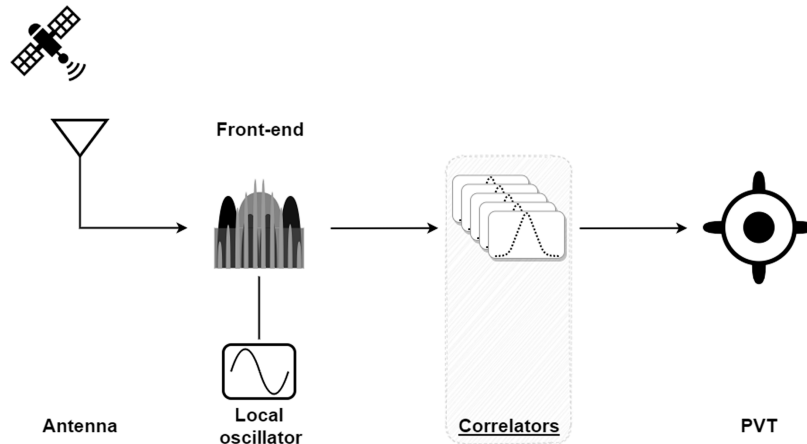


Fig. 1. Location of the correlation stage in the receiver architecture.

The narrow correlation technique was proposed as an alternative to the traditional one-chip spacing correlators to achieve high-precision measurement of the signal arrival time using code measurements. This technique is designed based on the idea that decreasing the correlator spacing allows the early-late tracking point to be closer to the peak of the correlation function [8]. Similar to the traditional early-minus-late (EML) discriminator, the narrow correlator discriminator is computed by subtracting the early and late correlator values after integration during at least one code period. The early and late code replicas are assumed to have a subchip code phase spacing, with a typical value of 0.1 chips, 5 times smaller than the traditional correlator spacing.

Without noise, the correlator spacing could be reduced even more, and smaller errors would be possible. However, small correlator spacing implies a wider pre-correlation bandwidth and a higher sampling rate. Moreover, increasing the data sampling rate results in more computational load to the receiver as the processing rate increases. Another disadvantage when reducing the correlation interval is the operating range of the discriminator. The receiver cannot track substantial changes in the LOS code delay and may lose code lock more frequently than a traditional wide correlator. Despite the limitations, the narrow correlation technique can bring more accuracy to the code-based measurements when multipath interference is present, and this technique has been widely adopted in the replacement of the traditional wide correlator.

Double delta correlator

The double delta ($\Delta\Delta$) correlator is another type of geometry-based architecture designed to improve the performance of the traditional code-tracking loop. The idea behind this technology is to use 2 pairs of correlators—very early, early, prompt, late, very late—to produce the correlation function. Different MMTs have been derived from this approach, namely, the early-late slope [9], the strobe correlator (SC) [10], and the high-resolution correlator (HRC) [11].

The early-late slope technique, also known as multipath elimination technology, uses 2 pairs of correlators to compute the slope of both sides of the ACF. The spacing of the second pair of correlators (E2, L2) is twice the spacing of the first pair (E1, L1). When the signal is multipath-free, both sides of the ACF will have approximately the same slope, and the prompt correlator will be near the LOS signal delay. However, when

multipath is present, the slopes will be different. The early-late slope exploits the slope angle to drive the prompt correlator to the peak of the ACF approximated by the intersection of the rising and falling slopes, thus compensating for the slope difference. As for the SC and the HRC, they both use the 2 pairs of correlators as a combination of 2 narrow correlators, one with a narrower early-late spacing than the other. The discriminator function of both techniques is similar ($D_{SC} = 2 \times D_{HRC}$) as well as their multipath envelope. The fundamental idea behind these techniques is to leverage a finer time resolution between the code replicas to detect and mitigate short-delay multipaths.

The double delta correlator technology has a better-reported performance than the EML-type techniques in case of medium to long delay multipath [11]. However, it cannot reject the effects produced by short-delay multipaths. Moreover, due to the second pair of correlators, the uncertainty region around the actual zero crossing is more substantial, so it is more sensitive to noise and can suffer from false lock problems. To leverage the different strengths of both geometry-based techniques, a C/N_0 -based 2-stage delay tracking technique was proposed as a combination of normalized EML and the HRC techniques [12]. The 2-stage delay technique initializes the tracking with the normalized EML tracking loop and switches to the HRC for fine tracking when the estimated C/N_0 meets certain criteria (e.g., C/N_0 greater than 33 dB-Hz).

Channel parameter estimation techniques

These techniques aim to discern the different components of the input signal to track the direct path component accurately. Each signal component (direct and indirect paths) is characterized by 3 parameters—amplitude α_n , time delay τ_n , and phase ϕ_n . The maximum likelihood (ML) principle or the inverse channel can be adopted to estimate the whole set of parameters of the incoming signal paths. Once the profile of the signal components is estimated, the contribution of the multipaths can be removed or taken into account to direct the tracking of the component of interest.

This class of algorithms computes the estimates that minimize the following function:

$$\varphi(\alpha_n, \tau_n, \phi_n) = \int_{t-T}^t \|z(t) - s(t)\|^2 dt \quad (9)$$

where $z(t)$ is the incoming signal and the estimate of the composite signal $s(t)$ is the summation of M signal components:

$$s(t) = \sum_{n=0}^{M-1} \hat{a}_n x(t - \hat{\tau}_n) e^{j\hat{\phi}_n} \quad (10)$$

Estimation is a crucial process for the performance of these methods. The more accurate the signal estimate is compared to the actual signal characteristics, the more reliable and accurate the multipath mitigation algorithms can be. To improve the estimates over time, some techniques incorporate adaptive capabilities to dynamically track changes in the channel parameters. As will be discussed, the environment is rapidly changing in real-world scenarios due to the dynamics between the receiver and the satellites, so the channel profile is always altering at different paces. This may be a key feature for deciding which algorithm to use for a given application.

Summary and discussion

Starting with the wide correlator, all developments at the correlator level were intended to bring robustness against interference with a trade-off in bandwidth (and noise) or complexity. Even the narrow correlator, which maintains the number of correlators of its predecessor, requires an increase in the pre-correlation bandwidth for a higher resolution in measuring the code delay. Together with higher sampling rates, noise is also an essential factor since a wider pre-correlation bandwidth allows more noise to pass through. Although current hardware has more capabilities, all modifications to the base architecture add complexity to the system and may compromise its real-time usability at some point.

In addition to the necessary conditions for the improved versions of the wide correlator, it is even more important to clarify their limitations in measuring the LOS code delay. Geometry-based techniques essentially explore the geometry of the ACF to find the code delay of the component of interest. Although the performance of these techniques is reasonable in environments with very little or no multipath, the error produced increases notoriously in dense multipath environments [4,5]. In the latter, the ACF peak may appear displaced relative to the LOS delay, inducing the DLL to track the delay of a composite signal instead of just the direct path component. Pseudorange measurement errors can reach 15 m using either the narrow correlator or the double delta technology for relative delays between 0.05 and 0.2 chips. In dense multipath scenarios or in situations where the LOS is greatly attenuated compared to its replicas, measurement errors can naturally be much worse.

Given the limitations of geometry-based techniques, more advanced methods have been proposed to improve the code-tracking loop performance in the presence of multipath. By obtaining the channel profile, it is possible to extract more information about the state of the channel and produce more accurate corrections for the delay of the correlators (or frequency of the code replicas). However, these techniques face several major challenges in achieving higher accuracy than traditional techniques on real data. The undeniable complexity of some algorithms keeps them from being a ubiquitous solution for commercial receivers [4,5].

Channel parameter estimation algorithms try to reach the performance upper limits in multipath mitigation. However, from the results presented in the literature, the maximum performance is sometimes restricted to certain conditions [e.g.,

few multipath and very attenuated relative to the LOS, channel stationarity, and optimal signal-to-noise ratio (SNR)] [4]. Each study presents a specific profile of performance tests for a given algorithm, making it difficult to compare with other algorithms. It thus becomes relevant to delimit the potentials and limitations of each one to clarify its applicability in the real world.

Channel Parameter Estimation Review

Channel parameter estimation techniques aim to retrieve statistical information from the incoming signal by estimating the parameters of the signal components. Within these techniques, 2 main strategies are presented in the literature to mitigate multipath interference. For instance, some methods opt to remove the distortion caused by multipath in the ACF [5,13], and others adopt a different discriminator function to track only the estimated LOS code delay [6,14]. This work focuses on channel parameter estimation techniques, which involve discerning between different signal components, enabling posterior multipath interference mitigation.

There are several degrees at which the channel estimation can happen, for instance, some techniques directly estimate the delay caused by multipath without estimating the complete profile of the channel, such as the a posteriori multipath estimation (APME) [15]. While some techniques are designed for single-antenna receivers, others only apply to dual-antenna receivers [16]. Moreover, some methods are implemented at the correlation level. In contrast, others operate at the observation level, for example, by taking advantage of the reduced multipath error in carrier phase measurements to correct pseudoranges, such as the code-minus-carrier (CMC) method [17]. Over the years, multipath mitigation has become an increasingly explored topic, and the literature is currently vast. In this sense, the article focuses on techniques that estimate the channel at the tracking channel level in single-antenna receivers.

To address multipath effects at the tracking level, we have categorized the estimation techniques into 2 distinct approaches: static and dynamic. Static estimation techniques, such as ML estimators, rely on efficient maximization strategies over the likelihood function to obtain optimal solutions. These methods perform remarkably well in scenarios with long-delay echoes, outperforming other techniques when prior information is unavailable. On the other hand, adaptive approaches for dynamic channels with continuously changing conditions show promise in incorporating prior channel knowledge and dynamically tracking channel parameters for enhanced multipath mitigation. Although the main focus of this work is adaptive techniques, a section dedicated to popular ML-based methods is presented to contextualize the parameter estimation topic and introduce the more advanced adaptive algorithms. Moreover, with the fast-growing use of machine learning methods, particularly for multipath mitigation in GNSS receivers, a section dedicated to those methods is also presented.

Maximum likelihood

Although the traditional DLL implementation can be seen as an approximation of a ML code delay estimator, in the presence of multipath, the resulting composite signal introduces bias into the DLL estimate, which translates to positioning errors [18]. By considering the multipath signal components in the

ML formulation, it is possible to estimate the full-channel profile by solving a nonlinear and multi-dimensional problem.

The multipath estimation delay lock loop (MEDLL) [19] attempts to reduce the complexity of the estimation problem by decomposing it into a single-dimensional one and converging to the ML solution iteratively. The MEDLL adopts a bank of correlators to obtain a detailed ACF and then uses the ML criterion to estimate the channel profile. The optimal solution for Eq. (9) is obtained by setting the derivatives $\frac{\partial \varphi}{\partial \tau_n}$, $\frac{\partial \varphi}{\partial \alpha_n}$, and $\frac{\partial \varphi}{\partial \phi_n}$ to 0. The system of equations to estimate the delay, amplitude, and phase of the i th signal component are the following [13], respectively:

$$\hat{\tau}_i = \max_{\tau} \mathbb{R} \left\{ \left(Y(\tau) - \sum_{\substack{n=0 \\ n \neq i}}^{M-1} \hat{\alpha}_n R_{\hat{q}q}(\tau - \hat{\tau}_n) e^{j\hat{\phi}_n} \right) e^{-j\hat{\phi}_i} \right\} \quad (11)$$

$$\hat{\alpha}_i = \mathbb{R} \left\{ \left(Y(\hat{\tau}_i) - \sum_{\substack{n=0 \\ n \neq i}}^{M-1} \hat{\alpha}_n R_{\hat{q}q}(\hat{\tau}_i - \hat{\tau}_n) e^{j\hat{\phi}_n} \right) e^{-j\hat{\phi}_i} \right\} \quad (12)$$

$$\hat{\phi}_i = \arg \left(Y(\hat{\tau}_i) - \sum_{\substack{n=0 \\ n \neq i}}^{M-1} \hat{\alpha}_n R_{\hat{q}q}(\hat{\tau}_i - \hat{\tau}_n) e^{j\hat{\phi}_n} \right) \quad (13)$$

A technique proposed in [20] by Van Nee can be used to solve the system of equations. A set of steps should be followed to estimate the parameters of each signal component iteratively. As the components are estimated, their contribution is removed from the correlation function to estimate the remaining signal components, as illustrated in Eqs. 11 to 13. The number of signal components should be previously estimated to prevent both under- or overestimation, which could introduce major estimation errors. Some steps need to be repeated until a stop criterion is fulfilled, indicating the convergence of the algorithm. The stop criterion could be empirically discovered, resulting in several loops through the steps that the algorithm needs to execute. Naturally, the complexity and execution time of the algorithm increases as more components need to be estimated. The computational complexity can be derived by analyzing the steps required to solve the equations [13,20].

Using as an example the use of MEDLL to estimate a channel with 2 paths (LOS + single echo), the computational cost starts with the calculation of correlation matrix $R_{\hat{q}q}$ requiring $N^2 L$ multiplications and $N^2(L-1)$ summations, where N is the number of correlators and L is the number of samples per code period defined by the sampling rate ($L \gg N$). Next, the following steps are:

1. Search for LOS in the initial ACF
 - (a) Search the delay of resulting ACF peak: $N - 1$ comparisons;
 - (b) Compute the amplitude: 2 multiplications, 1 summation, and 1 square root;
 - (c) Compute the phase: 1 arctang;
2. Subtract contribution of LOS in the initial ACF
 - (a) Reconstruct clean LOS ACF: $N + 2$ multiplications;
 - (b) Subtract LOS ACF from initial ACF: $2N$ subtractions;
3. Search for echo in the LOS-free ACF
 - (a) Search the delay of resulting ACF peak: $N - 1$ comparisons;
 - (b) Compute the amplitude: 2 multiplications, 1 summation, and 1 square root;
 - (c) Compute the phase: 1 arctang;
4. Subtract contribution of echo in the ACF
 - (a) Reconstruct echo ACF: $N + 2$ multiplications;
 - (b) Subtract echo ACF from initial ACF: $2N$ subtractions;
5. Search for LOS in the echo-free ACF
 - (a) Search the delay of resulting ACF peak: $N - 1$ comparisons;
 - (b) Compute the amplitude: 2 multiplications, 1 summation, and 1 square root;
 - (c) Compute the phase: 1 arctang;
6. Repeat steps 2, 3, 4, and 5 until a stop criterion is satisfied

The presented algorithm scales according to the number of paths to be estimated and the number of iterations to improve the estimates. Although pre-computing the correlation matrix reduces a lot of complexity (at the expense of memory), the iterative process of finding the optimal parameters following Eqs. 11 to 13 creates a set of nonparallelizable nested loops quickly increasing complexity.

In [5], the modified sequential maximum likelihood (MSML) algorithm was introduced, incorporating an initial detection stage for the signal components, followed by the sequential estimation of the profile of each component. This approach keeps the number of estimated parameters as low as possible, reducing the propensity for estimation degradation caused by estimating many parameters. Moreover, through the detection stage, the algorithm is able to optimize the iterative process of finding each signal path and its properties, thus reducing some of the complexity. Nonetheless, due to the large search space, the complexity is still substantial, so MSML was combined with a Wiener filter to track slow variations in the channel.

A subspace-based method was also proposed in [5] to enhance channel estimation by leveraging Doppler spectrum broadening in fast-fading channels. The approach starts by creating a low-resolution channel profile from correlator outputs at multiple Doppler frequencies. These outputs are used to form a signal covariance matrix, which is then refined using a high-resolution algorithm called multiple signal classification (MUSIC). The use of this algorithm was proven to be more precise in separating the LOS signal from multipath reflections,

improving the accuracy of TOA estimates in challenging environments. In [21], the Teager–Kaiser (TK) operator was derived for rectangular pulse shapes and extended to the root-raised cosine pulse shapes, and applied to the problem of multipath mitigation. A comparison was performed with MUSIC, where TK presents a similar performance with lower complexity but increased sensitivity to cross-correlation interference.

Without noise, interference from other signals, and using infinite pre-correlation bandwidth, these techniques have no limiting factor, so the accuracy should attain upper bounds. However, those ideal scenarios are never the case in reality, leading to performance degradation. Nonetheless, the algorithm outperforms the traditional wide correlator mainly in mid and long-delay multipath presence, achieving nearly null error in static conditions. As for short-delay multipath—around and below 0.1 chips—errors are maximum. The magnitude of those errors depends on many factors, such as pre-correlation bandwidth, number of correlators, or accuracy of the estimated number of multipaths. Reported results show a maximum error of 2 m when multipaths have relative delays around 0.1 chips, using a correlator spacing of 0.033 chips and 12-MHz (2-sided) bandwidth, also outperforming the traditional DLL [13].

The vision correlator (VC) [22] is an implementation of the MMT [23], where the latter is a derivation of the MEDLL. The MMT solves the computational complexity of the ML process by performing a nonlinear transformation of the multipath parameters. The fundamental principle in the VC algorithm is to measure the radio frequency properties of the chip transitions in fine detail. The VC samples are generated by filtering all the transitions over a time period. A new hardware circuitry is required to perform additional signal processing to extract the VC data. The noise is filtered by super-imposing successive chip transitions during a specific time interval to form an average chip transition shape. Then, the MMT algorithm processes an array of complex samples containing pulse shape information (along the length of a PRN chip) to produce an estimate of the LOS and multipath signals. Multiple versions of the reference function are used to estimate the best fit of the VC vector using an ML process. The PRN code chip transitions are very sensitive to multipath interference; therefore, deviations from the standard reference function can be identified as having closer delays than the traditional correlator. To detect such deviations, the MMT requires a reference shape function to compare the chip transitions with the incoming data. The reference function should include the radio frequency properties of the incoming signal as well as the eventual distortions caused by the receiver equipment. A long-term average of measured pulse shapes can be performed from a satellite signal with a good SNR to produce a good reference function. Alternatively, a high-quality simulator could be used for the same effect. After establishing the reference function, the MMT estimates the signal components from the VC signal.

The results show that the VC can remove the effects of multipath signals less than 10 m from the LOS signal—close-in multipath. A drawback of VC is the need for a previous estimation of the number of multipath components in the receiving signal. Similar to the MEDLL, achieving better performance with this technology requires higher sampling frequency and more correlators. The VC examines the transition area of the modulation code in the signal, so the greater the bandwidth, the more effective the process is. Moreover, robust estimations depend on good SNR, which may imply long-term integration. Although

these technologies enable maximum theoretical performance, their use is more suitable for static conditions due to nonstationarity [4,24]. Under rapid channel changes, the assumption of a constant channel profile during the observation interval may not hold, which can degrade the channel estimation accuracy. Additionally, the need to estimate the number of echos or rely on an empirical channel profile also conditions the use of these techniques in highly dynamic environments where the channels can be subject to very distinct conditions.

The reduced search space maximum likelihood (RSSML) delay estimator [25] performs a nonlinear curve fit on the ACF by searching for the best match from a set of reference ACFs. Each reference function is defined by a certain number of signal components, their amplitudes, delays, and phases. The classical RSSML implements a threshold-based peak detection method using the estimated noise variance to define the detection threshold value. While in the classic implementation, a delay window range of ± 2 chips is adopted, a reduced-complexity RSSML implementation was later proposed, which reduces the delay search points to ± 0.2 chips [26].

The coupled amplitude delay locking loop (CADLL) [27] is a sophisticated system that excels in estimating and tracking multipath sources. It operates by sequentially activating tracking units and leveraging information from the already active units to initialize newly started ones. Initially, CADLL utilizes a conventional tracking loop to lock onto the incoming signal and obtain a rough estimate of the LOS code phase position. Then, it gradually adds more units to the loop to track potential multipath signals. If a unit fails to track any multipath signal, it indicates the absence of multipath in the signal; however, if successful, CADLL dynamically incorporates new units to detect and handle additional multipath components. This unique process, guided by a tracking control block, ensures efficient searching and localization of new multipath signals, adapting the structure to match the actual number of multipaths present. In practical scenarios, CADLL typically identifies and tracks the strongest multipath signal first, followed by the subsequent strongest ones. Results show that the CADLL provides reliable estimation for the number of multipath components and their states using a coherent integration time equal to half of the data bit period for GPS L1 C/A. Under weak SNR scenarios—usually below 30 dB-Hz—the CADLL has difficulties estimating the channel parameters, producing less reliable estimates [28].

Adaptive techniques

In the realm of signal processing and communication systems, the ability to extract valuable information from noisy and dynamic environments is of great importance. Adaptive filtering, a powerful and versatile technique, has emerged as a breakthrough to address this critical challenge. Unlike conventional filtering approaches, which rely on fixed filter coefficients, adaptive filtering provides the capability to automatically adjust its parameters based on input data, allowing for continuous optimization and efficient adaptation to varying conditions. The primary objective of adaptive filtering is to enhance the quality of signals, suppress interference, and improve overall system performance in the presence of uncertainties, noise, and changing operating environments. It finds widespread applications across diverse fields, including wireless communications, audio processing, control systems, radar, sonar, and image processing, among others. A line of research in multipath mitigation resorts to well-known and established adaptive filtering techniques to

estimate the channel parameters [5,6,29,30]. This approach enables continuous tracking of changing parameters, thus being an appealing solution for both static and dynamic scenarios.

At the core of adaptive filtering lies the ability to estimate and track the unknown characteristics of a system or channel, such as its impulse response, frequency response, or channel parameters. This adaptability empowers the filter to dynamically respond to fluctuations, making it well suited for real-time processing and scenarios where traditional fixed filters may prove suboptimal. The iterative adjustment of filter coefficients is driven by an error signal that quantifies the difference between the desired output and the actual output. By minimizing this error signal, the adaptive filter continually refines its response, converging toward an optimal solution that best matches the dynamic nature of the input data. The filter output is given by:

$$y_i = \sum_{k=0}^K w_{ki} u_{i-k} = U_i^T W_i \quad (14)$$

where $U_i = [u_i, u_{i-1}, \dots, u_{i-K}]^T$ is the input data stream containing the current and past K input values, and the weight vector represented as $W_i = [w_{0i}, w_{1i}, \dots, w_{Ki}]^T$ at time i . The size of the filter is given by the number of weights, which also dictates the number of previous samples used to calculate the current filter output y_i . For each epoch, the filter output is compared with the desired filter response d_i to produce an error signal e_i , which is used to update the filter coefficients for the next iteration.

The adaptive process converges to the optimal solution by minimizing the mean squared error (MSE) signal. Assuming that e_i , d_i and u_i are statically stationary, the expected instantaneous MSE is given by:

$$\begin{aligned} \xi &= E[e_i^2] = E[(d_i - y_i)^2] \\ &= E[D_i^2] + W_i^T E[U_i^T U_i] W_i - 2E[U_i^T D_i] W_i \end{aligned} \quad (15)$$

For linear functions, the MSE is a quadratic function that contains a single global minimum without local minima. The point of minimum MSE represents the optimal filter coefficients. To reach the minimum error, the filter weights are updated iteratively by following the opposite direction of the gradient given by:

$$\nabla(\xi) \equiv \frac{\partial \xi}{\partial W} = \left[\frac{\partial \xi}{\partial w_0} \frac{\partial \xi}{\partial w_1} \dots \frac{\partial \xi}{\partial w_K} \right] = 2R_u W - 2\Gamma \quad (16)$$

where $R_u = E[U_i^T U_i]$ represents the input correlation matrix and $\Gamma = E[U_i^T D_i]$ is the cross-correlation between the desired response and the input signal components. The optimal solution for the filter coefficients, W^* , is obtained where the gradient is zero. Setting Eq. (16) to zero and solving for the weight parameters results in:

$$W^* = R_u^{-1} \Gamma \quad (17)$$

The cost function for the residual MSE is found by substituting Eq. (17) into Eq. (15) and can be represented by:

$$J_{\min}(W) = \text{tr}\{R_d - \Gamma^T R_u^{-1} \Gamma\} \quad (18)$$

where $R_d = E[D_i^T D_i]$. However, it is not possible to directly compute the optimal weights because the statistical properties

of both the desired and input signals are not always known. Instead, stochastic gradient (SG) algorithms go through an iterative process where R and Γ are estimated to obtain an approximation to the Wiener solution. Exploiting the gradient measurements enables the adaptive algorithm to find the minimum MSE in the quadratic function. Using the steepest (or gradient) descent algorithm, the weights are updated according to:

$$W_{i+1} = W_i - \mu \nabla_i \quad (19)$$

where μ is the step-size parameter (positive scalar) for controlling the convergence rate. The value of step size should be correctly chosen to enforce $J(W_i) < J(W_{i-1})$, thus allowing a systematic convergence to the minimum error. As for the gradient itself, it needs to be estimated at each iteration to form a feedback correction to the filter weights.

Least mean square

The least mean square (LMS) is an SG-based algorithm that is widely used as an adaptive filtering technique that plays a fundamental role in signal processing. The LMS algorithm provides an efficient and iterative approach to adjust the parameters of a system to minimize the MSE between the output and a desired response. At the core, the LMS algorithm leverages the principle of gradient descent to optimize the parameters of the system. By estimating the gradient, $\hat{\nabla}_i$, of the performance surface using the error signal e_i^2 at each iteration, the algorithm updates the parameters, aiming at continuously improving the performance of the system over time. The estimate of the gradient is computed as:

$$\hat{\nabla}_i = \begin{bmatrix} \frac{\partial e_i^2}{\partial w_0} \\ \vdots \\ \frac{\partial e_i^2}{\partial w_K} \end{bmatrix} = 2e_i \begin{bmatrix} \frac{\partial e_i}{\partial w_0} \\ \vdots \\ \frac{\partial e_i}{\partial w_K} \end{bmatrix} = -2U_i^T e_i \quad (20)$$

Given the estimate of the gradient, the recursion equation is formed for the LMS adaptive algorithm:

$$\begin{aligned} W_{i+1} &= W_i - \mu \hat{\nabla}_i \\ &\equiv W_i + \mu U_i^T [D_i - U_i^T W_i] \end{aligned} \quad (21)$$

where minimizing the error is equivalent to minimizing half of the error; hence, the term “2” in Eq. (20) disappears. As mentioned before, the choice of step size is critical to ensure the stability of the algorithm. The convergence is theoretically guaranteed if:

$$0 < \mu < \frac{2}{\lambda_{\max}} \quad (22)$$

where $\lambda_{\max}$ is the largest eigenvalue of the input correlation matrix R_u . Although setting the step size according to Eq. (22) may ensure stability, the speed of convergence depends on the ratio between the largest and smallest eigenvalues of R_u given by $\left(\rho = \frac{\lambda_{\max}}{\lambda_{\min}}\right)$ —convergence assured following Eq. (22) for small values of ρ . Moreover, variations on the statistics of the channel, i.e., nonstationary input with time-varying power,

combine with changes in the input correlation matrix, which consequently influences the step-size upper bound.

To account for possible variations in input statistics, the normalized least mean square (NLMS) algorithm was proposed, which updates the step size iteratively along with the filter coefficients. Building upon the foundation of the classic LMS, the NLMS algorithm performs a normalization that ensures that the step size used for parameter updates is proportional to the power of the input signal. By doing so, the NLMS algorithm enhances stability and convergence speed, making it well suited for applications in environments with varying input power levels. The time-variant step size is then given by:

$$\mu_i = \frac{\tilde{\mu}}{\varepsilon + \|U_i\|^2} \quad (23)$$

with $0 < \tilde{\mu} < 2$ and ε being a small positive constant to maintain the denominator greater than 0, summed with the input signal power during a time interval between iterations.

The complexity cost of LMS can be measured from the number of operations carried out in Eq. (21), where the filter weights are updated each epoch according to the error signal. Using N as the size of the ACF, the complexity cost can be divided into 4 parts:

1. $U_i^T W_i$
(a) N^2 multiplications and $N^2 - N$ summations;
2. $D_i - U_i^T W_i$
(a) N subtractions;
3. $\mu U_i^T [D_i - U_i^T W_i]$
(a) $N^2 + N$ multiplications;
4. $W_i + \mu U_i^T [D_i - U_i^T W_i]$
(a) N^2 summations;

The use of the LMS for adaptive filtering at the correlation stage can be found in [31], where the LMS was used for mitigating the multipath error estimated using the CMC measurement. By exploiting the dual-frequency operation mode and combining the code and carrier phase measurements, the CMC is used to approximate the multipath error. The LMS then uses the multipath error to adaptively adjust the coefficients of the filter to remove the multipath contribution from the intermediate frequency (IF) signal.

The LMS was also used in [5] as an adaptive equalizer inside the code-tracking loop with the purpose of GNSS multipath mitigation. The proposed architecture builds upon 2 blocks—adaptive channel compensation and LOS signal estimation blocks [32,33]. While the former filters the ACF, the latter estimates the LOS code delay from the output of the former block. The LOS code delay estimate is then feedback to the code numerical control oscillator for adjusting the code replicas. The ML estimate is used to produce the LOS code delay estimation under the assumption of a multipath-free model, and the LMS is employed to filter out the multipath components and converge to a clean ACF centered on the LOS code delay estimate.

The architecture for adaptive channel estimation, represented in Fig. 2, belongs to the inverse modeling class of adaptive filtering applications. The adaptive filter is designed to provide an inverse model that represents the unknown noisy system [34]. In addition to the LMS and its normalized version, the affine projection algorithm (APA) and the recursive least squares (RLSs) were also adopted as channel equalizers for comparison in terms of LOS code delay tracking accuracy, convergence time, complexity, and robustness against noise and channel dynamics. The fundamentals of both algorithms are addressed next.

Affine projection algorithm

The APA is an SG algorithm that performs a different estimate of the input correlation matrix and the cross-correlation between the desired response and the input signal components to form the required recursion formula for updating the weights. While the LMS simply uses the instantaneous values of u_i and

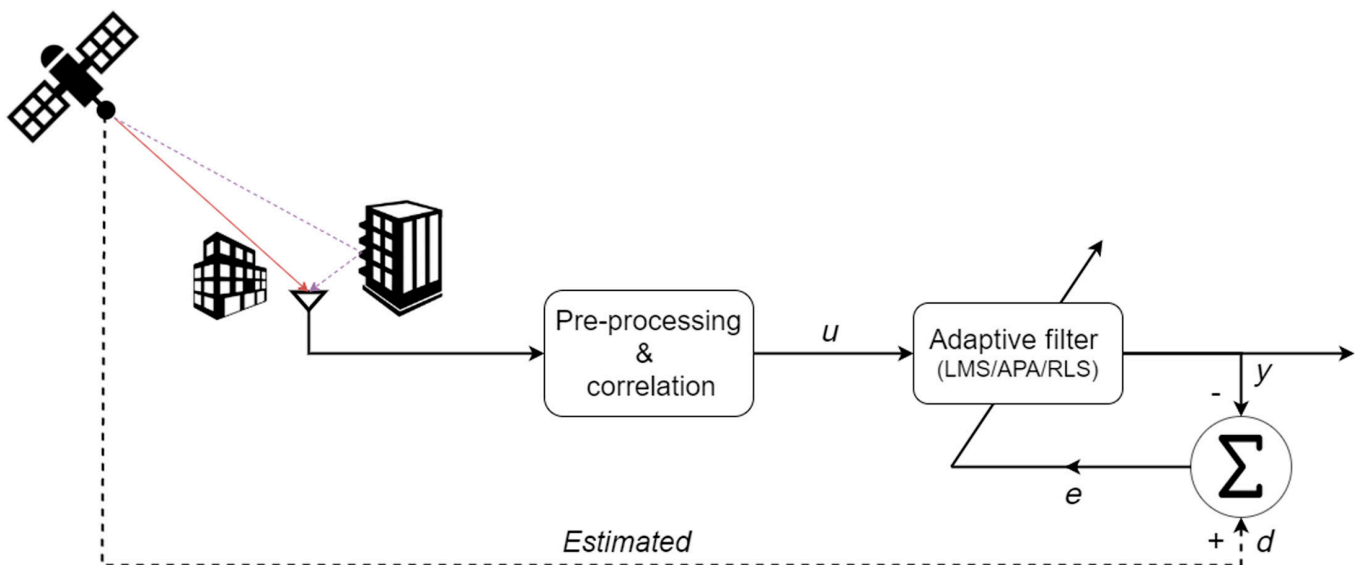


Fig. 2. Channel adaptive equalizer.

d_i to approximate R_u and Γ , the APA employs the L most recent inputs and observations. Denoting

$$U_i = [u_i, \dots, u_{i-(L-1)}] \quad \text{and} \quad D_i = [d_i, \dots, d_{i-(L-1)}] \quad (24)$$

the approximations are represented by:

$$\hat{R}_{u_i} = \frac{1}{L} U_i^T U_i \quad \text{and} \quad \hat{\Gamma}_i = \frac{1}{L} U_i^T D_i \quad (25)$$

There are multiple alternatives for obtaining such correlation approximations, each one dealing with the trade-off between computation complexity, convergence performance, and steady-state behavior. The approximations in 25 are then substituted into regularized Newton's recursion formula to produce the feedback correction for the weights:

$$\begin{aligned} W_{i+1} &= W_i - \frac{1}{2} (R_{u_i} + \epsilon I)^{-1} \nabla_i \\ &= W_i + \mu (R_{u_i} + \epsilon I)^{-1} [\Gamma_i - R_{u_i} W_i] \\ &= W_i + \mu U_i^T (U_i^T U_i + \epsilon I)^{-1} (D_i - U_i W_i) \end{aligned} \quad (26)$$

where ϵ is a regularization parameter—a small positive constant to avoid singularity.

Following the same complexity cost analysis presented for the LMS, the total complexity for the APA is $N^{2.4} + 3N^2 + 2N$ multiplications and $N^{2.4} + 3N^2 - N$ summations, assuming an $O(N^{2.4})$ complexity for the matrix inversion [35]. Similar to the LMS, APA was integrated into the adaptive channel compensation block in [5]. The relevance of APA arises because it was designed to improve the performance of other adaptive algorithms, especially those based on LMS. The APA reuses old data and leverages the correlation with the input data to accelerate convergence [36]. The trade-off between computation complexity and convergence speed motivated the implementation and analysis of this algorithm in [5,32,33].

Recursive least squares

The RLS algorithm, a variant of the least squares (LS) method, is designed to efficiently handle time-varying systems by recursively updating its filter coefficients as new data become available. In contrast to the LMS algorithm, which updates its coefficients incrementally based on the instantaneous error between the desired and the output signal, the RLS algorithm computes the LS solution for each incoming data batch. This computation involves the inversion of an autocorrelation matrix, which results in a higher computational load compared to the simpler weight update scheme of the LMS algorithm. However, this increased complexity grants the RLS algorithm a distinct advantage in scenarios where the channel conditions evolve rapidly or exhibit nonstationary behavior.

The RLS uses an exponentially weighted average of all instantaneous input correlation matrices as an approximation for the current input correlation matrix defined by:

$$\hat{R}_{u_i} = \frac{1}{i+1} \sum_{j=0}^i \lambda^{i-j} U_j^T U_j \quad (27)$$

The weight of each previous input correlation matrix on the current approximation depends on the scalar factor λ , which should be in the range of $0 \ll \lambda \leq 1$. More or less weight is given

to older input correlation matrices, depending on whether λ is closer to 1 or 0, respectively.

Using Eq. (27) in the regularized Newton's recursion formula to produce the feedback correction for the weights results in:

$$W_{i+1} = W_i + U_i^T \left(\sum_{j=0}^i \lambda^{i-j} U_j^T U_j + \epsilon I \right)^{-1} (D_i - U_i W_i) \quad (28)$$

Using the Sherman–Morrison–Woodbury formula, also known as the matrix inversion lemma, the recursion formula can be rearranged to speed up the computation of the inverse term:

$$\begin{aligned} W_{i+1} &= W_i + P_i U_i^T (D_i - U_i W_i), \\ P_i &= \lambda^{-1} \left[P_{i-1} - \frac{\lambda^{-1} P_{i-1} U_i^T U_i P_{i-1}}{1 + \lambda^{-1} U_i^T P_{i-1} U_i} \right] \end{aligned} \quad (29)$$

While the LMS-based algorithms represent the simplest and most easily applied adaptive algorithms, the RLS is known for its superior performance at the cost of increased complexity ($9N^2 + 3N + 1$ multiplications and $7N^2 - 3N$ summations, where $N = \text{ACF size}$). The RLS recursively finds the filter coefficients that minimize a weighted linear LS cost function relating to the input signal where the total error is computed from the beginning. The performance difference between the RLS and LMS in multipath mitigation was addressed in [37] as a continuation of the study started in [31]. From the setup presented in [31], the LMS was replaced by 4 RLS-based filters to compute the coefficients of a 32-stage finite impulse response (FIR) low pass filter designed to remove the multipath interference.

The RLS algorithm was also studied in [5,32,33] to benchmark its real-time time-variant parameter estimation capabilities compared to LMS and APA algorithms. The faster convergence and ability to track sudden channel profile changes are of the most importance to GNSS receivers, so the viability of integrating the RLS in practical scenarios was analyzed.

An inverse approach was followed in [38], compared to [5,32,33], where the channel impulse response (CIR) is approximated by a tap weight vector. The RLS is used to recursively update the weight vector, given that the desired signal is represented as the actual ACF, and the input signal is represented as the ideal ACF. From the estimated CIR, it is assumed that the direct signal component is known and separated from the multipath corrupted signal. Similarly in [39], from the CIR estimated using the LS, the impulse response of the optimized equalization filter is computed to annihilate the multipath—an approach also called deconvolution [40].

Bayesian filters

In modern data science and artificial intelligence (AI), accurate and robust information processing has led to the development of sophisticated techniques that enable systems to navigate through uncertain and dynamic environments. One such powerful methodology is Bayesian filtering, which provides a systematic framework for estimating the true state of a system in the presence of noisy and incomplete observations. Bayesian filtering represents a departure from traditional deterministic approaches, acknowledging the inherent uncertainty of real-world scenarios. Whether tracking the trajectory of a moving object, localizing a robot in an unknown environment, or tracking the delay of the LOS signal, observations often have

uncertainty due to various noise sources and errors. Bayesian filtering rises to this challenge by seamlessly integrating both prior knowledge and current observations, producing refined and continually updated estimates of the underlying system state.

The Bayes theorem provides a means to update the probability distribution of a hypothesis given new evidence arriving from measurements. By applying the Bayes theorem iteratively, Bayesian filtering algorithms maintain a dynamic cycle of belief refinement. This iterative process involves predicting the state of the system based on the previous estimate, updating the estimate using the latest observations, and repeating the cycle to enhance the understanding of the actual state progressively. Bayesian filtering encompasses a range of algorithms, each tailored to specific scenarios and system models. Prominent examples include the Kalman filter (KF), variants like the extended Kalman filter (EKF) and unscented Kalman filter (UKF), and the particle filter (PF). These algorithms enable systems to make informed decisions, respond adaptively to changing environments, and ultimately enhance their performance by accounting for uncertainty.

The Bayesian approach in filtering considers a discrete state-space model to account for prior information, where the evolution of states is modeled by the state equation stochastically. The state equation is represented as the prior distribution:

$$x_i \sim p(x_i | x_{i-1}) \quad \text{for } i \geq 1 \quad (30)$$

and the likelihood function illustrates the relationship between measurements z and states x as a probability distribution:

$$z_i \sim p(z_i | x_i) \quad \text{for } i \geq 1 \quad (31)$$

In the context of Bayesian filtering, the posterior distribution is fundamental, encapsulating essential statistical information about the state, particularly valuable in sequential estimation. Derived from Bayes' theorem, the posterior distribution integrates prior beliefs about the states with the likelihood of observed measurements. This integration provides a probabilistic depiction of evolving beliefs about the states, reflecting the dynamic uncertainty in the system over time:

$$p(\mathbf{x}_{0:i} | \mathbf{z}_{1:i}) = \frac{p(\mathbf{z}_{1:i} | \mathbf{x}_{0:i}) p(\mathbf{x}_{0:i})}{p(\mathbf{z}_{1:i})} \quad (32)$$

The posterior distribution is the probability distribution of the state vector $\mathbf{x}$ from time 0 to i given all observations $\mathbf{z}$ from time 1 to i . The likelihood function $p(\mathbf{z}_{1:i} | \mathbf{x}_{0:i})$ quantifies the probability of observing the sequence of measurements $\mathbf{z}$ given the state sequence $\mathbf{x}$, assessing how well the current state explains the observed measurements. The prior distribution $p(\mathbf{x}_{0:i})$ encapsulates beliefs about the state before incorporating measurements. The denominator $p(\mathbf{z}_{1:i})$ is the marginal likelihood, ensuring that the posterior distribution is a valid probability distribution by normalizing over all possible state combinations. The marginal distribution, also known as filtering distribution, at time i is a snapshot of the posterior distribution focused on the state at that specific time i , allowing the estimation of $\mathbf{x}_i$ [41].

From the filtering distribution, the optimal state estimates can be computed with respect to any criterion conditioned by

available measurements. Two popular estimators that operate within the Bayesian framework are the minimum mean squared error (MMSE) estimator and the maximum a posteriori (MAP) estimator, both being essential concepts in statistical estimation with different approaches. The MMSE estimator seeks to minimize the expected value of the squared difference between the estimated and the true value. Notably, the MMSE estimator does not explicitly incorporate prior distribution and focuses on minimizing expected squared error. On the other hand, the MAP estimator takes a Bayesian approach that explicitly incorporates prior information. It aims to find the state that maximizes the posterior probability, considering both prior distribution and the likelihood of the observed data. The non-Bayesian equivalent to the MMSE and MAP estimators are the LS and ML estimators, respectively.

In linear systems, the posterior distribution is conveniently formed by linear/Gaussian statistical models where the critical information about the state space (SS) can be directly extracted from the mathematical properties of these models. For instance, the well-known KF exploits these properties to efficiently estimate the state of a system given noisy observations. The update equations of the KF involve computing mean and covariance, which together serve as sufficient statistics for the posterior distribution in these specific models. However, in nonlinear systems, the posterior distribution cannot be characterized using the same properties, so optimal algorithms need to be modified or replaced by suboptimal algorithms to deal with the inherent nonlinearity. The EKF and the PF are examples of algorithms developed to extend the applicability of Bayesian filtering to nonlinear systems.

In GNSS receivers, the problem of CIR and LOS delay estimation is considered partially nonlinear due to the nature of both the signal propagation and the satellite–receiver dynamic. Next, the basic concepts of 2 popular algorithms—EKF and PF—that fall into the category of suboptimal solutions will be covered.

Extended Kalman filter

The KF, and by extension, its variant EKF, calculates the filtering distribution sequentially in 2 steps—prediction and update—where the current state is first predicted using the prior distribution, and then the measurements are introduced in the update to form the current distribution. The state prediction is performed using a state transitional matrix, which models the time evolution of the state variables:

$$\mathbf{x}_i = \mathbf{f}_{i-1} \mathbf{x}_{i-1} + \mathbf{v}_i \quad (33)$$

where $\mathbf{v}_i$ is a vector of variables generated from a zero-mean white Gaussian probability density function (PDF) and $\mathbf{f}_{i-1}$ is a nonlinear function from which the local linearization is realized in each epoch:

$$\mathbf{F}_i = \left. \frac{d\mathbf{f}_i(x)}{dx} \right|_{x=\hat{\mathbf{x}}_{i-1|i-1}} \quad (34)$$

The relation between measurements and states is represented by the measurement equation:

$$\mathbf{z}_i = \mathbf{j}_i \mathbf{x}_i + \mathbf{n}_i \quad (35)$$

where $\mathbf{n}_i$ also generated from a zero-mean white Gaussian PDF and $\mathbf{j}_i$ is a nonlinear function. The local linearization to form the measurement matrix is also achieved using the Jacobian:

$$\mathbf{J}_i = \left. \frac{d\mathbf{j}_i(x)}{dx} \right|_{x=\hat{\mathbf{x}}_{i|i-1}} \quad (36)$$

Through the linearization of the transition and measurement models, the EKF approximates the posterior distribution as a Gaussian, where the embedded statistical properties are computed sequentially with the Kalman equations. The set of equations that compose the prediction stage is:

$$\hat{\mathbf{x}}_{i|i-1} = \mathbf{F}_{i-1} \hat{\mathbf{x}}_{i-1|i-1} \quad (37)$$

$$\mathbf{P}_{i|i-1} = \mathbf{F}_{i-1} \mathbf{P}_{i-1|i-1} \mathbf{F}_{i-1}^T + \mathbf{Q} \quad (38)$$

where $\mathbf{P}$ is the state covariance matrix and $\mathbf{Q}$ is the process covariance matrix. The predicted stage is followed by the update stage:

$$\mathbf{S}_i = \mathbf{J}_i \mathbf{P}_{i|i-1} \mathbf{J}_i^T + \mathbf{R} \quad (39)$$

$$\mathbf{K}_i = \mathbf{P}_{i|i-1} \mathbf{J}_i^T \mathbf{S}_i^{-1} \quad (40)$$

$$\mathbf{P}_{i|i} = (\mathbf{I} - \mathbf{K}_i \mathbf{J}_i) \mathbf{P}_{i|i-1} \quad (41)$$

$$\hat{\mathbf{x}}_{i|i} = \hat{\mathbf{x}}_{i|i-1} + \mathbf{K}_i (\mathbf{z}_i - \mathbf{J}_i \hat{\mathbf{x}}_{i|i-1}) \quad (42)$$

where $\mathbf{S}$ is the residual covariance matrix representing the covariance of the innovation, $\mathbf{K}$ is the Kalman gain, and $\mathbf{R}$ is the measurement noise covariance matrix. The complexity cost of each processing epoch through Eqs. 39 to 42 is $N^{2.4} + 7N^2$ multiplications and $N^{2.4} + 9N^2 - 4N - 1$ summations, where N is the ACF size and $O(N^{2.4})$ is the complexity for the matrix inversion [35].

Although the EKF can cope with the nonlinearities in the system, the local Gaussian approximations might introduce considerable estimation errors that fail the requirements of a certain application where nonlinearity is extreme [42]. KFs have been widely adopted in GNSS receivers for coupling tracking channels to increase the overall tracking and positioning robustness [43]. The use of the EKF for tracking the synchronization parameters was first proposed in [44] as a closed-loop joint estimation of the code phase and multipath parameters. Although the proposed structure effectively estimates the channel parameters, the EKF was designed to operate in the pre-correlation domain, with short code lengths and high SNR. Due to complexity reasons, new approaches were introduced in the literature, proposing the use of the EKF in the post-correlation stage where the received signal is projected to a smaller vector space through a bank of correlators [14,30,45,46]. These works design the EKF to estimate both the LOS code delay and CIR from the ACF as an attempt to detect multipath and reduce its effect in code delay measurements.

Particle filter

Similarly to the EKF, the PF falls into the category of suboptimal algorithms in the Bayesian solution for the dynamic state

estimation problem in nonlinear systems. The PF provides a versatile and robust alternative to the EKF by tackling the state estimation with the posterior distribution representation as a set of discrete particles. Each particle serves as a potential state hypothesis, and the associated weights capture their likelihood given the observed measurements. This characteristic makes the PF particularly well suited for nonlinear systems and scenarios where the true distribution might exhibit relevant deviations from Gaussianity [42].

At the heart of the PF is the sequential importance sampling, where particles are propagated through the system dynamics and weighted according to their consistency with the measurements. The sequential importance sampling algorithm is based on the sequential Monte Carlo method, which serves as the basis for most sequential Monte Carlo filters proposed in the literature over the decades [47]. The sequential Monte Carlo method is used for implementing a recursive Bayesian filter with the key idea of representing the required posterior density function by a set of random samples (also known as particles) through which estimates are produced based on their weights. The set of N_s particles is represented as $\{\mathbf{x}_{0:i}^k, w_i^k\}_{k=1}^{N_s}$ and used to discretely approximate the characterization of the posterior PDF:

$$p(\mathbf{x}_{0:i} | \mathbf{z}_{1:i}) = \sum_{k=1}^{N_s} w_i^k \delta(\mathbf{x}_{0:i} - \mathbf{x}_{0:i}^k) \quad (43)$$

where the weights w^k are chosen using the importance sampling principle. This principle relies on resorting to an importance density $q(\mathbf{x})$ for sampling more efficiently:

$$\begin{aligned} E_p[s(\mathbf{x}_i)] &= \int q(\mathbf{x}_i) \left[\frac{p(\mathbf{x}_i)}{q(\mathbf{x}_i)} s(\mathbf{x}_i) \right] d\mathbf{x}_i \\ &= E_q \left[\frac{p(\mathbf{x}_i)}{q(\mathbf{x}_i)} s(\mathbf{x}_i) \right] \\ &\approx \frac{1}{N_s} \sum_{k=1}^{N_s} \frac{p(\mathbf{x}_i^k)}{q(\mathbf{x}_i^k)} s(\mathbf{x}_i^k) \quad \mathbf{x}_i \sim q(\mathbf{x}) \end{aligned} \quad (44)$$

with $s(\mathbf{x})$ being a function of the state vector and $q(\mathbf{x})$ chosen to have a higher probability distribution, where the probability distribution of $[p(\mathbf{x})s(\mathbf{x})]$ is higher, to attain the desired variance reduction $\text{Var}_q \left[\frac{p(\mathbf{x})}{q(\mathbf{x})} s(\mathbf{x}) \right] < \text{Var}_p[s(\mathbf{x})]$. Although sampling from $p(\mathbf{x})$ approaches a Gaussian distribution around $E[s(\mathbf{x})]$ as the number of samples increases, according to the central limit theorem [48], the practical implementation of this Monte Carlo method would be unfeasible due to complexity reasons. In summary, variance reduction is the key idea behind the importance sampling, where the sampling process occurs from a probability-weighted average of $\frac{p(\mathbf{x})}{q(\mathbf{x})}$, with the probability given by $q(\mathbf{x})$ instead of $p(\mathbf{x})$. The $q(\mathbf{x})$ density now represents where the particles are (i.e., in the SS), and their weight is given by:

$$w_i^k \propto \frac{p(\mathbf{x}_{0:i}^k | \mathbf{z}_{1:i})}{q(\mathbf{x}_{0:i}^k | \mathbf{z}_{1:i})} \quad (45)$$

From this formulation, the importance weights are computed over the complete state trajectory, increasing the computational complexity over time. Given the practical limitations

of using this algorithm, a recursive version—sequential importance sampling—was derived to avoid updating the weights from the beginning:

$$\begin{aligned} w_i^k &\propto \frac{p(\mathbf{z}_i|\mathbf{x}_{0:i}^k, \mathbf{z}_{1:i-1})p(\mathbf{x}_{0:i}^k|\mathbf{z}_{1:i-1})}{p(\mathbf{z}_i|\mathbf{z}_{1:i-1})} \\ &\propto \frac{q(\mathbf{x}_i^k|\mathbf{x}_{0:i-1}^k, \mathbf{z}_{1:i})q(\mathbf{x}_{0:i-1}^k|\mathbf{z}_{1:i-1})}{q(\mathbf{x}_i^k|\mathbf{x}_{0:i-1}^k, \mathbf{z}_{1:i})} \end{aligned} \quad (46)$$

where the weights are computed based on their last value [i.e., Eq. (45) at time $i - 1$], the likelihood, the prior, and the chosen importance density function. For more details on the factorization of the importance function and the posterior recursion, please see [47,49,50].

The selection of the importance density function plays a critical role in the performance of particle filters. The optimal choice is the filtering distribution itself $p(\mathbf{x}_i|\mathbf{x}_{0:i-1}^k, \mathbf{z}_{1:i})$, which guarantees minimum variance of the importance weights; however, finding the optimal importance density that closely approximates the true filtering distribution can be challenging, especially in complex and nonlinear systems [47]. One common suboptimal choice is the bootstrap filter [51], also known as sampling importance resampling, where the importance density is set equal to the transition prior, and the importance weights are computed from the likelihood function. While computationally efficient, this choice might not capture the true dynamics of the system accurately as the observations are not considered when generating particles [52]. While other suboptimal PF variants exist in the literature, emphasis will be on those applied to the channel estimation problem in GNSS receivers.

The fundamentals of particle filters cannot be completed without mentioning the resampling process. Particles represent potential states of the system. However, some may reveal poor consistency with measurements over time. Resampling is employed to ensure that particles with higher weights, indicative of a better fit to the observed data, are replicated, while those with lower weights are pruned, preventing the degeneracy of particles. In other words, resampling prevents the particle cloud from becoming too dispersed and emphasizes regions of the SS that align more closely with the true underlying dynamics. A common approach to address this problem is by measuring the sparsity of the samples, representing the level of degeneracy, and establishing a threshold to decide when to perform resampling. This approach was introduced in [53], which proposed the measurement of the effective sample size N_{eff} as a quantification of the degeneracy:

$$\hat{N}_{\text{eff}} = \frac{1}{\sum_{k=1}^{N_s} (w_i^k)^2} \quad (47)$$

where w_i^k is the normalized weight obtained in Eq. (46). When $\hat{N}_{\text{eff}}$ falls below a threshold, resampling is employed. Several resampling procedures can be found in the literature [53], where systematic resampling is the most popular due to the reduced complexity [54]. Despite the simplicity of some resampling techniques compared to others, this step introduces extra complexity to an already computationally demanding algorithm. The computational complexity is dominated by 3 main steps: (a) importance sampling—generating M_{pf} path delays for N_s particles with

a cost of $O(N_s M_{pf})$; (b) weight update—transforming particles into the measurement domain and updating weights, requiring $O(N_s N_{\text{corr}} n)$, where N_{corr} is the number of correlators and n is the number of samples per code period; and (c) covariance update—computing the weighted covariance of the particles states, with a cost of $O(N_s M_{pf}^2)$.

This is the main reason for the limited exploration of particle filters in GNSS receivers for channel estimation. In [6,7], the PF is introduced as a Bayesian solution for the tracking of synchronization parameters. Given the nonlinear/non-Gaussian nature of the channel estimation in GNSS receivers, the PF was designed to handle the nonlinear part of the observation model, while the KF deals with the linear part of it. In the solution referred to as Rao–Blackwellized PF, the PF estimates the code delay of the signal components in the correlation domain, and from the estimated delays, the KF estimates their complex amplitude. The performance of this solution was investigated in a dynamic multipath scenario and compared to the EKF.

Combining the PF with the KF unleashes the optimal solution for the linear estimation part while reducing the complexity of the overall estimation block. Moreover, given the complexity gap between the extended version of the KF and the PF, some works propose an integration of the EKF with the PF [55,56]. While the PF estimates the highly nonlinear part of the observation model, the EKF handles as many state parameters as possible to reduce complexity without compromising the accuracy of the algorithm.

In [57], a multipath particle filter algorithm is proposed to apply the particle filter to the baseband multipath signal model. Different resampling strategies and their respective parameter configuration are analyzed to solve the problem of particle degradation and diversity. Each algorithm variant was tested under both multipath-free and single echo scenarios and compared with the narrow correlation and MEDLL technologies in terms of 3-dimensional (3D) positioning error.

Machine learning

AI technology continues to integrate our lives and industries with advancing data-driven models capable of solving a broad range of tasks nowadays [58]. Given future projections indicating a massive transversal adoption of AI after success in diverse fields [59], the use of this technology in GNSS receivers for multipath mitigation has also been investigated [60–62]. From the receiver positioning layer down to the signal tracking level, the proposed machine learning techniques aim to tackle the limitations of current algorithms that rely on physics-based models. Model-based algorithms generally operate under the assumption of white Gaussian noise and depend on accurate physics-based models. Under complex real-world scenarios, these algorithms can be easily exposed to nonnominal conditions, such as interferences, that may severely degrade their performance [63]. Deep learning algorithms rely on data-driven methodologies rather than intricate, physics-based models. These algorithms process vast datasets to identify patterns and relationships, training multiple layers of a neural network (NN) in a computationally intensive process until satisfactory accuracy is achieved. NNs become particularly valuable when such physics-based models are either unavailable or impractical due to their complexity, the latter being the argument for channels with multipath interference.

The literature on machine learning applied to GNSS continuously grows with experiments on different methods used for the most diverse tasks, such as signal analysis and classification [64–68], environment recognition [69–72], and quality assessment [73–75]. For the problem of multipath interference, the different use cases for machine learning methods include the detection of multipath or signal blockage [76–78], the removal of multipath-induced distortion on the ACF [79,80], and channel estimation [81]. Similar to other MMTs from the literature, machine learning methods can be implemented in several processing stages of the receiver and generally use the same information to perform their task. While observation-level approaches typically use the pseudorange and carrier phase measurements, satellite elevation angles, and SNR [71,76,77,82], tracking-level approaches take the baseband signal samples and the correlation values as input [65,79–81].

A machine learning-based algorithm for multipath estimation and compensation is presented in [81]. A multilayer perceptron (MLP) with a hidden layer size of 8 was designed to receive as input the normalized and phase-corrected correlator results and output the channel profile. The LOS parameters are then fed to the conventional tracking loop to adjust the local replicas. The implementation was performed on the BBP2 GNSS ASIC leveraging from an integrated hardware accelerator correlator bank. Although limited to 2-path channel estimation under “infinite” bandwidth and no correlation peak distortions, the results show a multipath error envelope reduction from 75 m to 1.137 m compared to the traditional wide discriminator.

The work presented in [79] studies the use of deep learning to model the multipath effect on the ACF. The traditional correlator structure was replaced by an NN-based correlator scheme, producing and feeding the usual correlation values to the MLP models for regression and classification blocks, responsible for outputting the multipath-free ACF and a binary label on whether multipath is present or not, respectively. The output of the MLP models is used to calculate a weighted average between the raw ACF and the output of the regression block to produce the estimated ACF, which is then fed to the DLL discriminator. Simulation results show an advantage compared to the wide correlator and no substantial improvement compared to the narrow correlator under multipath interference. An extension of this work was presented in [80], where 2 independent and parallel MLP models for regression receive the correlation result from the convolution layer and output the noise-filtered and multipath-free complex ACF. The model was trained based on a single multipath-ray simulation to learn the properties of the multipath interference, and the results using the simulated multipath channel model from the German Aerospace Center (DLR) show an improvement in robustness to multipath-induced errors compared to the standard correlation for shorter integration times. The DLR multipath channel model was also used in [68] to train both an MLP and convolutional neural network (CNN) to detect the presence of multipath in the ACF. A 2D discrete Fourier transform is applied to the ACF from different Doppler frequencies and used as input for the NNs, which serves as a binary classifier—(0) no multipath or (1) multipath. Tests conducted show that decreasing the number of correlators, especially in the frequency domain, negatively affects the performance of the classifier. Additionally, it was demonstrated that the CNN produced more accurate results than the MLP.

Another correlator-level classifier is described in [65] to distinguish between LOS, non-line-of-sight (NLOS), and

multipath signals. This work is focused on classification accuracy across new scenarios after training. Performance is compared with that of an observation-level classifier using receiver observations, such as signal strength, satellite elevation angle, and code pseudorange. On the other hand, the proposed classifier uses 4 features as input: the ratio between the measured maximum correlation value and standard value, the mean and variance of the correlation peak delay, and the early late phase. The experiments in this work show a substantial performance degradation of observation-level classifiers when testing data are collected under different scenarios compared to training data. This drop in classification accuracy, also observed in [83], is justified by observation-level classifiers having a feature sensitivity issue (high scenario dependence). In contrast, the correlator-level classifier is shown to be more robust. Furthermore, it is emphasized that features at the correlator level may be more valuable due to the more profound relation with the complex nature of signal propagation.

In [84], a distributional loss optimization is used as a soft labeling mechanism for a regression CNN. Multipath parameters are estimated from the correlator output using a CNN for high-level feature extraction and a soft labeling approach, where target distributions are modeled as histograms, improving generalization and accuracy through a novel histogram loss function based on Kullback–Leibler divergence. Experiments demonstrate that this technique outperforms traditional regression methods, even at lower image resolutions, which is especially important for practical implementation in physical receivers with limited hardware. In [78], 2 supervised machine learning methods, SVM and NN, are used for NLOS multipath detection from the features of the multi-correlator ACF shape. The argument is presented that using correlation features extracted by SVM and NN yields a more robust solution to discriminate the NLOS than using SNR information.

The literature on channel estimation using machine learning methods is growing fast, with continuous improvements for complex scenarios as more datasets are generated and available for training. While a general picture of how machine learning methods are used for channel estimation at the correlator level is given with the mentioned works, recent and detailed reviews on machine learning applied to multipath mitigation can also be found in [61,62].

Summary and Discussion

Adaptive techniques present several advantages compared to both correlation-based and ML-based techniques. Correlation-based techniques are simple but depend on the ACF shape geometry, which often becomes distorted due to multipath. Essentially, the location of the ACF peak and the slope of each side drive these techniques to estimate the code error. Experiments show that distortions caused by echoes can cause tens to hundreds of meters of positioning error [85]. Regarding ML-based algorithms, adaptive techniques also show superior performance justified by the ability to adapt and track channel changes [4,57]. The medium-high complexity of some promising ML-based algorithms suggests that these algorithms are more suitable for static high-precision receivers with prior knowledge of the environment properties [4].

Adaptive non-Bayesian algorithms, i.e., LMS, NLMS, RLS, and APA, are more appropriate for dynamic environments where channel profile changes frequently. These algorithms offer

adaptability, updating their estimates in real time as new data become available, and do not require complex statistical models or iterative processes. LMS and NLMS are particularly well suited for resource-constrained conditions and real-time applications due to their low complexity. Their ability to track changes continuously allows them to estimate the channel effectively in slowly to moderate varying environments. However, they may struggle in rapidly varying channels, leading to higher steady-state errors and slower convergence. RLS provides a more robust solution for moderately varying channels due to its faster convergence and better performance in tracking rapid changes in the channel. It is, however, more computationally demanding than LMS or NLMS, making it less suitable for resource-constrained scenarios. Similarly, APA provides faster convergence than LMS-based adaptive filters, offering a middle ground between the LMS and RLS algorithms, balancing both convergence speed and computational complexity. Despite the requirement for more processing power and memory, which can be a limiting factor in real-time applications, this balance between computational complexity and convergence speed makes it also a viable choice for moderately varying channels.

Machine learning methods to mitigate multipath have received particular interest from the scientific community for a few decades. Since then, new models on different architectures have

been investigated to mitigate multipath and combat the limitations of physics-based models. While current literature and solutions show that machine learning is a powerful tool for complex problems, the application in GNSS receivers to mitigate multipath is still a topic that requires much attention from the scientific community to overcome current challenges. The dependence on a large volume of data that mainly cover complex scenarios (e.g., multipath-dense, low-SNR conditions) is crucial to guarantee the training of models to prepare them for real-world conditions. Although the experiments presented by most articles report better performance in multipath detection or even interference removal compared to traditional techniques, tests are often limited to controlled environments. Further developments are needed to ensure that exposure to different environments does not affect the reported performance. Additionally, computational complexity will be a determining factor for the mass adoption of these methods for channel estimation and multipath mitigation. In addition to several studies reporting the high-resolution dependence of the ACF, which implies a high number of correlators, the model itself also implies an extra computational cost. As with other techniques, computational demand is a limiting factor when implemented on resource-constrained low-power edge devices.

A summary of the differences between the analyzed algorithms is represented in Table 1, according to various metrics.

Table 1. Comparison between channel estimation algorithms

Criteria	ML-based algorithms	Adaptive algorithms (LMS/APA/RLS)	Bayesian filters (KF/EKF/PF)	Machine learning
Application	System identification Signal decomposition	System identification Channel equalization Adaptive filtering	State estimation Dynamic system tracking	System identification Filtering
Underlying model	Linear model		KF: Linear model EKF: Linearized model PF: Nonparametric	Nonparametric
Handling nonlinearity	Limited in nonlinear systems		KF: Limited in nonlinear systems EKF: Limited in highly nonlinear systems PF: Designed to handle nonlinearities	Designed to handle nonlinearities
Memory consumption	Moderate	Low to moderate	KF: Moderate EKF: Moderate to high PF: High	Moderate to High
Computational complexity	Moderate to high (iterative process)	Low to moderate	KF: Moderate EKF: Moderate to high PF: High	High
Noise robustness	Performance degradation in noisy conditions	Performance degradation in high noise conditions	Designed to handle noisy measurements	Can be trained to handle noisy measurements
Performance	Effective in static scenarios Dependent on correlation resolution and model accuracy	Effective in slowly to moderate varying scenarios with high C/N_0 Dependent on correlation resolution and filter configuration	KF, EKF: Effective in dynamic environments PF: Well suited for dynamic environments	Can be trained to handle dynamic environments

The Bayesian filters are distinguished in this category due to the ability to use prior information to estimate current and future states. Essentially, they leverage all available information about the underlying state from the beginning to produce the best estimate possible. Moreover, modeling the state evolution, as well as the relation between measurements and states, provides these algorithms with more knowledge about the dynamics of the system, which represents a clear advantage over other techniques. The advantage of modeling the system is accentuated when there is a possibility of measurements being interrupted or corrupted in extreme scenarios.

The superior capabilities of Bayesian solutions come with a complexity cost. Although a theoretically higher performance is expected, adopting these algorithms for real-time multipath mitigation is still an open topic, given diverse limitations. The PF presents itself as the most computationally demanding algorithm, partially due to the number of particles required to reach a certain accuracy. Furthermore, the required number of particles increases with the dimension of the SS. The Rao–Blackwellization approach allows for a decrease in the number of particles by reducing the SS handled by the PF while also allowing for a performance improvement due to the integration of the optimal solution for the linear part of the observation model, i.e., the KF. However, it is important to analyze how the complexity is affected by combining both algorithms. For example, using one instance of the KF per particle could easily become the major bottleneck of the algorithm, especially when the number of particles reported by some studies is in the order of thousands [57]. The same concerns extend to the works combining the PF and the EKF, where a clear indicator about the feasibility of executing these algorithms in real time is relevant to be given.

Regarding the PF capabilities for channel estimation and multipath mitigation tasks, there are some uncertainties about the robustness against mismodeling. Although some results indicate a reasonable degree of robustness, further tests are required to prove the same behavior in real-world data. Simulation tests often resort to 2-path scenarios with a single on/off echo, while the PF is (hard-coded) modeled for 2 signal components [6]. Estimating the number of paths is still an open research topic, and the resulting implications for the algorithm are not usually addressed. While other algorithms are referred to assist the PF in correctly modeling the number of paths [86], a complexity increase is inevitable, and a performance dependence on other algorithms exists. Nonetheless, using PFs in GNSS receivers is not limited to the correlation stage investigated here. For instance, the application of PF in the direct position estimation receiver architecture was proposed in [87]. This architecture was initially developed in [6], in which the position, velocity, and time estimate feedback were produced by an ML-based algorithm. In [87], the results show a performance improvement by replacing it with a PF.

The use of EKF alone, on the other hand, can be characterized as a medium-high complexity algorithm despite different uses of the filter that have been proposed. Although the EKF could be designed to estimate individual signal components similarly to the PF, the nonlinear space is increased, which does not favor using an algorithm that approximates the distributions as Gaussian. In fact, estimating a specific number of paths seems to be a strategy to justify using PFs for channel estimation. Instead, letting the position of correlators define the search points for all signal components is a choice that is more in line with the nature of the EKF. Furthermore, the EKF was revealed

to be more sensitive to mismodeling than the PF, so designing the EKF to operate in the same SS as the PF in [6] would be to replicate or even amplify the limitations of that solution.

Promising solutions adopting the EKF still face several challenges in multipath mitigation. Since approximations are local, accurate initialization of states and covariances is critical to ensure convergence to the optimal solution [6,30]. Reported results show the ability to estimate and track the LOS code delay as well as the CIR [30] when some assumptions hold. For instance, the LOS is always present and being the strongest path. Although this is a usual real-world scenario, very often, the LOS component is obstructed in dense multipath environments. Given this limitation, the combination between the EKF and the vector tracking architecture has been studied. The increased immunity of vector tracking to interference and receiver dynamics, along with the ability to bridge signal outages, compared with scalar tracking architectures, can complement the limitations of the EKF [88,89].

Future Directions

Existing solutions are often explored for different multi-correlator structures but neglect the impact of the integration step, particularly by extending the coherent integration periods [90]. Current implementations are usually limited to the data bit period and resort to noncoherent integration. However, the important phase information is lost in this procedure, resulting in direct and indirect signal component accumulation within the same correlation peak, corrupting the LOS code delay information and also making it impossible to explore new ideas such as motion compensation at the correlation level [91,92]. Techniques for enabling extended coherent integration periods, for instance, combining legacy and modern pilot signals or developing prediction on data bit streams [93], could substantially enhance existing multipath mitigation strategies and improve position, velocity, and time (PVT) estimates.

Building on the foundation of multi-correlator structures, channel estimation-based MMTs have shown promise, although they considerably increase the complexity and resource consumption of GNSS receivers. As a result, these methods have primarily been associated with high-precision and, in some cases, stationary receivers. Future research should focus on optimizing these techniques to enable broader adoption, extend the validation to other signals (e.g., L5 signals, more resilient to multipath [94]), and integrate these techniques with positioning algorithms commonly used in high-precision applications, such as precise point positioning (PPP). While much progress has been made in mitigating multipath effects in code-based observations, the application of channel estimation techniques to carrier phase measurements remains underexplored. Carrier phase observations, although less affected by multipath compared to code-based pseudoranges, i.e., up to a quarter of the observing wavelength [95], are essential for achieving high-precision positioning and particularly susceptible to multipath-induced cycle slips, which is problematic for solving integer ambiguities [85,96].

Standard techniques for mitigating multipath in high-precision receivers involve 3 strategies: (a) hardware-based approaches, such as advanced antenna designs, which reduce the impact of reflected signals by rejecting low-elevation signals or through polarization characteristics; (b) receiver-based approaches, suppressing the multipath effects at signal processing stage primarily using traditional geometry-based methods; and (c) postprocessing approaches, employing techniques such as sidereal filtering

(SF), SNR method, multipath hemispherical map (MHM), wavelet analysis, and empirical mode decomposition (EMD) to exploit the repetitive characteristics of multipath in time and spatial domains. Despite postprocessing methods demonstrating effectiveness in mitigating multipath errors, these approaches are generally more effective in static environments, with real-time mitigation remaining challenging.

Adaptive channel estimation techniques could complement these existing methods by providing more dynamic and real-time multipath mitigation, particularly for high-precision receivers with more computing capabilities. By accurately estimating the characteristics of both the direct and reflected signals, channel estimation methods could offer a more precise and adaptive solution to multipath errors in real time, even in challenging environments. Integrating these techniques with existing hardware and stochastic models could enhance the robustness and accuracy of carrier phase-based positioning solutions like PPP.

Machine learning represents another promising approach to enhance channel estimation techniques in the future, given the successful advances in other areas. Machine learning models could be used to estimate signal parameters dynamically and continuously update the models by learning from new scenarios in real time. Integration with other receiver architectures should also be explored, such as the vector tracking loop or direct position estimation architectures. Data fusion is another area of interest in GNSS positioning, where machine learning could be used to combine data from different sensors, building more robust solutions. Considering the growing number of low earth orbit (LEO) satellites being launched, NNs could also help optimize signal processing in this increasingly dense satellite environment, offering more effective mitigation of multipath effects. Within the LEO satellite navigation system and its current advancements, subspace methods have great potential due to the greater Doppler dynamics caused by the rapid motion of those satellites [97]. The fast-changing geometry of LEO satellites enhances the decorrelation between direct and reflected signals, opening an opportunity for subspace techniques, such as the one presented in [5], to be adopted for channel estimation.

Conclusion

Channel estimation techniques provide knowledge about the channel profile to which the receiver is subject for each signal. Through channel information, it is possible to discern between the direct path component of the signal—LOS—and the indirect path components—multipath—making it possible to track only the component of interest instead of a composite signal corrupted with interferences. The modeling of channel dynamics by Bayesian filters gives them superior capabilities in characterizing the channel and its changes over time. However, addressing nonstationarity proves to be challenging even with the Bayesian approach. The dynamic nature of the system introduces a level of complexity, particularly in modeling its unpredictable evolution. The inherent randomness in nonstationary conditions poses a highly demanding task for accurate characterization. While Bayesian filters provide adaptability, the intricacies of capturing dynamic shifts in these environments persist. This underscores the ongoing necessity for innovative methodologies to better encapsulate the stochastic nature of nonstationary systems and advance channel estimation for evolving and unpredictable scenarios. Apart from the nonstationarity problem, the solutions found in the literature still face several challenges when applied

to the traditional receiver architecture. Recent studies propose integrating Bayesian filters with the vector tracking architecture and the direct position estimation to handle their limitations and seek improvements over traditional techniques.

Geometry-based techniques, the traditional approach to code delay tracking, can lead to erroneous LOS code delay measurements when subject to dense multipath scenarios. Despite this limitation, this category of techniques, such as wide or narrow correlators, are still widely used in commercial receivers due to their reduced complexity. The results presented in the literature, through comparison with traditional techniques, confirm that the new techniques are very promising for dealing with the challenges of critical applications; however, more effort is needed to optimize these techniques for real-time scenarios. Particularly for machine learning algorithms, the hardware requirements are a challenge when deployment is targeted to embedded systems or low-power devices. Additionally, models that generalize typically require larger datasets, which may also imply large amounts of memory. While more robust techniques capable of mitigating multipath interferences emerge, it is imperative to further reduce their computational complexity due to the low feasibility of execution in real-time and resource-constrained applications.

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Data Availability

The data and materials that support this study are available from the corresponding author upon reasonable request.

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