

Coevolutionary Neural Dynamics Considering Multiple Strategies for Nonconvex Optimization

Jialiang Fan[†], Long Jin^{*†}, Peirong Li, Juntao Liu, Zhengguang Wu, and Wei Chen^{*}

Abstract: In practical applications, solving nonconvex optimization problems plays a crucial role. However, many practical applications often encounter perturbations that may affect solutions to relevant nonconvex problems. Such perturbations are typically unavoidable. Moreover, in the presence of perturbations, most algorithms for nonconvex optimization suffer from low solution accuracy and a tendency to become trapped in local optima. This paper proposes a coevolutionary neural dynamics considering multiple strategies (CNDMS) model to address this limitation. Firstly, a modified neural dynamics model with a dual-gradient accumulation term is constructed as a local search operator, effectively exploring the local optimal value in noise. Secondly, a modified opposition-based learning method is employed to generate improved candidate solutions based on the current solution, thereby ensuring population diversity throughout the search process. Additionally, a hybrid variation strategy is utilized to mutate the global optimal solution and reduce the probability of the proposed CNDMS model falling into local optima. The global convergence and robustness of the proposed CNDMS model are proven by theoretical analyses and further validated through numerical experiments and an engineering application.

Key words: nonconvex optimization problems; coevolutionary neural dynamics considering multiple strategies (CNDMS); coevolutionary mechanism; mutation strategies

1 Introduction

Nonconvex optimization is significant in science and engineering because many practical issues can be modeled as nonconvex optimization problems^[1–3]. Solving these problems is crucial for improving system performance^[4], resource utilization efficiency^[5], and

engineering design^[6]. Conventional optimization methods, such as gradient descent methods, are limited to local optimal solutions and do not guarantee global optimality for nonconvex problems. Moreover, they exhibit a high sensitivity to the selection of initial values. This may give rise to challenges and

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uncertainties when dealing with nonconvex optimization problems, necessitating the use of relaxation techniques^[7], approximation methods^[8], and piecewise linearization strategies^[9] to obtain convex approximations of the original problem. While these approaches streamline the solving process of problems, they may also entail sacrificing some characteristics inherent in the original issues.

Given the escalating complexity of problems, swarm intelligence algorithms have emerged as effective tools for addressing them^[10]. These algorithms adopt a simulation-based approach that harnesses the collective behavior and phenomena observed in nature to tackle intricate optimization problems, without relying on specific mathematical models^[11]. The distinctive advantage of this class of algorithms lies in the effective information interaction, transmission, and preservation within the population, thereby endowing it with a remarkable capacity for global search^[12]. However, the global search capability of swarm intelligence algorithms also exhibits certain limitations. For example, the obtained solution may lack accuracy, and minority groups tend to prematurely converge during the search process, resulting in local optima and compromising the quality of the solution^[13, 14]. The aforementioned issues may be solved by coordinating global exploration and local exploitation. To achieve the above goal, many current works introduced heuristic algorithms and traditional numerical methods into swarm intelligence algorithms as local search strategies, or improved the quality of the initial population before searching and the crossover operator of the related algorithm^[15].

In many practical problems, the obtained solutions need high precision and robustness in the process of solving^[16–18]. Unlike traditional numerical methods, neural dynamics (ND) methods show better local convergence and robustness^[19, 20]. Therefore, it is possible to combine ND models with the coevolutionary mechanism in swarm intelligence algorithms, and some research has been done on it. For example, Yan et al.^[21] first combined an ND method with the particle swarm optimization (PSO) algorithm to improve the global convergence and accuracy of the algorithm. Further, a wavelet function was introduced to enhance the global search ability to prevent the algorithm from falling into a local optimum prematurely. A collaborative ND optimization method, presented by Li et al.^[22], was employed for solving

nonconvex problems. To ensure population diversity during the solution process, Levy variation was utilized. Through theoretical analyses, it has been proven that the mechanism enhanced global search capability and improved the method's robustness. However, the above works neglected the perturbation in the process of solving. Therefore, in Ref. [23], a robust ND model was presented for solving nonconvex problems. In this model, linear noise, random noise, and constant noise are considered in the process of solving problems. Nevertheless, most of the above works used continuous ND models combined with the traditional PSO algorithm, which are hard to implement on hardware devices. A coevolutionary neural solution (CNS) was presented that combines a discrete simplified ND method with the traditional PSO algorithm to address nonconvex problems^[24]. This method considered the effects of constant and random noise and could effectively suppress them. In the above works, each ND model is used for local exploration and global information interaction through the collaborative mechanism inspired by swarm intelligence algorithms. However, the CNS model still exhibits limitations in resisting complex noise.

Inspired by the above works and considering that continuous ND models are challenging to implement in digital circuits and that noise is inevitable in practical applications, this paper proposes a modified neural dynamics (MND) model. Compared with existing ND models, the MND model incorporates a dual-gradient accumulation term, enabling a more robust and accurate local search under various types of noise. On this basis, a coevolutionary neural dynamics considering multiple strategies (CNDMS) model is further proposed. The proposed CNDMS model employs an alternating execution strategy, utilizing an MND model as a local operator and incorporating a coevolutionary mechanism inspired by swarm intelligence algorithms to search for the globally optimal solution. Additionally, to prevent premature convergence to local optima, a modified opposition-based learning method is employed based on existing solutions to generate superior candidate solutions^[25–27], while a hybrid variation strategy is utilized to help the proposed CNDMS model escape from local optima during the search process^[28]. Specifically, the research framework of this paper is presented in Fig. 1.

The rest of the paper is organized as follows. Section 2 presents the specific framework of the

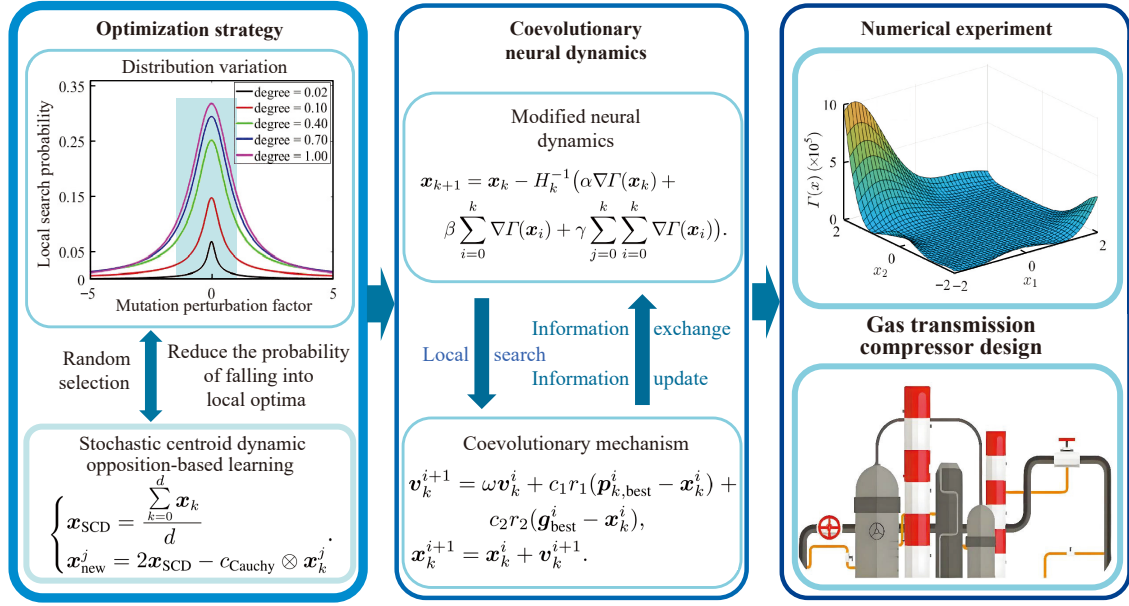


Fig. 1 Research framework of this paper.

proposed CNDMS model, including a description of the MND model as the local search operator and other strategies adopted to enhance the model's performance. Analyses on the global convergence and robustness of the proposed CNDMS model are given in Section 3. Numerical experiments, an application to the gas transmission compressor design problem, and relevant comparisons are presented in Section 4. At last, Section 5 concludes this paper. In addition, the contributions of the paper are as below.

(1) This paper proposes an MND model with a dual-gradient accumulation term, which is used as the local operator in the CNDMS model. Compared with existing discrete ND models, the proposed MND achieves more stable optimization performance under various types of noise.

(2) An opposition-based learning strategy and a hybrid mutation strategy are introduced to maintain population diversity and improve global search capability.

(3) Convergence and robustness of the proposed CNDMS model are theoretically proven, and its superior performance is validated by comparing benchmark and engineering problems.

2 CNDMS

In this section, the construction process of the CNDMS model is described. First, a general mathematical formulation of the nonconvex optimization problem to be addressed is briefly presented. Then, an MND

model is introduced, which serves as the local search operator within the CNDMS framework. Subsequently, the coevolutionary mechanism and other strategies designed to enhance global search capability are discussed. Finally, the pseudocode of the CNDMS model is provided.

2.1 Problem formulation

This paper focuses on nonconvex optimization problems with the mathematical structure as

$$\begin{aligned} \min \quad & \Gamma(\mathbf{x}), \\ \text{s.t., } & \mathbf{x} \in \mathcal{E} \end{aligned} \quad (1)$$

where $\Gamma(\cdot) \in \mathbb{R}$ is a smooth, twice differentiable, and nonconvex objective function; $\mathbf{x} \in \mathbb{R}^n$ is a decision variable to be solved; $\mathcal{E}$ denotes the constraint set of the optimization problem. This is a global optimization problem, where the nonconvex nature of the objective function makes it challenging to find the global optimum $\mathbf{x}$ that minimizes $\Gamma(\mathbf{x})$ within the feasible set $\mathcal{E}$.

2.2 Local operator

The process of solving nonconvex optimization problems requires both global exploration and local exploitation. Utilizing an effective local operator can enhance the solution's precision and accelerate the convergence^[29, 30]. Therefore, this work investigates the ND model further to improve local convergence capability.

To enhance the robustness of the model, an MND

model is constructed as

$$\mathbf{x}_{k+1} = \mathbf{x}_k - H_k^{-1} \left(\alpha \nabla \Gamma(\mathbf{x}_k) + \beta \sum_{i=0}^k \nabla \Gamma(\mathbf{x}_i) + \gamma \sum_{n=0}^k \sum_{i=0}^n \nabla \Gamma(\mathbf{x}_i) \right) \quad (2)$$

where $\nabla \Gamma(\mathbf{x}_k)$ is the gradient of the objective function $\Gamma(\mathbf{x}_k)$; H_k is the Hessian matrix corresponding to the function $\Gamma(\mathbf{x}_k)$; α , β , and γ are coefficients of the discrete-time model; the superscript $^{-1}$ is the inverse of a matrix. Parameters α , β , and γ control the current gradient, single accumulated gradient, and dual-accumulated gradient. Generally, α determines the descent intensity, while β and γ enhance stability and noise suppression through gradient accumulation. In practice, α is the dominant factor, and relatively smaller β and γ are used to balance convergence speed and robustness.

Compared with the original ND model, the MND model introduces a dual-gradient accumulation term, enhancing the model's noise rejection capability and maintaining higher solution accuracy under noisy conditions. The convergence and robustness analyses of the MND model described by Eq. (2) are provided in Section 3. For the nonconvex optimization problem described by Formula (1), its local optimal solution can be solved by the MND model described by Eq. (2), so the MND model can be regarded as a local operator.

2.3 Coevolutionary mechanism

The coevolutionary mechanism employed in swarm intelligence algorithms facilitates the interaction and transmission of the global information, thereby effectively enhancing the capacity of methods for global exploration.

In the search for the feasible region, the population has n individuals, each of which is m -dimensional, and its corresponding velocity is $\mathbf{v}_k$. During the search process, the position and velocity of each individual change, while the corresponding update formula in the coevolutionary mechanism is

$$\begin{cases} \mathbf{v}_k^{j+1} = \omega \mathbf{v}_k^j + c_1 r_1 (\mathbf{p}_{k,\text{best}}^j - \mathbf{x}_k^j) + c_2 r_2 (\mathbf{g}_{\text{best}}^j - \mathbf{x}_k^j) \\ \mathbf{x}_k^{j+1} = \mathbf{x}_k^j + \mathbf{v}_k^{j+1} \end{cases} \quad (3)$$

where $\mathbf{p}_{k,\text{best}}^j$ is the local optimal value of the k -th particle for the j -th iteration; $\mathbf{g}_{\text{best}}^j$ represents the global optimal value to the j -th iteration; c_1 and c_2 represent

the individual learning factor and the social learning factor, respectively; ω is the inertia weight; r_1 and r_2 are random numbers in the interval $[0, 1]$, which can enhance the randomness during the search process.

The performance of a single coevolutionary mechanism is limited when faced with some complex optimization problems. Therefore, this paper explores the following strategies to enhance the performance of the proposed CNDMS model.

2.4 Adaptive weight factor

In the process of evolution, the change of the weight factor ω is the key to the balance between exploration and exploitation. Inspired by the design of weight factors in previous literature, this paper designs the adaptive weight factor as

$$\omega = 1 - (\omega_{\max} - r_3 e^{j/j_{\max}})^{-4(\omega_{\max} - \omega_{\min})} \quad (4)$$

where $\omega_{\max}$ and $\omega_{\min}$ denote the maximum and minimum values of the inertial factor, respectively; j and $j_{\max}$ represent the current and maximum iterations, respectively; r_3 is a random number. Note that $\omega_{\max}$ is commonly set to 0.9, and $\omega_{\min}$ is set to 0.4, following widely adopted designs in swarm intelligence algorithms. A large inertia weight in the early stages facilitates global exploration by preserving momentum, while a small one in later stages promotes local exploitation and convergence. This dynamic adjustment strategy helps mitigate premature convergence and improve search efficiency in complex optimization landscapes.

2.5 Optimization strategy

The preservation of population diversity is a crucial concern. Maintaining a high level of population diversity during the optimization process facilitates an expedited search for the globally optimal value. Opposition-based learning is currently employed in numerous methodologies, and some works confirmed that candidate solutions generated through opposition-based learning are closer to the optimal solution compared to those randomly generated^[25]. The combination of Cauchy distribution and stochastic centroid dynamic opposition-based learning is utilized to guide the generation of solutions close to the optimal solution, effectively ensuring population diversity throughout the solving process. The dynamic opposite for the candidate solution $\mathbf{x}$ in the evolutionary process is calculated as

$$\begin{cases} \mathbf{x}_{\text{SCD}} = \frac{\sum_{k=0}^d \mathbf{x}_k}{d} \\ \mathbf{x}_{\text{new}}^j = 2\mathbf{x}_{\text{SCD}} - c_{\text{Cauchy}} \otimes \mathbf{x}_k^j \end{cases} \quad (5)$$

where $\mathbf{x}_{\text{SCD}}$ is an intermediate variable; $\mathbf{x}_{\text{new}}^j$ represents the newly generated candidate solution; d represents the quantity of particles selected in a random manner; c_{Cauchy} represents a Cauchy operator; $\otimes$ represents the inner product of vectors.

The work on evolutionary strategies has demonstrated through experiments that opting for a heavy tail distribution, as opposed to a light tail distribution, effectively enhances the probability of discovering the global optimal solution^[31]. The Cauchy distribution and the t -distribution combination helps the proposed CNDMS model gradually switch from the global search to local exploration. The greedy strategy allows fast convergence towards the optimal solution without disrupting the current solution. To make the proposed CNDMS model approach the optimal solution faster in the search process, Eq. (6) is used to define the threshold for selecting the update strategy alternately,

$$\rho = -\frac{\ln\left[1 - \left(\frac{j}{j_{\max} + 1}\right)^2\right]^3}{j_{\max}} \quad (6)$$

Such opposition-based learning and mutation strategies contribute to the CNDMS model by effectively preventing premature convergence and maintaining population diversity. Their incorporation enables the model to escape local optima and achieve more accurate global solutions, especially in challenging scenarios involving high-dimensional or deceptive fitness landscapes. For the proposed CNDMS model, the pseudocode is described in Algorithm 1.

3 Theoretical Analysis

The theoretical analyses in this section examine the global convergence and robustness of the proposed CNDMS model.

3.1 Convergence analyses of CNDMS model

This subsection investigates the global convergence of the proposed CNDMS model.

Lemma 1^[21] For the nonconvex optimization problem $\Gamma(x)$, the proposed CNDMS model ensures that the global optimal value does not diverge.

Algorithm 1 CNDMS model

Require: Fitness function $\Gamma(x)$, c_1 , c_2 , $\omega_{\min}$, $\omega_{\max}$, $j_{\max}$

Ensure: Optimal $\Gamma(x)$

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1:  $\mathbf{p}_{\text{best}} = \Gamma(\mathbf{x})$ ;
2:  $\mathbf{g}_{\text{best}} = \min(\mathbf{p}_{\text{best}})$ ;
3: for  $j = 1$  to  $j_{\max}$  do
4:   for  $k = 1$  to  $n$  do
5:      $\hat{\mathbf{x}}_k^j = \text{MND}(\mathbf{x}_k^j)$ ;
6:     if  $\Gamma(\hat{\mathbf{x}}_k^j) < \Gamma(\mathbf{p}_{k,\text{best}}^j)$  then
7:        $\mathbf{p}_{k,\text{best}}^j = \hat{\mathbf{x}}_k^j$ ;
8:     end if
9:     if  $\Gamma(\mathbf{p}_{k,\text{best}}^j) < \Gamma(\mathbf{g}_{\text{best}}^j)$  then
10:       $\mathbf{g}_{\text{best}}^j = \mathbf{p}_{k,\text{best}}^j$ ;
11:    end if
12:    if  $\text{Rand} \leq \rho$  then
13:      Update the current solution by Eq. (5) and determine
      whether it is a better solution;
14:    else
15:      The current global optimal solution undergoes
      mutation through the integration of Cauchy
      distribution and  $t$ -distribution, followed by
      evaluation and selection;
16:    end if
17:    The value of the nonlinear weight factor is calculated
    by Eq. (4);
18:    The current particle position and velocity are updated
    by Eq. (3);
19:    Restrict particle position  $\mathbf{x}_k^{j+1}$  and velocity  $\mathbf{v}_k^{j+1}$  to the
    feasible domain:  $\mathbf{x}_k^{j+1} = \text{Retract}(\mathbf{x}_k^{j+1})$ ,
     $\mathbf{v}_k^{j+1} = \text{Retract}(\mathbf{v}_k^{j+1})$ ;
20:  end for
21: end for

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Proof For the proposed CNDMS model, at the j -th iteration, if $\Gamma(\mathbf{p}_{k,\text{best}}^j) < \Gamma(\mathbf{g}_{\text{best}}^j)$, then $\mathbf{g}_{\text{best}}^j = \mathbf{p}_{k,\text{best}}^j$. Therefore, there is no increasing iterative process leading to divergence.

Lemma 2^[23] The proposed CNDMS model ensures that an infinite number of searches within the search space $\mathcal{E}$ will always find a point in the Borel subset B , where the Lebesgue measure $u[B]$ is greater than zero.

Proof The infimum of the search space $\mathcal{E}$ is denoted as κ , guaranteeing the existence of non-empty subsets within the search space and approximating the fitness associated with points in the search space to its infimum. In this context, consider the optimal region as

$$\mathfrak{J}_{\theta, \varpi} = \begin{cases} \{\mathbf{x} \in \mathcal{E} | \Gamma(\mathbf{x}) < \kappa + \theta\}, & -\infty < \kappa < +\infty \\ \{\mathbf{x} \in \mathcal{E} | \Gamma(\mathbf{x}) < R\}, & \kappa = -\infty \end{cases} \quad (7)$$

where $\theta > 0$ and $R < 0$. Hence, if there exists a Borel subset B in the search space $\mathcal{E}$ with a positive Lebesgue measure $u[B] > 0$, this subset contains an optimal space $\mathfrak{J}_{\theta, \varpi}$. The likelihood of not discovering a point within subset B after an infinite number of searches is zero.

Theorem 1 For the search space $\mathcal{E}$ of the problem, the proposed CNDMS model converges to the global optimal solution with probability one.

Proof For the objective function $\Gamma(\mathbf{x})$, its initial solution is randomly determined. Let O_ϵ represent the ϵ -neighborhood of the optimal solution $\mathbf{x}^*$ with $O_\epsilon = \{\mathbf{x} \in \mathcal{E} : \|\mathbf{x} - \mathbf{x}^*\| < \epsilon\}$. Let P_ϵ denote the probability of $\mathbf{x}_{\text{new}}$ falling within O_ϵ after a single operation with a non-zero value. The likelihood of the solution $\mathbf{x}_{\text{new}}$ falling into O_ϵ at least once within j iterations is given by $1 - (1 - P_\epsilon)^j$. Additionally, when the model is executed continuously, the result as Eq. (8) can be derived,

$$\lim_{j \rightarrow +\infty} [1 - (1 - P_\epsilon)^j] = 1 \quad (8)$$

In summary, the probability that the proposed CNDMS model converges to the global optimal solution is one when j tends to infinity. The proof is thus completed. ■

3.2 Convergence analyses of MND model

As for the proposed CNDMS model, this paper mainly uses the MND model described by Eq. (2) to improve the convergence and robustness, so the MND model described by Eq. (2) is analyzed primarily in this subsection.

Theorem 2 The MND model described by Eq. (2) is transformed to obtain a decoupled dynamic system as

$$\mathbf{g}_{k+1} - (1 - \alpha)\mathbf{g}_k + \beta \sum_{i=0}^k \mathbf{g}_i + \gamma \sum_{n=0}^k \sum_{i=0}^n \mathbf{g}_i + o(\iota^2) = \mathbf{0} \quad (9)$$

where $\mathbf{g}_k = \nabla \Gamma(\mathbf{x}_k)$ and $o(\iota^2)$ represents the truncation error, and ι denotes the time step.

Proof The MND model described by Eq. (2) can be described as

$$\mathbf{x}_{k+1} = \mathbf{x}_k - H_k^{-1} \left(\alpha \nabla \Gamma(\mathbf{x}_k) + \beta \sum_{i=0}^k \nabla \Gamma(\mathbf{x}_i) + \gamma \sum_{n=0}^k \sum_{i=0}^n \nabla \Gamma(\mathbf{x}_i) \right) \quad (10)$$

Using the Taylor expansion, we obtain the result as

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \iota \dot{\mathbf{x}}_k + o(\iota^2) \quad (11)$$

$\dot{\mathbf{x}}_k$ denotes the time derivative of $\mathbf{x}$ at the k -th step.

The combination of Eqs. (10) and (11) yields the subsequent result as

$$\iota \dot{\mathbf{x}}_k + o(\iota^2) = -H_k^{-1} \left(\alpha \nabla \Gamma(\mathbf{x}_k) + \beta \sum_{i=0}^k \nabla \Gamma(\mathbf{x}_i) + \gamma \sum_{n=0}^k \sum_{i=0}^n \nabla \Gamma(\mathbf{x}_i) \right) \quad (12)$$

It can be transformed into

$$\iota H_k \dot{\mathbf{x}}_k + o(\iota^2) = - \left(\alpha \nabla \Gamma(\mathbf{x}_k) + \beta \sum_{i=0}^k \nabla \Gamma(\mathbf{x}_i) + \gamma \sum_{j=0}^k \sum_{i=0}^n \nabla \Gamma(\mathbf{x}_i) \right) \quad (13)$$

Then, it is simplified as the subsequent form as

$$H_k \dot{\mathbf{x}}_k = - \frac{1}{\iota} \left(\alpha \nabla \Gamma(\mathbf{x}_k) + \beta \sum_{i=0}^k \nabla \Gamma(\mathbf{x}_i) + \gamma \sum_{n=0}^k \sum_{i=0}^n \nabla \Gamma(\mathbf{x}_i) + o(\iota^2) \right) \quad (14)$$

Furthermore, according to $\mathbf{g}_k = \nabla \Gamma(\mathbf{x}_k)$, it is converted into

$$\dot{\mathbf{g}}_k = - \frac{1}{\iota} \left(\alpha \mathbf{g}_k + \beta \sum_{i=0}^k \mathbf{g}_i + \gamma \sum_{n=0}^k \sum_{i=0}^n \mathbf{g}_i + o(\iota^2) \right) \quad (15)$$

This is derived by applying the difference formula to $\dot{\mathbf{g}}_k$, as shown in

$$\dot{\mathbf{g}}_k = \frac{\mathbf{g}_{k+1} - \mathbf{g}_k}{\iota} = - \frac{1}{\iota} \left(\alpha \mathbf{g}_k + \beta \sum_{i=0}^k \mathbf{g}_i + \gamma \sum_{n=0}^k \sum_{i=0}^n \mathbf{g}_i + o(\iota^2) \right) \quad (16)$$

Hence, it can be further transformed into

$$\mathbf{g}_{k+1} - (1 - \alpha)\mathbf{g}_k + \beta \sum_{i=0}^k \mathbf{g}_i + \gamma \sum_{n=0}^k \sum_{i=0}^n \mathbf{g}_i + o(\iota^2) = \mathbf{0} \quad (17)$$

The proof is completed. ■

Theorem 3 The steady-state calculation error $\lim_{k \rightarrow +\infty} \|\mathbf{g}_k\|_2$ of the MND model described by Eq. (2) is $o(\iota^2)$, when $\alpha + \beta = 1$, $\alpha > \beta$, and $\beta > \gamma$.

Proof Based on Theorem 1, the MND model described by Eq. (2) can be transformed into a linear system that includes a residual component. The element $\mathbf{g}_k^j$ denotes the j -th component of $\mathbf{g}_k$, where j belongs to the set $\{1, 2, \dots, m\}$. Consequently, the subsequent result can be derived as

$$\mathbf{g}_{k+1}^j - (1-\alpha)\mathbf{g}_k^j + \beta \sum_{i=0}^k \mathbf{g}_i^j + \gamma \sum_{n=0}^k \sum_{i=0}^n \mathbf{g}_i^j + o(\iota^2) = 0 \quad (18)$$

It can be deduced from Eq. (18) that

$$\mathbf{g}_k^j - (1-\alpha)\mathbf{g}_{k-1}^j + \beta \sum_{i=0}^{k-1} \mathbf{g}_i^j + \gamma \sum_{n=0}^{k-1} \sum_{i=0}^n \mathbf{g}_i^j + o(\iota^2) = 0 \quad (19)$$

Subtracting Eq. (19) from Eq. (18) yields

$$\mathbf{g}_{k+1}^j = (2-\alpha-\beta)\mathbf{g}_k^j - (1-\alpha)\mathbf{g}_{k-1}^j - \gamma \sum_{i=0}^k \mathbf{g}_i^j + o(\iota^2) \quad (20)$$

It can be deduced from Eq. (20) that

$$\mathbf{g}_k^j = (2-\alpha-\beta)\mathbf{g}_{k-1}^j - (1-\alpha)\mathbf{g}_{k-2}^j - \gamma \sum_{i=0}^{k-1} \mathbf{g}_i^j + o(\iota^2) \quad (21)$$

Then, by subtracting Eq. (21) from Eq. (20), there is

$$\mathbf{g}_{k+1}^j = (3-\alpha-\beta-\gamma)\mathbf{g}_k^j - (3-2\alpha-\beta)\mathbf{g}_{k-1}^j + (1-\alpha)\mathbf{g}_{k-2}^j + o(\iota^2) \quad (22)$$

Defining vectors $\mathbf{q}_{k+1}^j = [\mathbf{g}_{k+1}^j; \mathbf{g}_k^j; \mathbf{g}_{k-1}^j]$ and $\mathbf{q}_k^j = [\mathbf{g}_k^j; \mathbf{g}_{k-1}^j; \mathbf{g}_{k-2}^j]$, the aforementioned equation is transformed into the subsequent equation as

$$\mathbf{q}_{k+1}^j = A\mathbf{q}_k^j + o(\iota^2) \quad (23)$$

where

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (24)$$

with $a_1 = 3-\alpha-\beta-\gamma$, $a_2 = -(3-2\alpha-\beta)$, and $a_3 = 1-\alpha$. According to Eq. (23), it can be further derived that

$$\begin{aligned} \mathbf{q}_{k+1}^j &= A\mathbf{q}_k^j + o(\iota^2) \\ &= A(A\mathbf{q}_{k-1}^j + o(\iota^2)) + o(\iota^2) \\ &= A^2\mathbf{q}_{k-1}^j + o(\iota^2) \\ &\vdots \\ &= A^k\mathbf{q}_1^j + o(\iota^2) \end{aligned} \quad (25)$$

Based on the definition of matrix A and the Gelfand formula for the spectral radius^[32], it can be inferred that within a certain region, the absolute value of the real part of its eigenvalues is less than 1. Consequently, as k approaches infinity, A^k converges to zero. This implies that $\mathbf{q}_{k+1}^j = o(\iota^2)$. The result as Eq. (26) is obtained,

$$\lim_{k \rightarrow +\infty} \|\mathbf{q}_{k+1}^j\|_2 = o(\iota^2) \quad (26)$$

Based on the definition of $\mathbf{q}_{k+1}^j$, it is derived that the steady-state error $\lim_{k \rightarrow +\infty} \|\mathbf{g}_k\|_2$ of the MND model (2) is $o(\iota^2)$. Thus, the proof is completed. ■

Theorem 4 The steady-state calculation error $\lim_{k \rightarrow +\infty} \|\mathbf{g}_k\|_2$ for the MND model described by Eq. (2) under linear noise is $o(\iota^2)$.

Proof The MND model described by Eq. (2) with linear noise $\eta_k = \varepsilon k \iota$ is described as

$$\mathbf{g}_{k+1} = (1-\alpha)\mathbf{g}_k - \beta \sum_{i=0}^k \mathbf{g}_i - \gamma \sum_{n=0}^k \sum_{i=0}^n \mathbf{g}_i + \varepsilon k \iota \quad (27)$$

where ε is a real number.

It can be deduced that

$$\mathbf{g}_k = (1-\alpha)\mathbf{g}_{k-1} - \beta \sum_{i=0}^{k-1} \mathbf{g}_i - \gamma \sum_{n=0}^{k-1} \sum_{i=0}^n \mathbf{g}_i + \varepsilon(k-1)\iota \quad (28)$$

A result is obtained by subtracting the above two equations as

$$\mathbf{g}_{k+1} = (2-\alpha-\beta)\mathbf{g}_k - (1-\alpha)\mathbf{g}_{k-1} - \gamma \sum_{i=0}^k \mathbf{g}_i + \varepsilon \iota \quad (29)$$

The results as Eq. (30) are obtained by using the Z-transform,

$$\begin{aligned} z\mathbf{g}(z) - z\mathbf{g}(0) &= (2-\alpha-\beta)\mathbf{g}(z) - (1-\alpha)z^{-1}\mathbf{g}(z) - \\ &\quad \gamma \frac{z}{z-1}\mathbf{g}(z) + \frac{z\varepsilon\iota}{z-1} \end{aligned} \quad (30)$$

where z is a complex number and z^{-1} represents the unit delay operator.

Arranging Eq. (30) elicits

$$\mathbf{g}^j(z) = \frac{z^2(z-1)\mathbf{g}(0) + z^2\varepsilon\iota}{z^3 - z^2 - (2-\alpha-\beta)(z^2 - z) + (1-\alpha)(z-1) + \gamma z^2} \quad (31)$$

According to the final value theorem, there is

$$\lim_{k \rightarrow +\infty} \mathbf{g}_k = \lim_{z \rightarrow 1} (z-1)\mathbf{g}(z) = \mathbf{0} \quad (32)$$

The above derivation verifies that the steady-state error of the subsystem is zero with linear noise. Therefore, the steady-state error of the MND model described by Eq. (2) under the influence of linear noise is $o(\iota^2)$. The proof is completed. ■

Remark 1 Through a similar analysis, the steady-state error of the MND model described by Eq. (2) is also $o(\iota^2)$ under the influence of the constant noise.

4 Numerical Experiment and Application

To verify the effectiveness of the proposed CNDMS model, we test it on a series of benchmark test functions and a practical engineering design problem,

and compare it with the artificial protozoa optimizer (APO)^[33], the attraction-repulsion optimization algorithm (AROA)^[34], the non-linear population size reduction success history adaptive differential evolution with rank-based selective pressure (NL-SHADE-RSP)^[35], the Newton-Raphson-based optimizer (NRBO)^[36], and the proportional-integral-derivative-based search algorithm (PSA)^[37]. In the subsequent parts of this section, bolded values in the tables indicate that the corresponding method outperforms other comparative methods on the relevant metrics.

4.1 Numerical experiment

In this section, the Goldstein–Price function^[38] is used as a demonstration example. The function has two unknown variables, and its mathematical formula is shown as

$$\begin{aligned} \min \Gamma(\mathbf{x}) = & \left[1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + \right. \\ & \left. 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2) \right] \times \left[30 + (2x_1 - 3x_2)^2 \right. \\ & \left. (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2) \right], \\ \text{s.t., } & -2 \leq x_1, x_2 \leq 2 \end{aligned} \quad (33)$$

Here, $\mathbf{x} = [x_1; x_2]$ represents the variable to be solved.

The 3D figure of the function expressed by Formula (33) is shown in Fig. 2. Although the Goldstein–Price function expressed by Formula (33) has only one global minimum, many local minimums exist in the feasible region.

Figure 3 shows the convergence curves of six algorithms, including the CNDMS model for solving the Goldstein–Price function. It can be seen that the proposed model converges better than other algorithms, both with and without noise, and can converge accurately to the optimal solution.

In order to achieve the fairness of comparison, the number of populations in all algorithms is 50, the maximum number of iterations is set to 500, and the optimal value of each algorithm parameter is selected. To eliminate the chance brought by the experiments, six algorithms including the CNDMS model are independently executed 30 times on the benchmark function, and the average (AVG), best result (BEST), and standard deviation (STD) are selected as evaluation indexes. It can be seen from Table 1 that the proposed CNDMS model is worse than the AROA algorithm without noise, but the results are optimal under the influence of noise. Furthermore, apart from the above example, this paper conducts statistical experiments on additional benchmark functions^[39], and the corresponding results are presented in Table 2. The accuracy of the proposed CNDMS model is comparable to or superior to that of other algorithms for most benchmark test functions without noise, with only a slight deviation in performance observed for a few benchmark test functions. However, when constant noise is present, the accuracy of the proposed CNDMS model is inferior only to that of the NRBO algorithm in the Chichinadze function. In all other benchmark

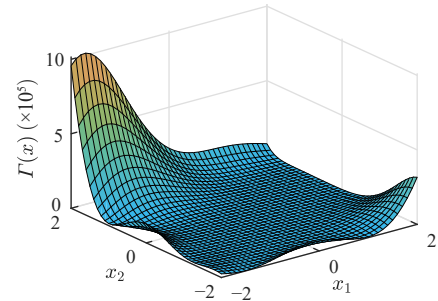


Fig. 2 3D figure of Goldstein–Price function expressed by Formula (33).

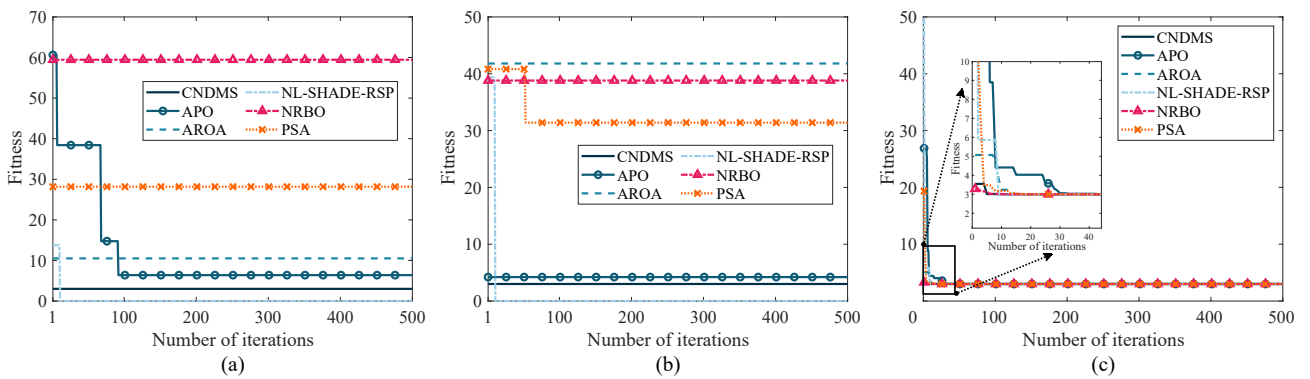


Fig. 3 Convergence changes of different algorithms on the Goldstein–Price function expressed by Formula (33) under different noise. (a) Linear noise environment. (b) Constant noise environment. (c) Noise-free environment.

Table 1 Statistical results of different algorithms on the Goldstein–Price function expressed by Formula (33) under different noise. The bold values mean that the algorithm achieves the best results under the same noise type.

Algorithm	Index	Goldstein–Price function		
		Linear noise	Constant noise	Noise free
CNDMS	AVG	3.00×10^0	3.00×10^0	3.00×10^0
	BEST	3.00×10^0	3.00×10^0	3.00×10^0
	STD	8.40×10^{-6}	1.64×10^{-6}	1.36×10^{-15}
APO	AVG	5.57×10^0	4.53×10^0	3.00×10^0
	BEST	3.03×10^0	3.02×10^0	3.00×10^0
	STD	2.23×10^0	1.34×10^0	1.29×10^{-15}
AROA	AVG	3.78×10^1	5.76×10^1	3.00×10^0
	BEST	3.59×10^0	4.85×10^0	3.00×10^0
	STD	2.88×10^1	2.63×10^1	8.92×10^{-16}
NL-SHADE-RSP	AVG	1.70×10^1	2.91×10^1	3.00×10^0
	BEST	3.26×10^0	3.66×10^0	3.00×10^0
	STD	1.37×10^1	2.55×10^1	1.36×10^0
NRBO	AVG	2.96×10^1	3.51×10^1	3.00×10^0
	BEST	3.23×10^0	4.05×10^0	3.00×10^0
	STD	2.85×10^1	2.33×10^1	2.99×10^{-15}
PSA	AVG	5.93×10^1	4.60×10^1	3.00×10^0
	BEST	8.79×10^0	3.52×10^0	3.00×10^0
	STD	5.19×10^1	3.05×10^1	3.73×10^{-15}

functions, the proposed CNDMS model outperforms other algorithms. Under the influence of linear noise, the performance of the proposed CNDMS model is slightly worse than that of the NRBO algorithm on three benchmark functions, and it achieves optimal performance on other benchmark functions.

4.2 Application to gas transmission compressor design

To verify the practicability of the proposed CNDMS model for practical problems, it is investigated on the gas transmission compressor design problem^[40]. The mathematical formulation of the gas transmission compressor design problem is defined as

$$\begin{aligned}
 \min F(\mathbf{x}) &= 8.61 \times 10^5 x_1^{\frac{1}{2}} x_2 x_3^{-\frac{2}{3}} x_4^{-\frac{1}{2}} + 3.69 \times 10^4 x_3 + \\
 &\quad 7.72 \times 10^8 x_1^{-1} x_2^{0.219} - 765.43 \times 10^6 x_1^{-1}, \\
 \text{s.t.,} \quad &x_4 x_2^{-2} + x_2^{-2} - 1 \leq 0, \\
 &20 \leq x_1 \leq 50, 1 \leq x_2 \leq 10, \\
 &20 \leq x_3 \leq 50, 0.1 \leq x_4 \leq 60
 \end{aligned} \tag{34}$$

In Eq. (34), $\mathbf{x} = [\hat{S}\hat{S}; \hat{C}\hat{R}; \hat{I}\hat{P}\hat{D}; \hat{G}\hat{F}\hat{R}]$, where $\hat{S}\hat{S}$, $\hat{C}\hat{R}$, $\hat{I}\hat{P}\hat{D}$, $\hat{G}\hat{F}\hat{R}$ denote station spacing, compression ratio, internal pipe diameter, and gas flow rate, respectively.

Figure 4 shows the convergence curves of the proposed CNDMS model and the other five algorithms in solving the gas transmission compressor design problem. In the absence of noise, the proposed CNDMS model is less effective than APO, AROA, NL-SHADE-RSP, and PSA. Under the influence of noise, the proposed CNDMS model is slightly better than other algorithms. In order to eliminate the chance brought by the experiments, six algorithms including the proposed CNDMS model are independently performed 30 times and statistically analyzed. The statistical results are shown in Table 3. Under the influence of noise, the effect of the proposed model is better than that of the other five algorithms, while the effect is opposite under the condition of no noise.

4.3 Application to pressure vessel design

To further evaluate the generalization performance of the proposed CNDMS model for real-world constrained engineering problems, it is applied to the pressure vessel design problem^[41]. This problem seeks to minimize the fabrication cost of a cylindrical pressure vessel under multiple nonlinear constraints involving material and welding costs and geometric limitations. The mathematical formulation is provided as

$$\begin{aligned}
 \min_{\mathbf{x}} \quad &f(\mathbf{x}) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + \\
 &\quad 3.1611x_1^2x_4 + 19.8621x_1^2x_3, \\
 \text{s.t.,} \quad &g_1(\mathbf{x}) = x_1 - 0.0193x_3 \geq 0, \\
 &g_2(\mathbf{x}) = x_2 - 0.0095x_3 \geq 0, \\
 &g_3(\mathbf{x}) = \pi x_3^2x_4 + \frac{4}{3}\pi x_3^3 - 750 \times 1728 \geq 0, \\
 &g_4(\mathbf{x}) = 240 - x_4 \geq 0, \\
 &0.0625 \leq x_1 \leq 2, \quad 0.0625 \leq x_2 \leq 2, \\
 &x_3 \geq 10, \quad 0 \leq x_4 \leq 240
 \end{aligned} \tag{35}$$

where $\mathbf{x} = [T_s, T_h, R, L]^T$ denotes variables representing shell thickness, head thickness, inner radius, and cylinder length, respectively. $g_1(\mathbf{x})$, $g_2(\mathbf{x})$, $g_3(\mathbf{x})$, and $g_4(\mathbf{x})$ are the physical constraints for the pressure vessel design.

In the experiments, the population size is set to 50, and the maximum number of iterations is fixed at 300. The same three noise environments are considered as in previous experiments. Figure 5 illustrates the convergence curves of all compared algorithms under different noise types, and Table 4 summarizes the corresponding statistical performance over 30 independent runs.

Table 2 Statistical results of different algorithms on benchmark functions under different noise. The bold values mean that the algorithm achieves the best results under the same noise type.

Function	Noise	Index	CNDMS	APO	AROA	NL-SHADE-RSP	NRBO	PSA
Ackley 1	Noise free	AVG	4.44×10^{-16}	1.57×10^{-1}	4.44×10^{-16}	4.44×10^{-16}	4.44×10^{-16}	4.48×10^{-1}
		BEST	4.44×10^{-16}	3.64×10^{-2}	4.44×10^{-16}	4.44×10^{-16}	4.44×10^{-16}	1.25×10^{-1}
		STD	0	5.39×10^{-2}	0	0	0	2.35×10^{-1}
	Constant noise	AVG	4.44×10^{-16}	3.63×10^0	3.28×10^0	1.64×10^1	7.80×10^{-1}	3.70×10^0
		BEST	4.44×10^{-16}	3.03×10^0	2.62×10^0	1.16×10^1	1.79×10^{-1}	2.85×10^0
		STD	0	2.98×10^{-1}	4.49×10^{-1}	1.42×10^0	3.19×10^{-1}	5.13×10^{-1}
	Linear noise	AVG	4.44×10^{-16}	5.08×10^0	6.01×10^0	1.64×10^1	1.36×10^0	4.94×10^0
		BEST	4.44×10^{-16}	4.13×10^0	5.14×10^0	1.27×10^1	5.41×10^{-1}	4.21×10^0
		STD	0	4.13×10^{-1}	4.95×10^{-1}	1.13×10^0	5.35×10^{-1}	3.92×10^{-1}
Ackley 2	Noise free	AVG	-2.00×10^2	-2.00×10^2	-2.00×10^2	-2.00×10^2	-2.00×10^2	-2.00×10^2
		BEST	-2.00×10^2	-2.00×10^2	-2.00×10^2	-2.00×10^2	-2.00×10^2	-2.00×10^2
		STD	0	0	0	0	0	7.46×10^{-15}
	Constant noise	AVG	-2.00×10^2	-1.99×10^2	-1.99×10^2	-1.88×10^2	-1.97×10^2	-1.99×10^2
		BEST	-2.00×10^2	-2.00×10^2	-2.00×10^2	-1.99×10^2	-2.00×10^2	-2.00×10^2
		STD	0	7.87×10^{-1}	3.16×10^{-1}	5.87×10^0	2.32×10^0	3.25×10^{-1}
	Linear noise	AVG	-2.00×10^2	-1.96×10^2	-1.86×10^2	-1.88×10^2	-1.96×10^2	-1.93×10^2
		BEST	-2.00×10^2	-1.99×10^2	-1.96×10^2	-1.97×10^2	-1.99×10^2	-1.99×10^2
		STD	0	1.96×10^0	6.36×10^0	5.29×10^0	2.27×10^0	4.28×10^0
Modified Ackley	Noise free	AVG	-4.59×10^0	-4.59×10^0	-4.59×10^0	-4.59×10^0	-4.59×10^0	-4.59×10^0
		BEST	-4.59×10^0	-4.59×10^0	-4.59×10^0	-4.59×10^0	-4.59×10^0	-4.59×10^0
		STD	1.81×10^{-15}	1.81×10^{-15}	9.90×10^{-16}	1.81×10^{-15}	1.81×10^{-15}	1.81×10^{-15}
	Constant noise	AVG	-4.59×10^0	-4.36×10^0	-4.38×10^0	1.15×10^0	-3.88×10^0	-4.47×10^0
		BEST	-4.59×10^0	-4.59×10^0	-4.59×10^0	-3.33×10^0	-4.56×10^0	-4.59×10^0
		STD	2.89×10^{-3}	2.06×10^{-1}	1.90×10^0	2.25×10^0	4.49×10^{-1}	1.50×10^{-1}
	Linear noise	AVG	-4.59×10^0	-2.04×10^0	1.42×10^0	8.32×10^{-1}	-3.10×10^0	-1.19×10^0
		BEST	-4.59×10^0	-4.50×10^0	-4.47×10^0	-4.24×10^0	-4.50×10^0	-4.52×10^0
		STD	1.81×10^{-15}	1.17×10^0	2.62×10^0	2.25×10^0	8.72×10^{-1}	1.57×10^0
Branin	Noise free	AVG	3.98×10^{-1}	3.98×10^{-1}	3.98×10^{-1}	3.98×10^{-1}	3.98×10^{-1}	3.98×10^{-1}
		BEST	3.98×10^{-1}	3.98×10^{-1}	3.98×10^{-1}	3.98×10^{-1}	3.98×10^{-1}	3.98×10^{-1}
		STD	0	3.39×10^{-10}	3.04×10^{-13}	0	0	0
	Constant noise	AVG	3.98×10^{-1}	4.30×10^{-1}	9.88×10^{-1}	1.18×10^0	4.48×10^{-1}	7.41×10^{-1}
		BEST	3.98×10^{-1}	4.00×10^{-1}	4.23×10^{-1}	4.39×10^{-1}	3.99×10^{-1}	3.98×10^{-1}
		STD	3.94×10^{-4}	4.43×10^{-2}	7.54×10^{-1}	7.56×10^{-1}	5.27×10^{-2}	6.99×10^{-1}
	Linear noise	AVG	3.99×10^{-1}	5.04×10^{-1}	2.07×10^0	7.40×10^{-1}	9.63×10^{-1}	1.46×10^0
		BEST	3.98×10^{-1}	4.00×10^{-1}	3.99×10^{-1}	4.03×10^{-1}	4.26×10^{-1}	4.41×10^{-1}
		STD	1.90×10^{-3}	1.49×10^{-1}	9.84×10^{-1}	3.53×10^{-1}	4.13×10^{-1}	9.08×10^{-1}
Chichinadze	Noise free	AVG	-4.29×10^1	-4.29×10^1	-4.29×10^1	-4.29×10^1	-4.29×10^1	-4.29×10^1
		BEST	-4.29×10^1	-4.29×10^1	-4.29×10^1	-4.29×10^1	-4.29×10^1	-4.29×10^1
		STD	3.61×10^{-14}	3.61×10^{-14}	3.61×10^{-14}	2.89×10^{-14}	1.36×10^{-1}	1.70×10^{-1}
	Constant noise	AVG	-4.25×10^1	-4.25×10^1	-4.26×10^1	-3.74×10^1	-4.26×10^1	-4.26×10^1
		BEST	-4.25×10^1	-4.28×10^1	-4.29×10^1	-4.26×10^1	-4.29×10^1	-4.29×10^1
		STD	5.56×10^{-14}	9.38×10^{-2}	1.07×10^{-1}	4.70×10^0	1.64×10^{-1}	1.47×10^{-1}
	Linear noise	AVG	-4.25×10^1	-4.18×10^1	-3.98×10^1	-3.72×10^1	-4.24×10^1	-3.98×10^1
		BEST	-4.25×10^1	-4.25×10^1	-4.25×10^1	-4.29×10^1	-4.28×10^1	-4.25×10^1
		STD	1.13×10^{-6}	1.14×10^0	3.52×10^0	4.40×10^0	2.11×10^{-1}	3.11×10^0
Deckkers-Aarts	Noise free	AVG	-2.48×10^4	-2.48×10^4	-2.48×10^4	-2.48×10^4	-2.48×10^4	-2.48×10^4
		BEST	-2.48×10^4	-2.48×10^4	-2.48×10^4	-2.48×10^4	-2.48×10^4	-2.48×10^4
		STD	3.70×10^{-12}	2.02×10^{-11}	3.64×10^{-12}	0	3.44×10^{-12}	3.51×10^{-12}
	Constant noise	AVG	-2.48×10^4	-2.38×10^4	-2.42×10^4	-6.14×10^3	-1.79×10^4	-2.39×10^4
		BEST	-2.48×10^4	-2.47×10^4	-2.48×10^4	-2.43×10^4	-2.47×10^4	-2.48×10^4
		STD	1.05×10^{-6}	7.84×10^2	5.05×10^2	2.40×10^4	6.92×10^3	1.11×10^3

(to be continued)

Table 2 Statistical results of different algorithms on benchmark functions under different noise. The bold values mean that the algorithm achieves the best results under the same noise type. (continued)

Function	Noise	Index	CNDMS	APO	AROA	NL-SHADE-RSP	NRBO	PSA
Deckkers-Aarts	Linear noise	AVG	-2.48×10^4	-1.99×10^4	-6.22×10^3	-8.33×10^3	-1.60×10^4	-6.43×10^3
		BEST	-2.48×10^4	-2.47×10^4	-2.35×10^4	-2.47×10^4	-2.45×10^4	-2.38×10^4
		STD	2.23×10^{-8}	3.76×10^3	2.29×10^4	2.89×10^4	8.14×10^3	1.24×10^4
Dixon-Price	Noise free	AVG	9.86×10^{-32}	2.22×10^{-8}	2.22×10^{-2}	2.15×10^{-29}	4.29×10^1	1.33×10^{-1}
		BEST	9.86×10^{-32}	1.03×10^{-14}	9.86×10^{-32}	2.14×10^{-29}	1.25×10^{-14}	3.33×10^{-27}
		STD	0	6.70×10^{-8}	1.22×10^{-1}	1.01×10^{-30}	3.19×10^{-1}	2.71×10^{-1}
	Constant noise	AVG	1.28×10^{-3}	1.23×10^2	2.18×10^2	2.30×10^2	1.69×10^2	5.14×10^2
		BEST	1.23×10^{-4}	1.71×10^1	2.41×10^0	9.38×10^0	1.61×10^1	8.06×10^0
		STD	7.55×10^{-4}	7.73×10^1	2.67×10^2	1.58×10^2	1.51×10^2	8.87×10^2
	Linear noise	AVG	5.99×10^{-3}	3.09×10^2	3.50×10^3	2.00×10^2	1.38×10^3	1.87×10^3
		BEST	7.34×10^{-4}	4.90×10^1	1.32×10^2	1.65×10^1	5.84×10^0	1.00×10^2
		STD	2.56×10^{-3}	2.21×10^2	2.72×10^3	1.56×10^2	1.17×10^3	1.54×10^3
Exponential	Noise free	AVG	-1.00×10^0	-1.00×10^0	-1.00×10^0	-1.00×10^0	-1.00×10^0	-1.00×10^0
		BEST	-1.00×10^0	-1.00×10^0	-1.00×10^0	-1.00×10^0	-1.00×10^0	-1.00×10^0
		STD	0	0	0	0	0	2.92×10^{-17}
	Constant noise	AVG	-1.00×10^0	-9.41×10^{-1}	-7.90×10^{-1}	-9.49×10^{-1}	-1.97×10^2	-8.20×10^{-1}
		BEST	-1.00×10^0	-9.86×10^{-1}	-9.33×10^{-1}	-9.84×10^{-1}	-2.00×10^2	-9.59×10^{-1}
		STD	0	2.48×10^{-2}	7.21×10^{-2}	2.01×10^{-2}	2.32×10^0	6.77×10^{-2}
	Linear noise	AVG	-1.00×10^0	-9.41×10^{-1}	-8.40×10^{-1}	-9.55×10^{-1}	-1.96×10^2	-8.35×10^{-1}
		BEST	-1.00×10^0	-9.90×10^{-1}	-9.66×10^{-1}	-9.84×10^{-1}	-1.99×10^2	-9.43×10^{-1}
		STD	0	2.33×10^{-2}	7.08×10^{-2}	2.01×10^{-2}	2.27×10^0	5.33×10^{-2}
Generalized Schwefel	Noise free	AVG	-1.14×10^4	-9.06×10^3	-9.98×10^3	-1.09×10^4	-5.12×10^3	1.02×10^4
		BEST	-1.19×10^4	-1.05×10^4	-1.11×10^4	-1.20×10^4	-6.54×10^3	1.13×10^4
		STD	2.93×10^2	4.47×10^2	5.56×10^2	4.57×10^2	6.42×10^2	5.08×10^2
	Constant noise	AVG	-1.15×10^4	-6.24×10^3	-7.28×10^3	-8.32×10^3	-4.74×10^3	-6.84×10^3
		BEST	-1.20×10^4	-7.13×10^3	-8.69×10^3	-9.29×10^3	-9.51×10^3	-7.98×10^3
		STD	3.51×10^2	3.95×10^2	5.89×10^2	4.25×10^2	1.37×10^3	7.37×10^2
	Linear noise	AVG	-1.18×10^4	-3.72×10^3	-6.35×10^3	-8.29×10^3	-1.26×10^4	-3.35×10^3
		BEST	-1.24×10^4	-5.14×10^3	-7.89×10^3	-9.25×10^3	-1.26×10^4	-4.55×10^3
		STD	3.62×10^2	4.55×10^2	8.98×10^2	4.40×10^2	1.16×10^0	4.24×10^2
Shekel 5	Noise free	AVG	-1.02×10^1	-1.02×10^1	-1.02×10^1	-9.14×10^0	-9.75×10^0	-5.56×10^0
		BEST	-1.02×10^1	-1.02×10^1	-1.02×10^1	-1.02×10^1	-1.02×10^1	-1.02×10^1
		STD	6.33×10^{-15}	4.38×10^{-7}	1.66×10^{-9}	2.06×10^0	1.32×10^0	3.41×10^0
	Constant noise	AVG	-1.02×10^1	-2.51×10^0	-1.40×10^0	-3.32×10^0	-1.02×10^1	-9.11×10^{-1}
		BEST	-1.02×10^1	-4.15×10^0	-4.32×10^0	-4.55×10^0	-1.02×10^1	-1.49×10^0
		STD	1.48×10^{-3}	8.15×10^{-1}	8.84×10^0	6.20×10^{-1}	6.32×10^{-3}	2.93×10^{-1}
	Linear noise	AVG	-6.62×10^0	-1.57×10^0	-6.26×10^{-1}	-3.17×10^0	-7.28×10^0	-7.88×10^{-1}
		BEST	-1.02×10^1	-4.09×10^0	-1.55×10^0	-4.27×10^0	-1.02×10^1	-1.98×10^0
		STD	2.97×10^0	7.24×10^{-1}	3.08×10^{-1}	5.93×10^{-1}	2.03×10^0	4.20×10^0
Sphere	Noise free	AVG	0	0	0	5.32×10^{-50}	0	2.08×10^{-14}
		BEST	0	0	0	2.91×10^{-75}	0	5.17×10^{-17}
		STD	0	0	0	2.72×10^{-49}	0	3.28×10^{-14}
	Constant noise	AVG	0	9.98×10^2	1.23×10^3	1.03×10^3	3.16×10^1	1.22×10^3
		BEST	0	6.32×10^2	1.03×10^3	8.88×10^2	0	1.03×10^3
		STD	0	9.41×10^1	7.72×10^1	6.36×10^1	2.89×10^1	9.70×10^1
	Linear noise	AVG	0	1.04×10^3	1.22×10^3	1.02×10^3	8.13×10^1	1.20×10^3
		BEST	0	9.16×10^2	1.02×10^3	8.41×10^2	0	1.01×10^3
		STD	0	5.45×10^1	8.85×10^1	5.83×10^1	9.37×10^1	8.18×10^1

From Fig. 5, it is evident that the proposed CNDMS model achieves rapid convergence and superior stability, especially in noisy environments. In the linear

noise setting, the CNDMS model obtains the best average value (5.87×10^3) and demonstrates the lowest variance (1.50×10^2), outperforming other

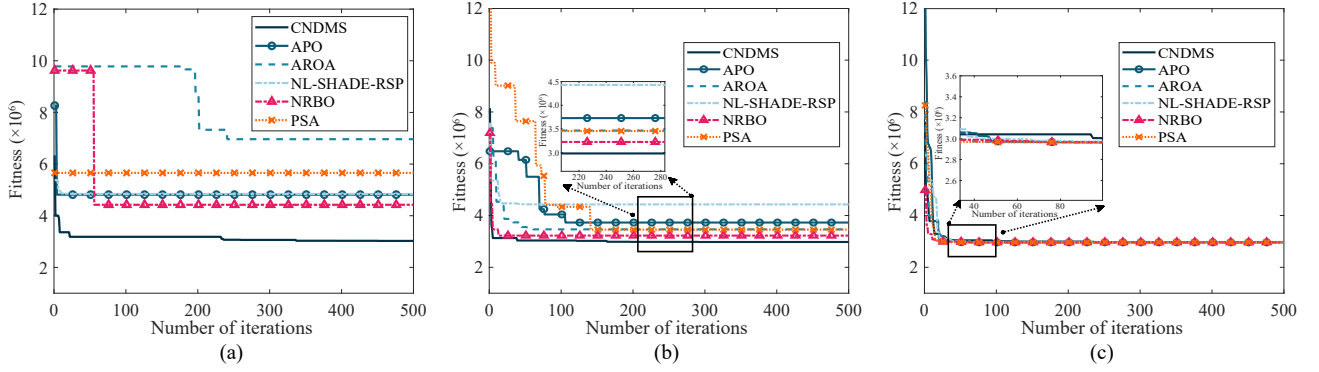


Fig. 4 Convergence changes of different algorithms on the gas transmission compressor design problem (34) under different noise. (a) Linear noise environment. (b) Constant noise environment. (c) Noise-free environment.

Table 3 Statistical results of different algorithms on the gas transmission compressor design problem described by Formula (34) under different noise. The bold values mean that the algorithm achieves the best results under the same noise type.

Algorithm	Index	Gas transmission compressor design problem		
		Linear noise	Constant noise	Noise free
CNDMS	AVG	3.22×10^6	3.18×10^6	3.14×10^6
	BEST	2.98×10^6	2.99×10^6	3.01×10^6
	STD	1.75×10^5	1.72×10^5	9.80×10^4
APO	AVG	4.62×10^6	3.56×10^6	2.97×10^6
	BEST	3.10×10^6	3.20×10^6	2.97×10^6
	STD	8.38×10^5	2.32×10^5	3.76×10^{-2}
AROA	AVG	7.94×10^6	5.69×10^6	2.97×10^6
	BEST	4.29×10^6	3.19×10^6	2.97×10^6
	STD	1.68×10^6	2.60×10^6	2.50×10^{-2}
NL-SHADE-RSP	AVG	6.13×10^6	6.02×10^6	2.97×10^6
	BEST	3.86×10^6	3.44×10^6	2.97×10^6
	STD	1.16×10^6	1.21×10^6	1.76×10^{-1}
NRBO	AVG	5.41×10^6	3.63×10^6	2.97×10^6
	BEST	3.04×10^6	3.22×10^6	2.97×10^6
	STD	1.45×10^6	3.60×10^5	4.43×10^3
PSA	AVG	7.30×10^6	3.64×10^6	2.97×10^6
	BEST	3.52×10^6	3.41×10^6	2.97×10^5
	STD	1.94×10^6	1.24×10^5	1.86×10^3

methods such as APO, AROA, NL-SHADE-RSP, NRBO, and PSA. Under constant noise, the advantage of the CNDMS model remains. Although all methods reach comparable results in the noise-free scenario, AROA achieves the highest solution accuracy. The CNDMS model exhibits the most robust performance across various noise environments, highlighting its adaptability and stability.

5 Conclusion

This paper proposes the CNDMS model for solving

nonconvex optimization problems under complex noise environments. The proposed CNDMS model searches for the local optimal value through a modified neural dynamics model and obtains the global optimal solution through global information interaction. In particular, the modified neural dynamics model integrates a dual-gradient accumulation term to improve local convergence capability and robustness. In addition, to avoid premature convergence and maintain population diversity, the opposition-based learning method and mutation strategies based on Cauchy and t -distribution are incorporated, thereby improving global search performance. Theoretical analyses have demonstrated the convergence and robustness of the proposed CNDMS model, while extensive comparisons with counterpart models on various benchmark functions and engineering design problems have validated its superior convergence accuracy and robustness.

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Declaration of competing interest

The authors have no competing interests to declare that are relevant to the content of this article.

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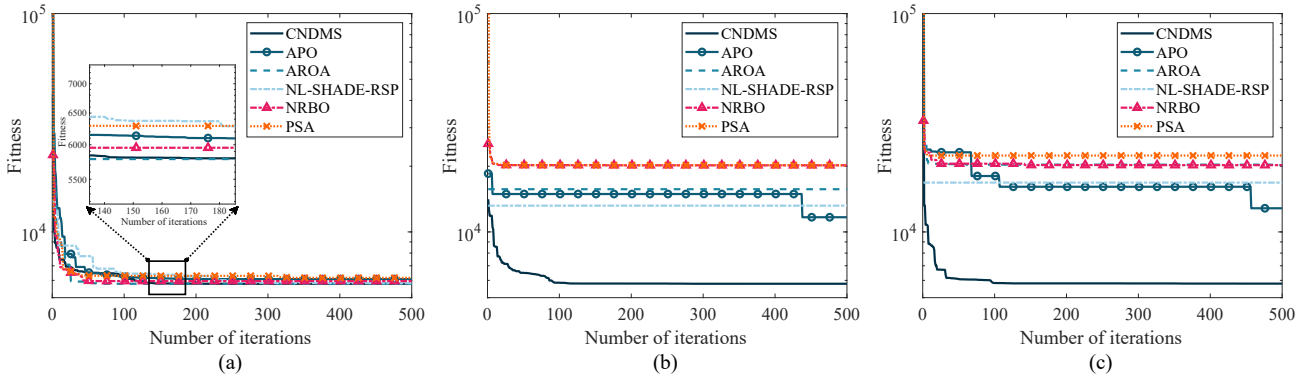


Fig. 5 Convergence changes of different algorithms on the pressure vessel design problem described by Formula (35) under different noise. (a) Linear noise environment. (b) Constant noise environment. (c) Noise-free environment.

Table 4 Statistical results of different algorithms on the pressure vessel design problem described by Formula (35) under different noise.

Algorithm	Index	Pressure vessel design problem		
		Linear noise	Constant noise	Noise free
CNDMS	AVG	5.87×10^3	5.88×10^3	5.84×10^3
	BEST	5.79×10^3	5.79×10^3	5.79×10^3
	STD	1.50×10^2	1.40×10^2	8.02×10^1
APO	AVG	1.28×10^4	1.17×10^4	5.89×10^3
	BEST	8.32×10^3	8.06×10^3	5.79×10^3
	STD	2.51×10^3	2.40×10^3	1.15×10^2
AROA	AVG	1.92×10^4	1.88×10^4	5.79×10^3
	BEST	7.09×10^3	9.64×10^3	5.79×10^3
	STD	3.06×10^3	2.65×10^3	2.22×10^{-10}
NL-SHADE-RSP	AVG	1.41×10^4	1.79×10^4	5.88×10^3
	BEST	8.73×10^3	9.17×10^3	5.81×10^3
	STD	4.70×10^3	8.32×10^3	6.46×10^1
NRBO	AVG	1.91×10^4	1.85×10^4	6.32×10^3
	BEST	7.48×10^3	7.41×10^3	5.79×10^3
	STD	2.79×10^3	3.22×10^3	5.15×10^2
PSA	AVG	2.09×10^4	1.88×10^4	6.14×10^3
	BEST	1.35×10^4	1.03×10^4	5.84×10^3
	STD	1.75×10^3	2.98×10^3	2.43×10^2

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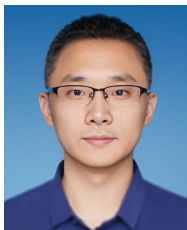
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