

Mechanical properties and fractal damage constitutive model of hole-bearing limestone under conventional compression

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Abstract: The development of underground dissolution leads to the formation of numerous cavities in limestone, thereby affecting the stability of karst mountain. Conventional compression tests were performed on rocks with different hole number and diameters to investigate the peak strength, crack damage behavior, energy characteristics and failure modes under different confining pressures (σ_3) and dissolution rates (η). The results indicate that the uniaxial compressive strength and crack damage stress (σ_{cd}) of hole-bearing limestone are independent of the dissolution rate (η). Under triaxial compression, the peak strength of hole-bearing limestone can be described by a linear Mohr-Coulomb criterion. The cohesion (c) and crack damage ratio (σ_{cd}/σ_p) increase with the increase of dissolution rate (η), with the maximum increases being 14.04% and 14.1%, respectively. The peak strength (σ_p), frictional angle (ϕ), and energy storage coefficient (K_e) decrease with the increase of dissolution rate (η), with the maximum decreases being 14.11%, 12.86%, and 9.72%, respectively. At the same dissolution rate, the pore diameter (d) exerts a more significant influence on the variations of various parameters in hole-bearing limestone than the pore number (n). σ_p , σ_{cd} , σ_{cd}/σ_p , and K_e all increase with increasing confining pressure (σ_3). The failure mode of hole-bearing limestone is not affected by hole number, and exhibits a conical shear failure without penetrating through the specimen when the hole diameter is greater than 15mm. A fractal damage constitutive model was developed for hole-bearing limestone, and the consistency between theoretical predictions and experimental results substantiates the validity of the proposed model. The concluding remarks can provide a basis for further research on the hole-bearing limestone and karst landslides.

Keywords: hole-bearing limestone; fractal dimension; crack damage stress; energy characteristic; constitutive model

1 Introduction

As complex geological bodies, rocks contain many different forms of discontinuities that affect the stability of slopes^[1-5]. Recent studies have analyzed the characteristics of rocks with variously shaped holes. Lin et al.^[6] utilized the discrete element method to investigate the mechanical properties and fracturing patterns of rock samples with two circular holes, and further studied the impact of joint parameters on the rock mass's mechanical behaviors. By applying complex function theory, Tan et al.^[7] investigated the stress distribution in rock masses with intricately shaped holes, improved the calculation method of the mapping function, proposed a new mapping function objective function, and simplified the calculation process. Han et al.^[8] conducted numerical studies on the shear performance of rock-like specimens with cracked holes, and discussed the effects of hole shape, connector angle, and combination on shear deformation and crack behavior. Previous studies have broadly analyzed rocks and rock-like materials with various cavity shapes, but there is a deficiency in research on vertical karst fractures observed in karst landslides.

A multitude of hydrogeological surveys have shown that limestone contains different internal structures due to natural geological processes^[9-11]. Karst mountains are

widely distributed on the Earth's surface, and in recent years, there have been numerous landslide disasters in karst mountains, such as the Pusa landslide^[12-13], Jiweishan landslide^[14-15], Jianshanying landslide^[16], Jiguanling landslide^[17-18], Yanchihe rockslide^[19], Kaiyang Phosphorite Mine rockfalls^[20], Zengziyan cliff^[21], Madaling landslide^[22], Zongling landslide^[23], Baiyan landslide^[24]. Karstification can lead to the expansion of fractures and pores in the rock mass, thereby affecting the mechanical properties of the rock mass. For instance, the landslide mass of Jiweishan Mountain contains a large number of vertical karst conduits (Fig. 1). These features substantially degrade rock-mass strength^[25-27], thereby increasing the instability of the mountain. In the figure, σ_1 denotes the axial compressive stress, and σ_3 denotes the confining pressure.

Scholars have extensively researched the instability and failure mechanisms of fractured rocks through experimental studies^[28-29]. For instance, Guo et al.^[26] discovered that the uniaxial compressive strength and elastic modulus of karst limestone decrease with the increased solubility, and the strain energy dissipation becomes greater. Yin et al.^[30-31] deeply explored the shear behavior of marl under different structural surface dip angles and normal stress conditions, finding that the shear strength and failure mode of the rock are greatly influenced by the fracture dip angle, initial stress level,

and unloading rate. Li et al.^[32] studied the effect of confining pressure and the angle of rock bridges on the fracture response and mechanical characteristics of marble exhibiting parallel fissures under conditions of triaxial compression. Wang et al.^[33–34] employed acoustic emission (AE) method and computed tomography (CT) system to analyze the mechanisms of macroscopic and microscopic fracturing, considering the study of fracture evolution and instability behavior of rock containing fractures and cavities to be of great significance for the safety of underground constructions. Li et al.^[35] disclosed the mechanical properties and failure mode of cavity rocks when subjected to simultaneous static and dynamic stresses. Yang et al.^[36–37] studied the effects of hole diameter and confining pressure on mechanical properties and failure mode of hollow sandstone samples. Dzik et al.^[38], Fakhimi et al.^[39], and Choens et al.^[40] used AE method and uniaxial, biaxial, and other testing methods to explore the effects of cavities or holes on the peak strength, fracture evolution, and energy characteristics of rocks. Energy dissipation is the essence of rock failure, and in view of this, Wang et al.^[41–42] studied the effects of stress amplitude on the fracture and energy evolution of pre-cracked granite under uniaxial incremental fatigue loads and presented an energy-based early-stage concrete chemo-thermo-mechanical model, confirming that the evolution of mechanical damage is driven by the conjugate elastic-plastic damage energy release rate. Chen et al.^[43–44] revealed the fracture propagation properties and energy damage characteristics of carbonaceous slate.

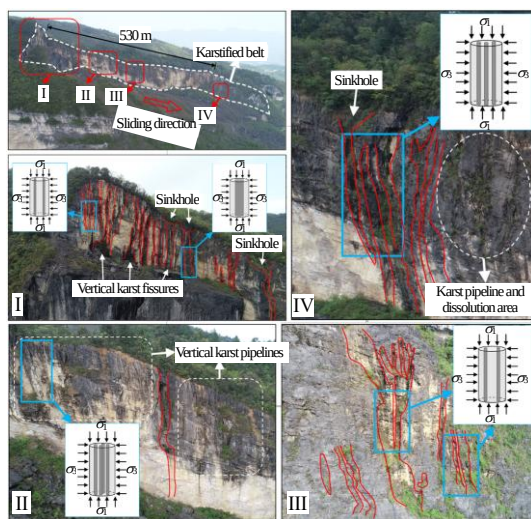


Fig.1 Typical erosion signs and vertical karst fissures of landslide source area (Jiweishan landslide) ^[14]

Although many studies have examined rocks with holes, most focus on idealized cavity geometries or only evaluate peak strength and stress–strain responses. Vertical dissolution conduits in karst slopes generate localized stress concentration and multi-source cracking behaviors that cannot be fully characterized by conventional indicators. In addition, existing damage models rarely integrate fractal crack evolution into a unified constitutive framework, limiting their ability to reflect the coupled

mechanical–energy–fracture process.

To bridge these gaps, this study conducts systematic conventional compression tests on limestone specimens containing vertical axial holes to simulate karst dissolution conduits, quantitatively couples strength parameters, energy storage/dissipation coefficients, and 3D fracture fractal dimensions to reveal a coordinated “mechanical–energy–geometric” failure mechanism, and proposes a reduced-parameter fractal damage constitutive model that explicitly links peak strength and fractal evolution, improving both interpretability and applicability under engineering conditions. This work aims to provide new experimental evidence and theoretical support for understanding failure behaviors of karst limestone and evaluating the instability potential of karst mountain slopes.

2 Experimental methodology

2.1 Engineering background

The karst topography prevalent in China's southwestern mountainous areas features an intricate system of fractures and conduits, which significantly reduce the stability of mountains, making them highly prone to large-scale landslides. Taking the Jiweishan landslide in Wulong, Chongqing as an example (Fig. 1), the exposed limestone shows clear signs of corrosion and karst development, which has led to a major collapse and sliding disaster ^[14–15]. As depicted in Fig. 1, vertical karst conduits erode from the top to the base of the mountain, with the dissolution rate of these pipelines being a critical factor affecting the stability of the mountainous body. In light of such engineering projects, a rock-like specimens containing holes were fabricated to explore the mechanical characteristics of limestone with different hole diameters and hole number.

2.2 Test method

As shown in Fig. 2, the limestone used for the experiment was sourced from Jiweishan in Wulong, Chongqing, located in southwestern China. The limestone samples were all taken from the same block without obvious macroscopic defects, ensuring the consistency of the experiment. In Fig.2, n denotes the hole number, d denotes the hole diameter, η denotes the degree of dissolution.

The specimens are drilled into cylindrical shapes with a diameter of approximately 50 mm and a length of about 100 mm. The hole number (n) is set to vary from 1 to 5. The hole diameter (d) is 6, 9, 12, 15, and 18 mm. To reduce the influence of variability on the results, three replicate samples were employed in each experiment, and the final results were averaged. As shown in Fig. 3, the multifunctional rock triaxial test system (Rock 600-50 HT PLUS) and the rock rheological testing machine (WADJ-600) were employed to carry out triaxial tests and uniaxial test respectively.

In accordance with the variation patterns of stratum stress ^[45], the confining pressure σ_3 in the experiment is applied at 0, 4, 6, and 8 MPa. In uniaxial compression ($\sigma_3 = 0$ MPa), axial displacement was established at a loading rate of 0.02 mm/min in the axial direction until the

specimen was destroyed. In triaxial compression ($\sigma_3 = 4, 6, 8$ MPa), the confining pressure was loaded to the predetermined value at a rate of 3 MPa/min, and then the axial displacement was loaded at a rate of 0.02mm/min until specimen failure.

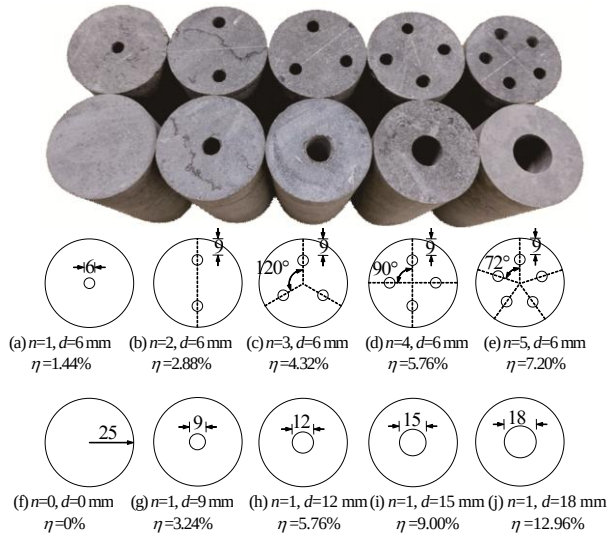
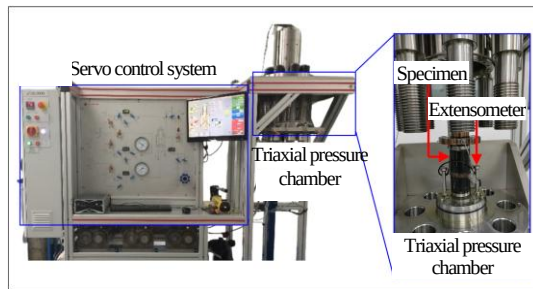


Fig.2 Intact specimen and specimen with axial fissures



(a) Rock 600-50 HT PLUS multifunctional rock triaxial test system



(b) The rock rheological testing machine (WADJ-600)

Fig.3 Experimental apparatus for triaxial and uniaxial compression

As shown in Fig. 4, the 3-D fracture networks of the failed hole-bearing limestone specimens were detected by the SIEMENS-SOMATOM oscilloscope X-ray CT scanning system. The parameters of the X-ray CT scan are as detailed: the voltage from the source was set at 130 kV, the current emitted by the source was 80 μ A, the distance between the object and the source was 200 mm, the distance from the detector to the source was 500 mm, and the angle of rotation was 0.18° . In the present experiments, the slice interval thickness was 0.75 mm, and 134 slices were obtained for each fractured sample. Additionally, each X-ray CT image was 512×512 pixels in size. CT

image segmentation and ROI extraction were conducted using Avizo. To ensure reproducibility and avoid subjective interference, a standardized segmentation workflow was adopted. First, the Otsu global thresholding algorithm embedded in Avizo was applied to determine the initial grayscale separation between the pore regions and the rock matrix. The threshold value was then slightly adjusted based on the valley point of the grayscale histogram to avoid over-segmentation of micro-defects. For quantitative analysis, a fixed region of interest (ROI) with a size of 141×141 pixels was centrally cropped from each CT slice to maintain statistical consistency across all specimens. The same ROI extraction parameters were applied to every sample to ensure comparability of the reconstructed 3D fracture networks.



(a) Scanning device

(b) Control console

Fig.4 3-D X-ray computed tomography (CT) scanning system

To enhance practical application in engineering contexts, a correlation was established linking the dissolution rate to the number and diameter of holes. The dissolution rate (η) is calculated as the proportion of the hole volume relative to the total sample volume.

Table 1 illustrates the dissolution rates for rock samples with varying numbers and diameters of holes. The volume of holes V_h :

Table 1 The dissolution coefficients (η) of hole-bearing limestone with different hole numbers and diameters

n	d/mm	$\eta/\%$
5	6	7.20
4	6	5.76
3	6	4.32
2	6	2.88
1	6	1.44
0	0	0.00
1	9	3.24
1	12	5.76
1	15	9.00
1	18	12.96

$$V_h = \frac{\pi n H d^2}{4} \quad (1)$$

The volume of the sample V_s :

$$V_s = \frac{\pi H D^2}{4} \quad (2)$$

Dissolution rate η :

$$\eta = \frac{V_h}{V_s} = n \frac{d^2}{D^2} \quad (3)$$

where D represents the diameters of the sample; and H denotes the height of rock sample.

3 Experimental results

3.1 Stress-strain behavior

Fig. 5 and Fig. 6 respectively display the stress-strain curves of hole-bearing limestone with different hole number and hole diameters. σ_1 represents the principal stress ε_1 and ε_3 represent the axial strain and radial strain, respectively. The positive strain corresponds to compressive deformation, and negative strain corresponds to expansive deformation. Fig. 5 (a) and Fig. 6 (a) illustrate that, during uniaxial compression testing, the hole-bearing limestone exhibits a swift decline in axial load-bearing capacity after peak strength. This behavior indicates that the limestone samples with holes display pronounced

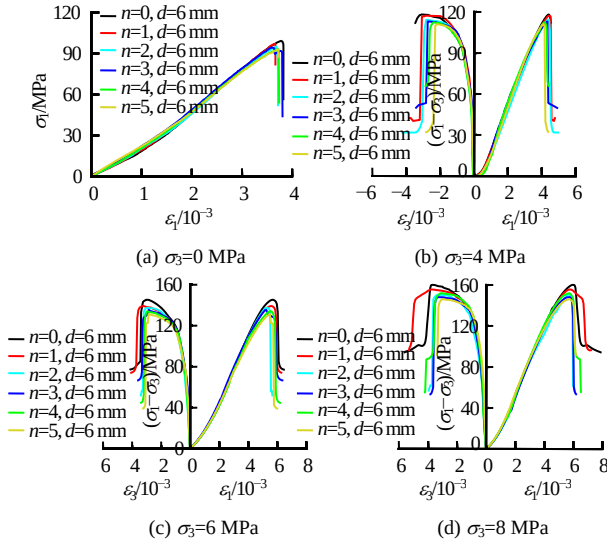


Fig.5 Stress-strain curves of hole-bearing limestone with different hole numbers

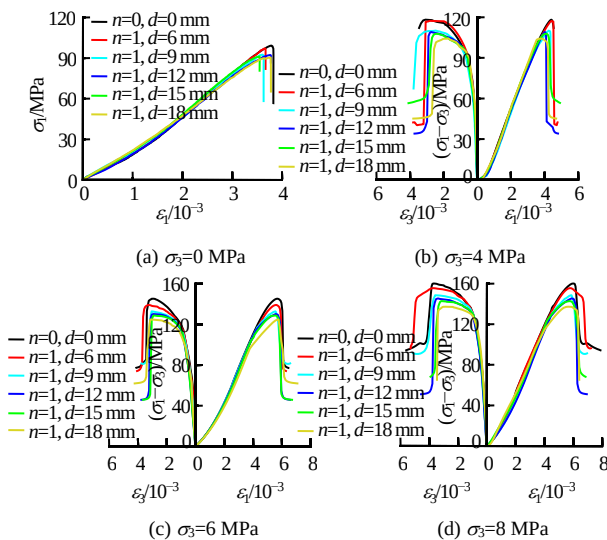


Fig.6 Stress-strain curves of hole-bearing limestone with different hole diameters

brittleness, and fail through an axial splitting mechanism. It can be observed that before reaching the peak strength, the axial holes have a small impact on the deformation behavior of limestone samples. After peak strength, the specimens display a predominantly brittle deformation behavior that is not significantly associated with the number or diameter of the holes. The peak strengths under different dissolution rates are listed in Fig. 7.

To deeper analyze the influence of different factors on peak strength, a multiple linear regression model was established for peak strength. In this model, n , $(d/D)^2$, and σ_3 were regarded as independent variables, while σ_p was considered as the dependent variable. The model formula is:

$$\sigma_p = 98.84 - 1.036n - 89.676(d/D)^2 + 6.74\sigma_3 \quad (4)$$

The correlation coefficient R^2 is 0.952, indicating a good model fit. The regression coefficients for n , $(d/D)^2$, and σ_3 are -1.036 , -89.676 , and 6.740 , respectively. This means that the hole number (n) exerts an insignificant effect on σ_p , $(d/D)^2$ imparts a significant negative change to σ_p , and confining pressure (σ_3) has a considerable positive impact on σ_p . Furthermore, Fig. 7 shows that under uniaxial compression, the peak strength is not significantly affected by the hole number and diameters ($\sigma_p = 4.1$ MPa), with a maximum variation of 4.14%, the maximum reduction in peak strength is 14.11% when σ_3 is 4, 6, 8 MPa. In Fig.7 $\Delta\sigma_p$ denotes the range of peak strength of specimens under the same confining pressure; $\sigma_{p, \max}$, $\sigma_{p, \min}$ represent the maximum peak strength and minimum peak strength of the tested specimens under the identical confining pressure, respectively.

The impact of σ_3 on σ_p of hole-bearing limestone was studied using the linear Mohr-Coulomb criterion, which can be represented by the following equation^[46]:

$$\sigma_p = \frac{2c \cos \phi + \sigma_3 (1 + \sin \phi)}{1 - \sin \phi} \quad (5)$$

where c is cohesion and ϕ is frictional angle.

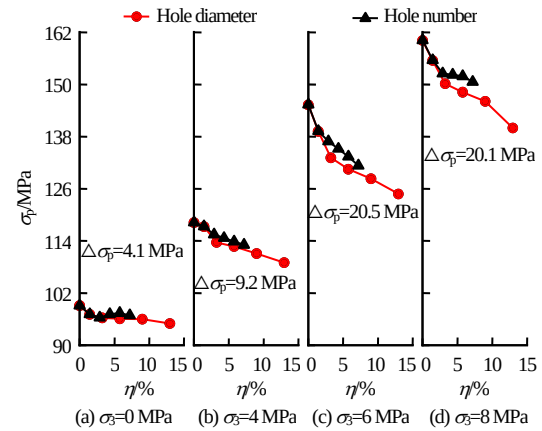


Fig.7 Peak strength of hole-bearing limestone with different dissolution coefficients ($\Delta\sigma_p = \sigma_{p, \max} - \sigma_{p, \min}$)

As shown in Fig. 8, the correlation coefficients R^2 are larger than 0.9, indicating that the peak strength of hole-bearing limestone satisfies the Mohr-Coulomb law.

As shown in Fig. 9, when the hole diameter (d) ranges from 0 mm to 18 mm, the frictional angle (ϕ) decreases from 50.62° to 44.11° , a reduction of 12.86%. The cohesion (c) increases from 17.09 MPa to 19.49 MPa, an increase of 14.04%. With different numbers of holes, as the hole number increases from $n=0$ to $n=5$, the frictional angle (ϕ) decreases from 50.62° to 47.49° , a reduction of 6.28%, and the cohesion (c) increases from 17.09 MPa to 18.12 MPa, an increase of 6.03%.

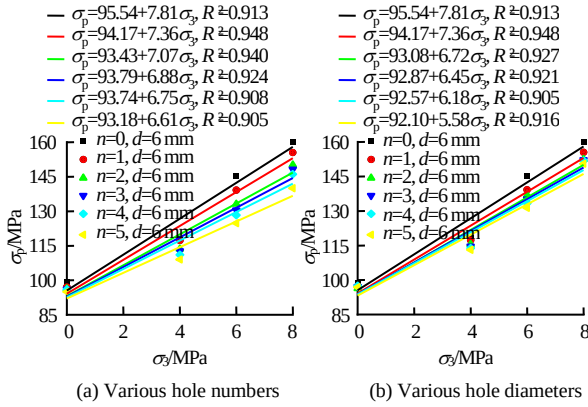


Fig.8 Effects of confining pressure on peak strength of hole-bearing limestone by the linear Mohr-Coulomb criterion

In summary, it can be observed that under the same dissolution rate (η), the hole diameter (d) has a greater impact on the changes in peak strength (σ_p), frictional angle (ϕ), and cohesion (c) than the hole number (n). This is because holes with larger diameters have a more significant effect on stress distribution than those with smaller diameters, as they provide more space for crack initiation and propagation, leading to a greater propensity for rock failure. It is worth noting that under the same dissolution rate (η), the dominant influence of pore diameter (d) compared to pore number (n) is governed by stress concentration rather than merely the geometric void ratio. A larger single pore generates a higher localized stress concentration factor around its boundary, leading to the formation of an expanded tensile-shear disturbance zone that facilitates earlier crack initiation and directional propagation. In contrast, multiple small pores disperse the weakening effect spatially, resulting in a lower peak local stress gradient and a less dominant crack path control. Consequently, specimens with larger d exhibit a more pronounced reduction in σ_p , ϕ and c .

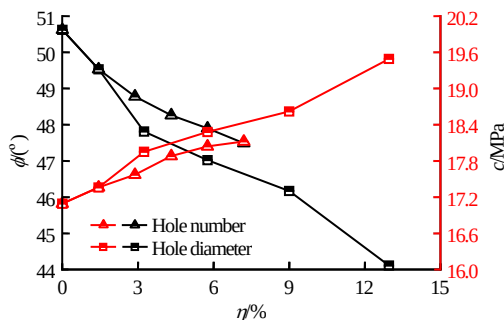


Fig.9 Effect of dissolution coefficients on the cohesion and internal friction angle of hole-bearing limestone

3.2 Crack damage behavior

Fig. 10 illustrates the volumetric strain curve in the hole-bearing limestone. The relationship between volumetric strain (ε_v), axial strain (ε_1), and circumferential strain (ε_3) is $\varepsilon_v = \varepsilon_1 + 2\varepsilon_3$. The characteristic of volumetric strain is an initial stage dominated by compaction behavior, followed by a stage dominated by dilation behavior. The process of rock compression is usually divided into five stages, defined by five stress points, namely the initial stress (σ_0), crack closure stress (σ_{cc}), crack initiation stress (σ_{ci}), crack damage stress (σ_{cd}), and peak stress (σ_p). The crack damage stress (σ_{cd}) of the specimen is defined as the corresponding axial deviatoric stress at which the specimen's volumetric deformation transitions from compaction-dominated to dilation-dominated [47-48]. In the stable crack growth stage ($\sigma_{ci} < \sigma \leq \sigma_{cd}$), new stable-growing cracks begin to form within the rock. In the initiation of macro-scale shear failure stage ($\sigma_{cd} < \sigma \leq \sigma_p$), the internal cracks in the rock start to expand disorderly, indicating the imminent onset of macroscopic shear failure [46, 49-50]. In the volumetric strain curve, when the compaction rate and the shear dilation rate become equal, an inflection point occurs in the volumetric strain curve, and the stress corresponding to the inflection point is σ_{cd} .

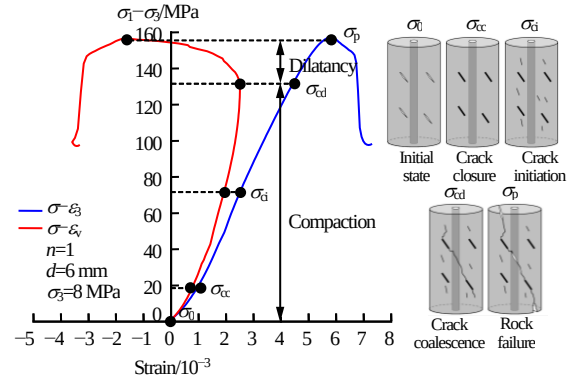


Fig.10 Crack damage stress and stress-volumetric strain curves of tested limestone

As shown in Fig. 11, there is no clear pattern of change in crack damage stress (σ_{cd}) under different dissolution rates (η). The crack damage ratio (σ_{cd} / σ_p) increases as η increases. At the same dissolution rate, the hole diameter has a greater enhancing effect on σ_{cd} / σ_p than the hole number, with the maximum increase being 14.1%. Fig. 12 shows that σ_{cd} / σ_p gradually increases with confining pressure.

A multiple linear regression analysis was performed on crack damage stress (σ_{cd}), and the following formula was obtained:

$$\sigma_{cd} = 55.955 + 0.15n + 7.683(d/D)^2 + 9.123\sigma_3 \quad (6)$$

The correlation coefficient R^2 is 0.993, indicating a good model fit. The regression coefficients for the hole number (n), $(d/D)^2$, and confining pressure (σ_3) are 0.150, 7.683, and 9.123, respectively. This means that the hole number (n) and $(d/D)^2$ do not have a significant

impact on σ_{cd} , while confining pressure (σ_3) has a significant positive impact on σ_{cd} .

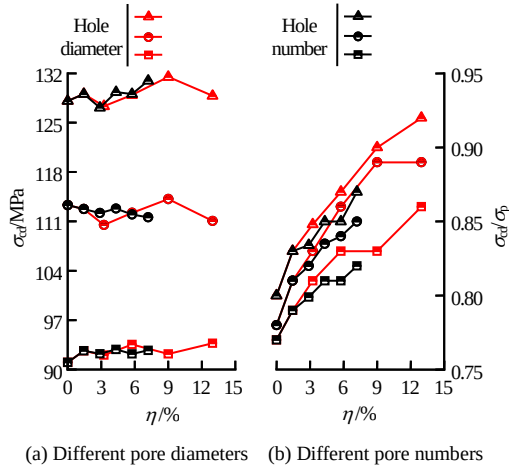


Fig.11 Crack damage stress (σ_{cd}) and crack damage ratio (σ_{cd} / σ_p) of hole-bearing limestone with different dissolution coefficients

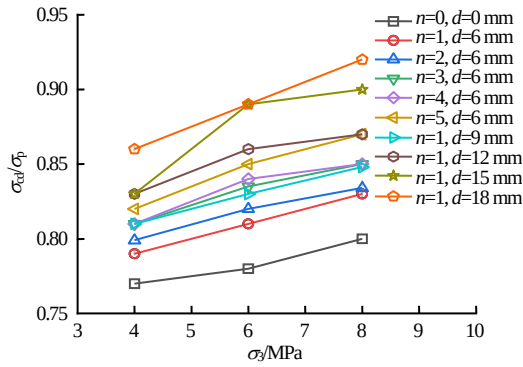


Fig.12 Crack damage ratio (σ_{cd} / σ_p) of hole-bearing limestone under different confining pressures.

The multiple linear regression result of crack damage ratio (σ_{cd} / σ_p) is:

$$\sigma_{cd} / \sigma_p = 0.724 + 0.009n + 0.763(d/D)^2 + 0.011\sigma_3 \quad (7)$$

The correlation coefficient R^2 is 0.941, indicating a good model fit. The regression coefficients for the hole number (n), $(d/D)^2$, and confining pressure (σ_3) are 0.009, 0.763, and 0.011, respectively. This means that the hole number (n), $(d/D)^2$, and confining pressure (σ_3) all have a significant positive impact on σ_{cd} / σ_p .

3.3 Nonlinear characteristic of energy

The total absorbed energy U under conventional compression can be formulated as [51]:

$$U = U_1 + U_3 = U_e + U_d \quad (8)$$

where U_1 and U_3 correspond to the energy contributions from axial stress and confining pressure, respectively. U_e and U_d correspond to the elastic strain energy and the dissipated energy, respectively. The

expressions for U_1 and U_3 are detailed as follows[51]:

$$\begin{aligned} U_1 &= \int_0^{\varepsilon_1^t} \sigma_1 d\varepsilon_1 = \sum_{i=1}^j \frac{1}{2} (\sigma_1^i + \sigma_1^{i+1}) (\varepsilon_1^{i+1} - \varepsilon_1^i) \\ U_3 &= 2 \int_0^{\varepsilon_3^t} \sigma_3 d\varepsilon_3 = \sum_{i=1}^j (\sigma_3^i + \sigma_3^{i+1}) (\varepsilon_3^{i+1} - \varepsilon_3^i) \end{aligned} \quad (9)$$

where j signifies the number of divisions on the stress-strain graph, i marks the dividing location, ε_1^t and ε_3^t are the axial strain and circumferential strain, respectively.

The elastic strain energy U_e under conventional compression is presented as [52]:

$$U_e = \frac{1}{2E} [\sigma_1^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_3 + \sigma_3^2)] \quad (10)$$

where E and μ denotes the elastic module and Poisson's ratio during the linear elastic stage, respectively. Given $\sigma_3 = 0$ MPa in Eqs. (4) and (5), the total strain and elastic strain energies under uniaxial compression are determinable.

When $\sigma_3 = 6$ MPa, the U , U_e and U_d of the selected specimens are shown in Fig. 13. It is evident that larger hole quantities and hole diameters lead to higher dissipated energy and lower elastic strain energy. The capability of rocks to store elastic energy is a key factor of further damage. Therefore, the study of the ability to stored elastic energy helps to assess the stability of rocks under compression. To delve into the capacity of energy storage under varying confining pressures, $K_e = U_e / U$ and the energy dissipation coefficient, $K_d = U_d / U$ have been delineated for a rigorous analysis of this problem[53]. Obviously, $K_d = 1 - K_e$. As illustrated in Fig. 14 and Fig. 15, referring to relevant literature[54], K_e at 85% of the peak stress value was selected to analyze the energy storage capacity of limestone.

From Fig. 14, it can be observed that as the hole diameter and the hole number increase, the energy storage coefficient (K_e) shows a decreasing trend, indicating that the axial holes reduce the energy storage capacity, with the maximum reduction being 9.72%. Furthermore, at the same dissolution rate, the hole diameter has a greater impact on K_e than the hole number, causing a greater reduction in K_e . In Fig.14, ΔK_e denotes the range of energy storage coefficient for multiple parallel specimens under the same confining pressure; $K_{e, \max}$, $K_{e, \min}$ represent the maximum and minimum values of the energy storage coefficient under the identical confining pressure respectively. Fig. 15 shows that K_e increases with the increase of confining pressure, but the ranking of K_e values remains unchanged under different confining pressures.

The multiple linear regression result of energy storage coefficient (K_e) is expressed as

$$K_e = 0.922 - 0.004n - 0.549(d/D)^2 + 0.004\sigma_3 \quad (11)$$

The correlation coefficient R^2 is 0.950, indicating a good model fit. The regression coefficients for the hole number (n), $(d/D)^2$, and confining pressure (σ_3) are -0.004 , -0.549 , and 0.004 , respectively. This means that

the hole number (n) and $(d/D)^2$ have a significant negative impact on K_e , while confining pressure (σ_3) has a significant positive impact on K_e .

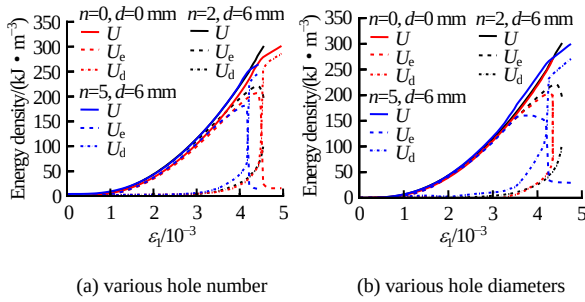


Fig.13 Energy evolution process of hole-bearing limestone specimens under conventional compression

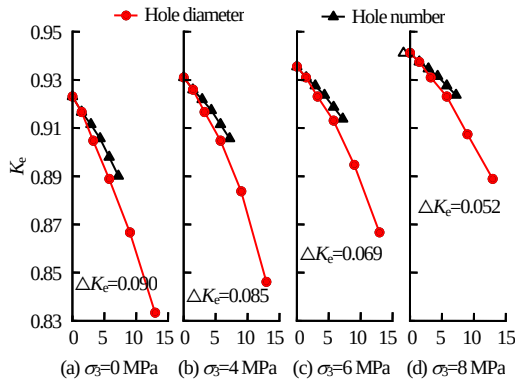


Fig.14 Energy storage coefficients (K_e) of hole-bearing limestone with different dissolution coefficients ($\Delta K_e = K_{e,max} - K_{e,min}$)

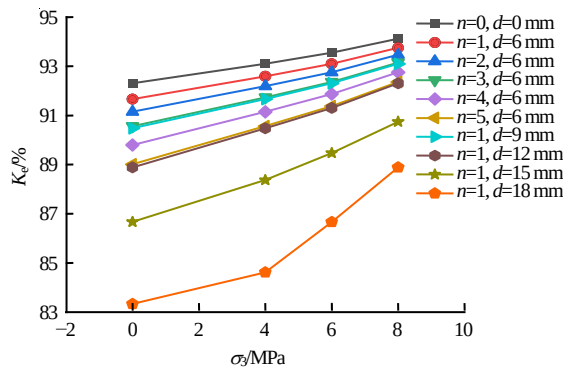


Fig.15 Energy storage coefficients (K_e) of hole-bearing limestone under different confining pressures.

3.4 Failure modes and fractal characteristics

As shown in Fig. 16, referring to relevant studies^[55], to distinguish the failure modes of the rock samples in this paper, a classification of cracks is defined based on the geometric shape and propagation pattern, including tensile crack (T_c), shear crack (S_c), secondary tensile crack (S_{tc}), secondary shear crack (S_{sc}), far-field tensile crack (F_t), far-field shear crack (F_s), surface spalling (S_p). In addition, the rupture angle (α) is defined based on the shear crack (S_c).

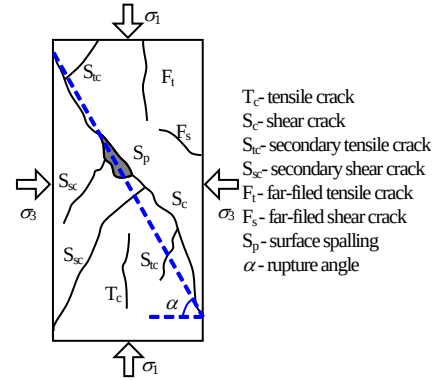


Fig.16 Classification of cracks for hole-bearing limestone samples under conventional compression^[55]

The failure modes of rock samples under confining pressures of 0, 4, 6, and 8 MPa are basically the same, so the characteristics when $\sigma_3=8$ MPa are presented. Fig. 17 shows the variation law of the rupture angle (α) and dissolution rate (η). It can be seen that the rupture angle decreases with the increase of dissolution rate. The rupture angle of intact specimen ($\eta=0$) reaches its maximum, approaching 80° . The influence of hole diameter on rupture angle is greater than the hole number.

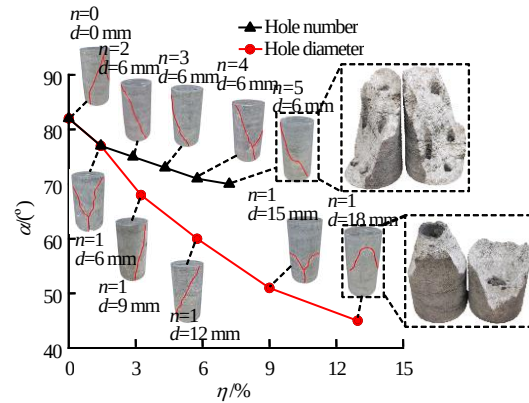


Fig.17 Rupture angle and failure mode of hole-bearing limestone with different dissolution coefficients ($\sigma_3=8$ MPa)

Fig. 18 illustrates the ultimate breakdown patterns and 3-D fracture networks of hole-bearing limestone under $\sigma_3=8$ MPa. From Fig. 18(a)–18(f), it can be observed that there is no significant change in the rock failure characteristics with the increase in hole number, with the shear fractures that penetrate through the top and bottom of the specimen being predominant. The crack propagation of rock samples with different hole number mainly exhibits shear failure through the specimens, resulting in surface spalling and fragmentation. During compression, the tip of the initial crack continuously nucleates and propagates to form the primary crack. Subsequently, multiple secondary cracks form along the direction of the primary crack, leading to macroscopic fracture and failure under external loads. From Fig. 18(g)–18(h), it can be seen that when the hole diameter is less than 12mm, the shear failure plane penetrates through the

top and bottom of the specimen. When the hole diameter is greater than 15 mm, the specimen exhibits a conical shear failure without penetrating through the specimen. When the hole diameter increases, the propagation path of cracks becomes more complex, and secondary tensile cracks decrease. This is because larger hole provides more space for crack propagation, thereby affecting the propagation path and ultimate shape of cracks.

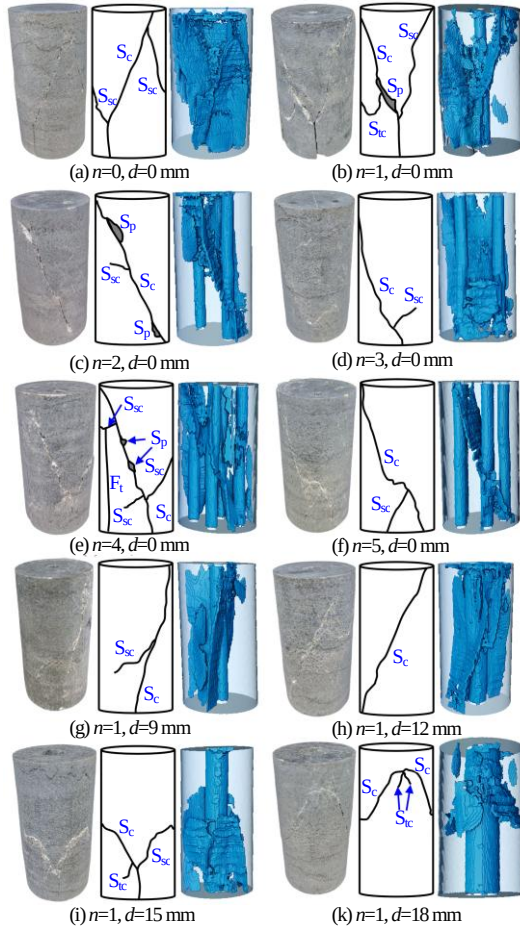


Fig.18 The ultimate failure patterns of hole-bearing limestone under compression ($\sigma_3 = 8\text{MPa}$).

By comparing the energy storage coefficient K_e with the final rupture geometry, it can be observed that specimens exhibiting lower K_e values tend to develop more tortuous crack paths with smaller rupture angles. This indicates that insufficient elastic energy accumulation leads to a dispersed, multi-source propagation mode, which is more energy dissipative and corresponds to the conical or non-penetrative fracture morphology in Fig. 18. Therefore, the transition in failure mode is essentially an energy-driven process, where the redistribution of elastic and dissipated energy governs the geometric evolution of fracture surfaces.

Additionally, Fig. 17 shows the fracture surface morphology of the shear failure surface of hole-bearing limestone. When $n=5$ and $d=6$ mm, the rupture angle is large, the shear fracture surface is rough and undulating, accompanied by a large amount of debris detachment. When $n=1$ and $d=18$ mm, the rupture angle is smaller, the sample was not penetrated and exhibited conical shear

failure, with only a few debris distributed.

Fractal dimension is commonly used to describe the complex internal structures of objects. The box-counting method has gained widespread recognition among scholars due to its stability, accuracy, and simplicity in calculating fractal dimensions^[56]. Different scales $\delta = L_0/2^{n+1}$, corresponding to square regions of various sizes, reflect different numbers of fracture lines N_δ (where L_0 is the initial statistical scale). By using a log-log plot, the linear fit's slope represents the two-dimensional fractal dimension D_e , with the calculation formula given as:

$$D_e = \lim_{\delta \rightarrow 0} \frac{\lg N_\delta}{-\lg \delta} \quad (12)$$

Based on the definition and calculation method of capacity dimension in fractal geometry, the 3D fractal capacity dimension D_s for fracture surfaces is obtained as follows:

$$D_s = -\lim_{\delta \rightarrow 0} \frac{\lg N_\delta}{\lg \delta} \quad (13)$$

where N_δ represents the number of fracture surfaces at the corresponding scale. Unlike the two-dimensional case, where squares are used, in three dimensions, cubes with edge length δ are used to cover the fracture surfaces. The number of fracture surfaces within this region with an area greater than 2δ is counted. The range for the two-dimensional fractal dimension D is^[57], while the 3D fractal dimension D_s ranges from^[58]. The calculated three-dimensional fractal dimension D_s varies with the number of holes as shown in Fig. 19.

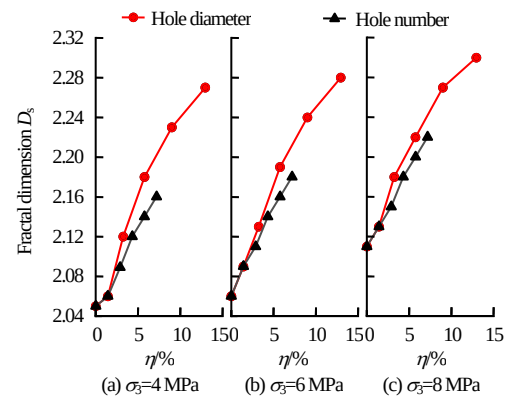


Fig.19 Fractal dimension of hole-bearing limestone under different dissipation rates and confining pressures.

According to Fig. 19, the fractal dimension (D_s) increases as the dissipation rate (η) increases, and an increase in hole diameter (d) is more likely to lead to an increase in fractal dimension compared to an increase in the hole number (n). A higher fractal dimension indicates that fracture paths exhibit stronger self-similarity and structural refinement, directly reflecting increased complexity in fracture morphology. This phenomenon arises because the growth in hole number and hole diameter generates additional stress concentration zones, forcing fracture propagation paths to adopt more tortuous and irregular trajectories, ultimately forming

microstructural characteristics with multiscale hierarchical details. Furthermore, the multiple linear regression result of fractal dimension (D_s) is:

$$D_s = 1.954 + 0.023n + 1.8(d/D)^2 + 0.016\sigma_3 \quad (14)$$

The correlation coefficient R^2 is 0.960, indicating a good model fit. The regression coefficients for the hole number (n), $(d/D)^2$, and confining pressure (σ_3) are 0.023, 1.8, and 0.016, respectively. This means that the hole number (n), $(d/D)^2$, and confining pressure (σ_3) have significant positive impacts on D_s .

4 Fractal damage constitutive model

When rock is subjected to external stress, microcracks randomly initiate from initial defects, propagate, and gradually coalesce, leading to a decline in rock strength and stiffness. The damage evolution is dominated by irreversible microstructural changes and exhibits multi-scale fractal characteristics. The distribution and propagation paths of microcracks are influenced by material heterogeneity. If the rock is divided into statistical unit elements, its damage degree can be quantitatively characterized using fractal dimension. Therefore, fractal damage mechanics theory can describe the cumulative effects of cracks within a unit: the isotropic damage variable is defined by the evolution rate of fractal dimension (reflecting crack complexity), while the anisotropic damage tensor is constructed through statistical analysis of crack orientations. Based on a thermodynamic framework, the constitutive model is realized through the degradation of the damage-elastic tensor^[59]:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda \\ \lambda & \lambda + 2\mu & \lambda \\ \lambda & \lambda & \lambda + 2\mu \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} \quad (15)$$

where λ and μ are the Lamé constants, whose relationships with the elastic modulus (E) and Poisson's ratio (ν) are expressed as:

$$\left. \begin{aligned} \lambda &= \frac{Ev}{(1+\nu)(1-2\nu)} \\ \mu &= \frac{E}{2(1+\nu)} \end{aligned} \right\} \quad (16)$$

In conventional triaxial test ($\sigma_2 = \sigma_3$), the result can be derived according to Eq. (14):

$$\sigma_1 = E\varepsilon_1 + 2\nu\sigma_3 \quad (17)$$

The relationship between damage variables, stress, and fractal dimension is defined as^[59]:

$$\xi = \frac{\sigma_1 - \sigma_0}{D_s} \frac{D_s - D_0}{\sigma_p - \sigma_0} \quad (18)$$

where ξ is damage rate, $\sigma_0 \leq \sigma_1 \leq \sigma_p$, σ_0 and D_0 are the axial stress and fractal dimension of rock at the onset of compressive loading, respectively.

The elastic modulus (E) and Poisson's ratio (ν) after holed rock damage can be written as:

$$\left. \begin{aligned} E &= \sum_{i=0}^{\infty} a_i \xi^i E_0 \\ \nu &= \sum_{i=0}^{\infty} b_i \xi^i \nu_0 \end{aligned} \right\} \quad (19)$$

where $a_0 = b_0 = 1$; E_0 and ν_0 are elastic modulus and Poisson's ratio of intact limestone without damage, a_i and b_i are material parameters reflecting damage.

Zhao^[59] recommends neglecting the third and higher order terms of ϕ . Related research^[36] indicates that the Poisson's ratio of holed rocks is minimally affected. Therefore, in this study, the elastic modulus (E) and Poisson's ratio (ν) are retained up to a linear term. The equation can be rewritten as:

$$\left. \begin{aligned} E &= (1 + a_1 \xi + a_2 \xi^2) E_0 \\ \nu &= (1 + b_1 \xi) \nu_0 \end{aligned} \right\} \quad (20)$$

Therefore, the constitutive model can be expressed as:

$$\sigma_1 = (1 + a_1 \phi + a_2 \phi^2) E_0 \varepsilon_1 + 2(1 + b_1 \phi) \nu_0 \sigma_3 \quad (21)$$

According to Eq. (13), when $n = 0$, $d = 0$ mm and $\sigma_3 = 0$ MPa, D_0 is adopted as 1.954. According to the stress-strain curves in Fig. 5 and Fig. 6, the values of E_0 and ν_0 were determined to be 30 GPa and 0.2, respectively. The peak strength (σ_p) and fractal dimension (D_s) of other working conditions are calculated by Eq. (4) and Eq. (13), respectively. Combining Eq. (18) and (21), the following equation can be obtained:

$$\xi_1 (\sigma_1)^2 + 2\xi_2 \sigma_1 + \xi_3 = 0 \quad (22)$$

where $\xi_1 = a_2 E_0 \varepsilon_1 k^2$, $k = (D_s - D_0) / (D_s \sigma_p)$, $D_s = 1.954 + 0.023n + 1.8(d/D)^2 + 0.016\sigma_3$, $\sigma_p = 98.840 - 1.036n - 89.676(d/D)^2 + 6.74\sigma_3$, $\xi_2 = 2b_1 \nu_0 \sigma_3 k + a_1 E_0 \varepsilon_1 k - 1$, $\xi_3 = E_0 \varepsilon_1 + 2\nu_0 \sigma_3$.

Therefore, based on Eq. (22), the following solutions can be obtained:

$$\sigma_1 = \frac{-\xi_2 \pm \sqrt{\xi_2^2 - 4\xi_1 \xi_3}}{\xi_1} \quad (23)$$

Obviously, since the relevant variables satisfy physical constraints, the solution to Eq. (22) exists only under specific parameter conditions, under which the fractal-damage constitutive model yields an analytical solution. As demonstrated in Eq. (21), the constitutive equation adopts a hyperbolic form. Meanwhile, Eqs. (4) and (14) further reveal that parameter k incorporates both fractal dimension (D_s) and peak strength (σ_p), thereby explicitly accounting for both hole diameter (d) and hole number (n). Additionally, it should be noted that the constitutive model in this study is similar to that reported by Zhao^[59], used for predicting mechanical behavior before peak strength. Compared to the research by Zhao^[59], the number of model parameters has been reduced from four (a_1 , a_2 , b_1 , b_2) to three (a_1 , a_2 , b_1).

The triaxial test results were utilized to validate the proposed constitutive model. As shown in Fig. 20, the model curves show well agreement with the experimental results, with $R^2 \geq 0.99$. As shown in Fig. 21, the model parameters (a_1 and b_1) under different pore diameters exhibit good consistency, while those under varying hole number show a certain degree of scatter ($\sigma_3 = 4$ MPa and $\sigma_3 = 6$ MPa), yet they generally display an overall decreasing trend. Totally, the proposed model effectively

captures the mechanical behavior of hole-bearing limestone under different dissolution rates and confining pressures.

5 Conclusions

This study investigated the peak strength, crack damage behavior, and energy characteristics of hole-bearing limestone with different hole diameters and numbers through conventional compression tests. A fractal damage constitutive model for hole-bearing limestone was established. The main conclusions are drawn as follows:

(1) The uniaxial compressive strength of hole-bearing limestone is not significantly affected by the hole number and diameters. The peak strength (σ_p) of hole-bearing limestone can be characterized by the linear Mohr-Coulomb criterion. As the equivalent dissolution rate (η) rises, the maximum reduction in internal friction angle (ϕ) reaches 12.86%, while cohesion (c) increases by 14.04%. Under the same dissolution rate, the hole diameter has a greater impact on the changes in σ_p , ϕ , and c than the hole number.

(2) Crack damage stress (σ_{cd}) is independent of the hole number (n) and hole diameter (d), but increases with increasing confining pressure (σ_3). The crack damage ratio (σ_{cd}/σ_p) increases with the increase in dissolution rate (η) and confining pressure (σ_3), with the maximum increase being 14.1%. Under the same dissolution rate (η), the hole diameter has a greater impact on the variation of σ_{cd}/σ_p than the hole number.

(3) The energy storage coefficient (K_e) decreases with increasing dissolution rate (η), with the maximum reduction being 9.72%, and increases with increasing confining pressure (σ_3). Under the same dissolution rate, the hole diameter has a greater impact on the variation of K_e than the hole number.

(4) The rupture angle decreases (α) with increased dissolution rate (η) and is more affected by hole diameter (d). Rock samples with different hole diameters exhibit conical shear failure when $d \geq 15$ mm. Rock samples with different hole number still exhibit shear cracks that penetrate the specimen. Under the same dissolution rate, the hole diameter has a greater impact on the changes in fractal dimension (D_s) than the hole number. Good agreement between theoretical predictions and experimental measurements verifies the validity of the proposed fractal-damage constitutive model.

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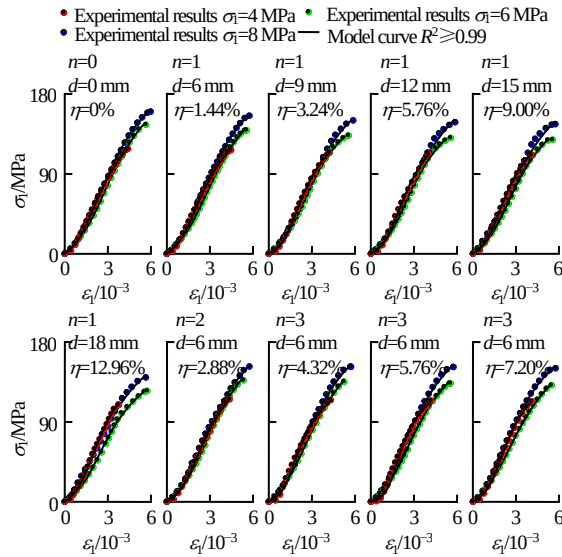


Fig.20 Comparison of stress-strain curves of hole-bearing limestone under triaxial compression obtained from the experiment and the proposed models

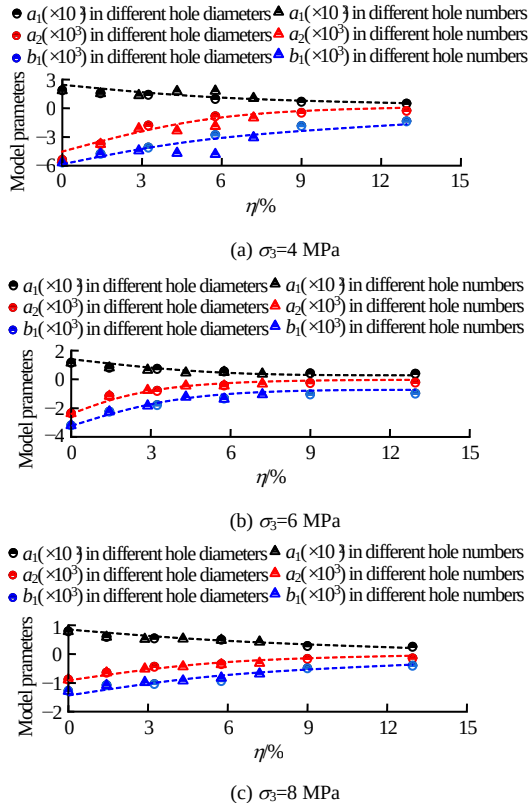


Fig.21 Model parameters of hole-bearing limestone under different dissolution rates and confining pressures

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