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ANALYSIS AND EXPERIMENTAL STUDY ON THE JUMPING CHAIN

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Abstract A freely falling chain from a cup at a certain height can jump. The process can be divided into two parts: a stable suspension and an accelerating procedure. Variational principle and force analysis demonstrate that the shape of the stable suspension is an inverted catenary. The requirement of the jumping and the parameters to describe the jumping catenary have been studied in detail, and experiments have been conducted to verify the theoretical analysis. The physical picture of the falling chain could be useful in certain falling systems, providing valuable insights into the dynamical system.

Key words jumping chain; catenary; classical mechanics

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1 Introduction

There is a very interesting physical phenomenon as shown in Fig. 1: a coiled but not entangled chain is put in a cup. When the end of the chain is pulled out of the cup, it will fall freely along the wall of the cup. At a certain point, usually in a sudden moment, the chain can jump up and remain in a steady arc shape which can last for a certain duration in the air. This experiment is often demonstrated in college physics classes as an example of an intriguing classical mechanical problem. The reader can also observe the details of the jumping in the supplementary video or on the website[1]. In this paper, we attempt to analyze the pattern and related topics such as the height of the jump and the height of the fall, etc. Experiments will be conducted and the data will be analyzed to verify our theoretical analysis[2].



Fig. 1 The jumping chain

In fact, the phenomenon is an application of classical mechanics in string, rope or fluid physics[3]. The jumping rope is a familiar example of a slender structure that interacts with the fluid through which it moves[4]. The chain is usually

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suspended and swung by a mechanical apparatus, and the pattern is well studied in the literature[5]. This is because an important class of truly 1D problems involves the motion of lines or filaments embedded in 3D space, such as vortex lines in defect lines in liquid crystals, type-II superconductors, or polymers in imposed flows. Nonlinear partial differential equations describe the dynamics, and the twisting, breaking, reconnecting, or the knotting are the essential topological parts of the chain dynamics[6]. However, the jumping chain studied in this paper is not suspended but jumps from a higher platform, falling freely in the space under the attraction of gravity. The damping from the viscosity of the air seems to be a sub-dominant effect in the whole process. The gravity and the tension inside the chain play an important role. The subtle point is that although the matter is moving from the higher position to the lower ground, the system is not a fluid or a falling stream. A detailed study will reveal the different dynamics.

In all, the paper is organized as follows. The phenomenology and dynamics of the jumping chain will be analyzed in Sec. 2. The experimental verification of our analysis will be presented in Sec. 3. The conclusion is given in Sec. 4.

2 Phenomenology and Theoretical Analysis

2.1 Phenomena

First, let's describe how the jumping phenomenon takes place. As shown in Fig. 1, the chains are coiled layer by layer inside a cup, and they are kept from becoming entangled (It is difficult to keep all steel balls untangled in the preparation, and it is one of the dominant disturbing factors of the experiment.). Then the cup is placed at a certain height. A small length of chain is taken out of the cup and allowed to fall freely. In the actual operation, we also tried to use a soft and non-retractable rope to do the same experiment. The rope can barely jump up, but the phenomenon is rather obscure, so the thin metal chain composed of steel balls is adopted in our experiment. The steel balls are loosely connected, and the chain is non-retractable. It means that the chain does not have flexural capacity. This is the key difference between the chain and the soft rope. The reason will be discussed in the following context.

The experiments show that the chain jumps to an appropriate higher position from the cup if the cup is set at some specific heights. Due to the complex way of chain winding in the cup, the starting point of the jump keeps changing, so the shape of the jumping from the cup to the highest point is very complicated. As for the pattern between the highest point and the falling off point on the ground, the chain can maintain an almost stable shape near the highest point on the top. The shape keeps shaking near the ground, but this does not affect the stable shape on the top.

Summarizing our observations, we think this physical phenomenon can be divided into two sub-processes: the accelerating process and the stable suspension process. Thus our analysis on the phenomenon is set out as shown in Fig. 2, in which point A is the beginning of the suspension and the end is point B, which is the collision point on the ground. The point C is the starting point of the jumping in the cup where the elements of the chain are static. The part from C to A is the accelerating process. Note that as dis-

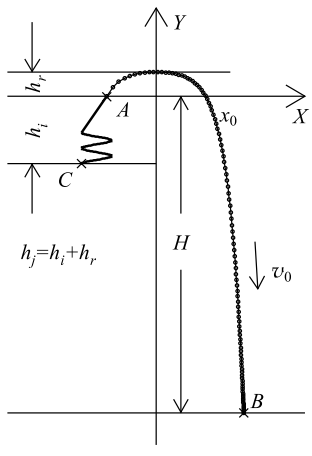


Fig. 2 The sketch map of our analysis on the jumping chain

cussed above, point B keeps shaking, and point C also varies quickly. At the same time, the height of Point C is decreasing. The definitions of the variables such as h_j, h_i etc. are given in the following context. One can see that the stable suspension is obviously of the most concern, and according to the following analysis, we find that this is actually the key point to comprehend the phenomenon. Thus we analyze the stable suspension in the following subsection.

2.2 Analysis on the shape of the stable Suspension

At first glance, the jumping chain appears like a parabolic curve. However, the truth is not so simple as expected. A free-falling mass point follows a parabolic trajectory. However, for the non-retractable chain, each element on the suspended chain falls at the same velocity, only the directions vary. Based on our observations, the coiled chain in the cup almost moves to the ground at a constant rate. Thus it is natural to assume that the suspended chain can maintain a constant velocity v_0 for an appropriate interval as an ideal case. Then, in Δt time, there is a part of the chain with mass $\lambda v_0 \Delta t$ (λ is denoted as the line density) in the cup which is accelerated from 0 to v_0. At the point A, the chain feels the dragging tension from the chain at rest in the cup as follows:

$$T_0 = \lim_{\Delta t \to 0} \frac{\lambda v_0^2 \Delta t}{\Delta t} = \lambda v_0^2 \tag{1}$$

It should be noted that this is merely the ideal case, and the following study will show that the actual tension at the end of A will be less than λv_0^2. At point B, the chain element with velocity v_0 collides with the ground. If the collision is elastic, the chain will rebound from the ground with velocity v_0. Similar analysis gives that the impulsive force to the ground is $2\lambda v_0^2$. Of course, the collision is not completely elastic due to the dragging. We shall provide the specific method for dealing with the impulsive force at point B later.

We find that, very interestingly, the shape of the suspended chain is actually an inverted catenary. The exploration of the catenary is an important progress in the history of classical mechanics and mathematics. Our discovery in fact compensates a block in the catenary theory. Here we briefly show the derivation of the catenary and explain the reason for the same shape of the suspended chain. Suppose the shape of the chain is $y = y(x)$, the solution of the catenary is to find a $y(x)$ such that the potential or center of gravity of the chain is at its lowest point. Using the variational principle, the action of the whole system is the center of gravity in the vertical direction

$$S = -\int \lambda g (y - c) \sqrt{1 + y'^2}\, \mathrm{d}x = \int \mathcal{L}\, \mathrm{d}x \tag{2}$$

where g is the acceleration of gravity and c is an arbitrary constant, y' is the derivative of y. $\mathcal{L}$ is the Lagrangian of the system

$$\mathcal{L} = -\lambda g (y - c) \sqrt{1 + y'^2} \tag{3}$$

The motion for $\mathcal{L}$ gives the non-linear differential equation of the catenary

$$(y - c) y'' - y'^2 - 1 = 0 \tag{4}$$

The solution is

$$y = c + \frac{a}{2} \left(\mathrm{e}^{\frac{x}{a} + b} + \mathrm{e}^{-\frac{x}{a} - b} \right) \tag{5}$$

where a, b are the parameters determined by the inputs of the catenary. The approach described above is the standard derivation of the catenary. Note that here the suspension chain are hanged at any height, thus an additional constant c is introduced. In fact, we notice that, although the equation of motion Eq. (4) is nonlinear, it has a discrete symmetry and $y(x)$ can be transformed

$$y(x) \to Y(x) = -y(x) + 2c \tag{6}$$

$Y(x)$ is also a solution of the catenary equation. Despite its lack of practical application, the inverted catenary solution is still a valid solution of the catenary equation. This solution is an unstable extremum point of variation, which has been largely overlooked in the literature. Nevertheless, our research has revealed that this inverted solution is indeed the exact shape of the suspended chain.

For the jumping chain, the potential energy is the same as that of the catenary. Therefore

only the kinetic term needs to be included in the action

$$S=\int \mathcal{L}\mathrm{d}x$$
$$=\int\left(\frac{1}{2}\lambda\sqrt{1+y'^2}v_0^2-\lambda g y\sqrt{1+y'^2}\right)\mathrm{d}x \tag{7}$$

By comparing Eq. (3) and assuming that v_0^2 is a constant, we can observe that the Lagrangian of the jumping chain is identical to that of the catenary, with the only difference being that c is taken as a constant

$$c=\frac{v_0^2}{2g} \tag{8}$$

Due to the given conditions, the solution will be a convexity, specifically, an inverted catenary

$$y=c-\frac{a}{2}(\mathrm{e}^{\frac{x}{a}}+\mathrm{e}^{-\frac{x}{a}}) \tag{9}$$

The difference between the deviation of an ordinary catenary and a jumping chain is that the chain is constantly in motion. The elements of the chain is continually jumping out of the cup and falling on to the ground, thus changing the mass of system. This may lead some readers to be skeptical about the validity of variational principle. However, the same differential equation of motion can be derived by the force analysis. As shown in Fig. 3, the tension in the jumping chain is denoted as $T=T(x)$ and the convention of the forward direction is defined as from point A to point B. Every element of the chain is moving in a curvilinear motion with the same speed, so the tangential acceleration is zero, and the infinitesimal tension $\mathrm{d}T$ cancels the tangential component of gravity

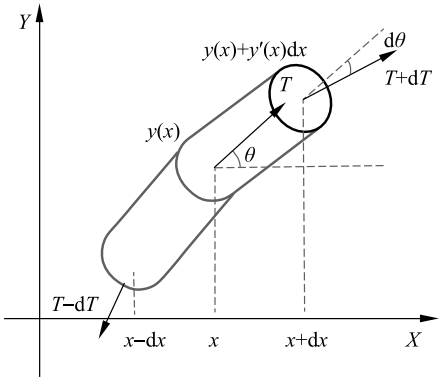


Fig. 3 The force analysis on the elements of the jumping chain

$$\mathrm{d}T=\lambda g\sqrt{1+y'^2}\,\mathrm{d}x\,\frac{y'}{\sqrt{1+y'^2}}=\lambda g\,\frac{\mathrm{d}y}{\mathrm{d}x}\mathrm{d}x$$
$$=\lambda g\,\mathrm{d}y \tag{10}$$

Thus the tension in the chain can be expressed as

$$T=T_A+\lambda g y \tag{11}$$

This is a very interesting result that the tension in the stable jumping chain changes along with the height of the elements, which is the same as the tension in the ordinary catenary. To analyze the normal component force of the infinitesimal chain imposed by tension from the curvature of the chain, we can look to the catenary or hanging rope for comparison. In the case of the ordinary catenary, the normal component of the tension is canceled by the gravity, but in the case of the jumping chain, the normal component force provides the centripetal force together with the gravity. The geometrical configuration is shown in Fig. 3 where we investigate the centripetal force acting on the two chain elements $x-\mathrm{d}x\to x$ and $x\to x+\mathrm{d}x$. From this, we can easily derive the equation for the centripetal force

$$\frac{2\lambda\sqrt{1+y'^2}\,\mathrm{d}x v^2}{\rho}=2T\,\mathrm{d}\theta+$$
$$2\lambda g\sqrt{1+y'^2}\,\mathrm{d}x\,\frac{1}{\sqrt{1+y'^2}}$$

where ρ is the radius of curvature at point x. By substituting ρ, $\mathrm{d}\theta$, and tension T, we can get

$$\frac{2\lambda|y''|v_0^2\mathrm{d}x}{1+y'^2}=\frac{2(T_A+\lambda g y)|y''|\mathrm{d}x}{1+y'^2}+2\lambda g\,\mathrm{d}x \tag{12}$$

Due to the convexity of the jumping chain

$$|y''|=-y'' \tag{13}$$

Eq. (12) are reduced to

$$-(T_A+\lambda g y-\lambda v_0^2)y''+\lambda g y'^2+\lambda g=0 \tag{14}$$

Given that T_A is the variable to be solved, it is convenient to define

$$T_A-\lambda v_0^2=-\lambda g c \tag{15}$$

Then the differential equation of y is

$$(y-c)y''-y'^2-1=0 \tag{16}$$

Proved the same catenary equation again.

Although the analysis of force is a bit more complicated than that of the variational principle of mechanics, the physical picture is easier to understand because it is an analysis of forces. We can also see that the forces at the two ends of the catenary and the jumping chain are different. In case of the ordinary catenary, the tensions at the two ends are dragging the chain. Whereas, in the case of the jumping chain, the tension at point A is dragging the chain, while the tension at point B is pushing the chain. Of course, this is the ideal case; the jumping process will be further analyzed in the following.

2.3 Analysis on the jumping

First, let's begin with the explanation for why $T_A \neq \lambda v_0^2$. $T_A = \lambda v_0^2$ corresponds to the ideal case that each chain element instantaneously accelerates from 0 to v_0. In contrast, the actual situation is that the chain is loosely coiled and the chain element needs an interval for the acceleration, making T_A smaller than λv_0^2. Analytically, if $c=0$, the shape of the suspended chain line is

$$y = -\frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right) \tag{17}$$

then

$$y \leqslant -a \tag{18}$$

It is evident that $y>0$ solution does not exist, as indicated by Eq. (16), which is a non-linear differential equation. Therefore, if the coordinate system shown in Fig. 2 is used a $y>0$ solution requires a non-zero c. From Eq. (15), it is clear that T_A must be less than λv_0^2. As shown in Fig. 2, the acceleration process for the upper elements in the stable suspended shape is added. And the height of the acceleration jumping up process from point C to point A is measured as h_i. The jumping height of the stable suspended chain line is h_r. The total jumping height is thus the sum of h_i and h_r

$$h_j = h_i + h_r \tag{19}$$

Along with the conventions mentioned above, the falling height (the height of the cup) of the chain is denoted as H and the width of the jump is $2x_0$ as shown in Fig. 2. We will further investigate the relationship between these quantities for the jumping chain and attempt to conduct experiments to verify our analysis.

The tension at point A is

$$T_A = \lambda v_0^2 - \lambda g c \tag{20}$$

According to the Eq. (11), the tension T from point A to point B is

$$T = \lambda v_0^2 + \lambda g y - \lambda g c \tag{21}$$

As discussed previously, the elastic collision implies that the force imposed from the ground to the chain at point B will be $2\lambda v_0^2$. This may appear suspicious to some readers, as a deceleration process is needed at point B. When the chain finally falls on the ground, the tension should be less than $2\lambda v_0^2$. However, we found that the chain elements can slightly bounce on the ground, thus the actual impulse to the ground should be greater than λv_0^2. To account for the force with more precision, a constant coefficient α can be introduced

$$-\alpha\lambda v_0^2 = \lambda v_0^2 - \lambda g H - \lambda g c \tag{22}$$

The value of α would be greater than 1 and less than 2. It should be noted that tension at the point B should be negative according the direction defined above. Since falling height H is much larger than the jumping height h_j, the tension fluctuation at point B is not sensitive to stable suspension above. Then

$$v_0^2 = \frac{1}{\alpha+1} g (H + c) \tag{23}$$

This is the relation between the falling height and the velocity of the suspended chain. The jumping height $y(x=0)$ of the stable suspended chain is

$$h_r = c - a \tag{24}$$

Then all the exploration focuses on the unknown parameter c and α, which will be analyzed in the next section when discussing the experimental verification. Before that, let us analyze the accel-

eration process.

As the velocity of the chain element changes over time during the acceleration process, it seems difficult to do the analysis. However, the time parameter can be eliminated after some substitutions. The velocity of accelerating elements is denoted as v. According to Fig. 2 and Eq. (11), the tangential acceleration should be

$$\lambda v dt \frac{\mathrm{d}v}{\mathrm{d}t} = \lambda v \mathrm{d}v = \mathrm{d}T - \lambda g \mathrm{d}y \tag{25}$$

Since the chain elements are loosely connected, the normal acceleration dynamical equation can be disregarded. It can be seen that the time dependence dt can be eliminated. This is the mathematical formulation of the transformation from gravitational potential energy to kinetic energy. Carry out the integration on both sides, we have

$$\begin{aligned}\int_0^{v_0} \lambda v \mathrm{d}v &= \int_0^{T_A} \mathrm{d}T - \int_{-h_i}^{0} \lambda g \mathrm{d}y \\ &= T_A - \lambda g h_i \end{aligned} \tag{26}$$

At point A

$$\frac{1}{2}\lambda v_0^2 = \lambda v_0^2 - \lambda g c - \lambda g h_i$$

namely

$$h_i = \frac{v_0^2}{2g} - c \tag{27}$$

Substitute v_0 from Eq. (23), we can get the relation between h_i and H

$$h_i = \frac{H - (2\alpha + 1)c}{2(\alpha + 1)} \tag{28}$$

From above equation, the requirement for the jumping can also be got. The jump can happen only when h_i greater than zero, thus the condition for the jumping is

$$H > (2\alpha + 1)c \tag{29}$$

The total jumping height is

$$h_j = h_i + h_r = \frac{H + c}{2(\alpha + 1)} - a \tag{30}$$

It is evident that the jumping height basically is linearly correlated with the falling height H, with additional parameter c, a and α to be determined.

3 Experimentation and Verification

3.1 Analysis and execution of the experiment

Through the theoretical analysis in Sec. 2, we can verify the relation through experimentation. Nevertheless, it is important to bear in mind that all the theoretical analysis is based on ideal conditions, and there are certain limitations in the actual experiment, such as

(1) Ideally, the chain in the cup should be loosely connected and freely coiled. Unfortunately, the elements in the chain inevitably become tangled, which prevents the chain from maintaining homogeneity during the acceleration and stable suspension processes. We discovered that this is the primary source of disturbance in the experiment. Even a small knot can cause a significant variation of the stable suspension.

(2) Ideally, the jumping point C and falling point B should remain fixed in order to obtain a stable suspension. However, these two points tend to shake during the falling process, making it difficult to accurately measure the shape. Nevertheless, since the shape of the catenary is an exponential function in which y increases exponentially with x, the shaking chain almost forms a straight line at the bottom. This makes it challenging to measure the correlation between the shape and the exponential curve. Furthermore, due to the upper two points, it is difficult to precisely determine the positions of points A, B and C. Generally, only the total jumping height h_j and falling height H can be measured.

Though there are these difficulties, it is sure that

$$H \gg c, a \tag{31}$$

and the jumping height h_j, a and c are at the same order. Based on this, we can assume that the actual shape of the jumping chain resembles an inverted catenary. The experiment should then test the linear relation between h_j and H (Eq. (30)) and the linear relation between v_0^2 and H (Eq. (23)). Both relations could be used to

evaluate the parameter α. which will validate our analysis in Sec. 2.

The apparatus used in our experiment is illustrated in Fig. 4. We drew a height calibration on the wall, which can be seen in panel A of Fig. 4. The chain and the cup are also depicted in the figure, and the details of the chain are provided in the Table 1. The experiment was conducted as follows: The chain was pulled out of the cup from a height of 200cm, then gradually lowered by 5cm each time. When the height reached 100cm, it became increasingly difficult to lower the chain further. Therefore, we chose to lower the chain by 10cm for heights lower than 100cm, until the height reached 40cm. At each specific height, a cellphone was used to record the jumping chain, while the camera was placed at the same height at the top of the jumping chain (see panel E of Fig. 4). Due to the presence of various disturbances in the experiment, multiple videos were taken in order to select the best one for the data collection.

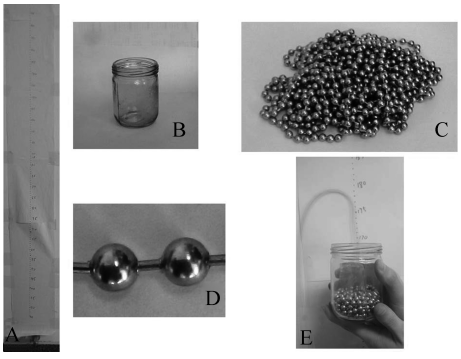


Fig. 4 The apparatus of the experiment: A, the height calibration on the wall; B, the cup; C, the chain; D, the chain elements, E, the screenshot

Table 1 The setup of the exepriment

Diameter of the steal ball	5±0.5mm
Length of the steal ball	3±0.5mm
Length of the chain	8.4±0.1m
Density λ	0.036±0.03kg/m
Height of the cup	95±0.5mm
Diameter of the cup	66±0.5mm

In principle, catenaries with different c and a can exist when the requirements discussed in Sec. 2 are satisfied. However, since the chain needs to jump from the cup, the shape of the jumping chain depends on specific initial conditions, namely, the diameter of the cup, the depth of the cup at the beginning of the jump, etc. Thus though the shape of the jumping at a specific height is not in a single parameters set, We found that c, a are at the same order of the diameter of the cup. Thus the linear relation could be found in the following analysis.

The data collection was carried out as follows. Each frame of the chosen video was examined, and the best moment with the highest jumping height and stable shape was identified. The falling height and jumping height were then recorded based on the frame at the best moment. To measure v_0, the time interval between a specific height of the chain left in the cup and the final moment was recorded. With these measured raw data, we can analyze the relation between the physical quantities discussed in the Sec. 2

3.2 Analysis on the data

As discussed in the previous subsection, the linear relation between h_j and H (Eq. (30)) and the linear relation between v_0^2 and H (Eq. (23)) can be used to verify our analysis. The corresponding results are shown in Fig. 5, with the left panel displaying the recorded jumping heights and falling height, and the right panel showing the calculated velocity v_0 of the stable suspension shape and falling height. Note that due to the presence of various disturbances in the experiment, only data with falling height higher than 100cm (black points) were chosen for the analysis. As can be seen in Fig. 5, the linear relations are realized and the slopes give the values of the α_1(left panel) and α_2(right panel)

$$\alpha_1 = 2.6^{+5}_{-1.3}, \quad \alpha_2 = 2.5^{+7.0}_{-0.7} \tag{32}$$

We can see that the errors are quite large and the central values exceed 2. However, given the difficult setup of the experiment, these two measurements on α are acceptable. Note that although α_1 and α_2 come from the same series of deriva-

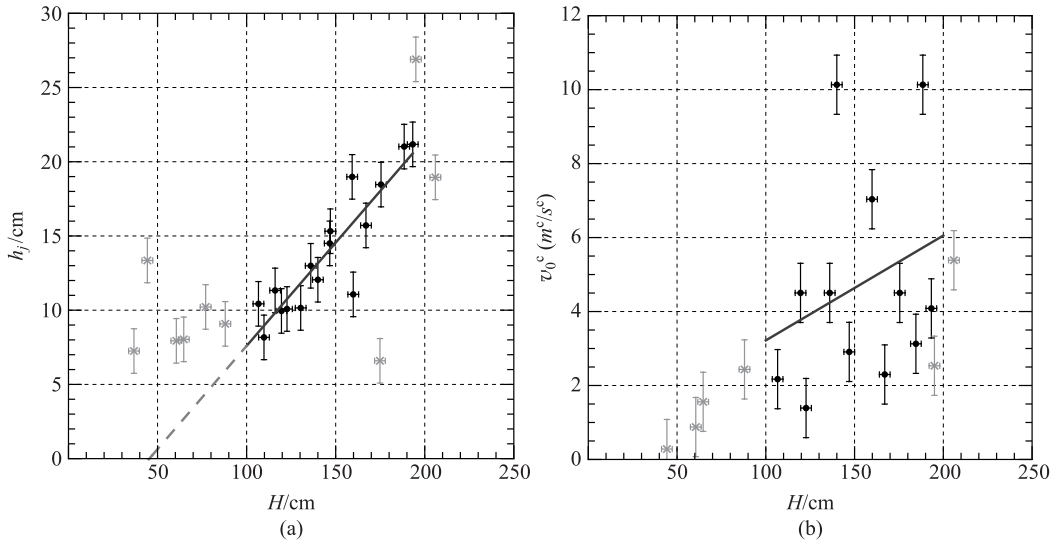


Fig. 5 The data analysis of the linear relations

(a) The recorded results between the jumping height and falling height; (b) the velocity v_0^2 of the stable shape and the falling height

tions, Eq. (30) and Eq. (23) are assumed to the independent with each other. Thus, we can conclude that α_1 and α_2 are in agreement with each other. Another interesting point is that the extension line in the left panel intersects the H coordinate axis at a point greater than 0. This point should correspond to the required height of the jumping condition in Eq. (29), although this is only a rough estimation.

Finally, we can explain why the soft rope cannot effectively jump. The elements of the chain are connected loosely, and the rope cannot achieve this condition. This is also the reason why the catenary cannot be realized by the soft rope. Specifically, the soft rope lacks the non-flexural capacity and the absence of disturbance from normal force, making the jumping impossible.

4 Conclusion

The jumping of a free-falling chain is an intriguing phenomenon, and this paper provides a detailed analysis of the physics behind it. The analysis are divided into two parts: the stable suspension and the jumping. Both variational principle and force analysis demonstrate that the stable suspension is an inverted catenary. The parameters that describe the phenomenon are studied and tested through experiments, The requirements for the jumping are discussed. The measurements of the parameter α meet our expectations. The physical picture of the falling chain could be useful in some falling systems, and the inverted catenary may be a complement to classical mechanics.

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