

INTERFERENCE OF ELECTROMAGNETIC WAVES IN THE BACKGROUND OF THE GRAVITATIONAL WAVES

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Abstract Based on the relationship between proper distance and coordinate distance, the geometrical phenomenon caused by the passing gravitational waves can not be observed locally. The electromagnetic wave equations in the background gravitational waves are studied. We find that the expansion and contraction of wave lengths are always synchronous with the objects it measures. The background of the gravitational waves leads to dissipation and dispersion in the propagation of electromagnetic wave. The phase of the gravitational waves control the dissipation term and dispersion term in the telegrapher's equation. The linearly polarized laser beam propagating in the direction of the incoming gravitational waves can give a possible measurement on the local metric. In case of the pulsed beats passing by, the relaxation time is greater than the period of the gravitational waves, thus the detector may only show a signal of the modulation of the beats. Finally we proposed a non-local interference experiment to detect the high-frequency gravitational waves. It is similar to the measurement of redshift caused by gravitation. Together with the ordinary detector, it will give us further and mutually measurements of the gravitational waves.

Key words gravitational wave; interference; telegrapher's equation; general relativity

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Introduction

The detection of gravitational waves (GWs) by the Laser Interferometer Gravitational-wave Observatory (LIGO) opened a new era of gravitational-wave astronomy^[1]. The precise wave shape of the first GWs signal is consistent with the waves shape produced from the coalescence of two black holes predicted by the numerical simulation in the theory of the general relativity^[2]. Later, several other GWs events were observed and studied in detail by the community. Most of these events were identified as the merging of binary black hole or neutron star^[3-6], i. e. the de-

tection of GW170814 provides an interesting approach of probing the polarization modes of the GWs^[5], GW170817 and GRB 170817A showed a very tight bound on the speed of the GWs^[7,8]. As a new window to explore our universe, the GWs play an important role in modern cosmology, such as black hole physics^[9], tests of strong gravitation field regime, other extended theory of gravitation and the early universe^[10]. Based on ingenious experiments and advanced technologies, an extraordinary amount of analysis has been carried out towards higher precision tests of relevant issues.

Besides LIGO experiment, there are several different types of the GWs detectors aiming at different frequency band of GWs. Pulsar timing

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arrays (PTAs)^[11-13] detect GWs in the lower-frequency band. Meanwhile, Advanced LIGO^[14], Advanced Virgo^[15] and KAGRA^[16], these ground-based interferometers detect the GWs in the high-frequency band ($10 \sim 10^4$ Hz), possessing higher sensitivity over a broader frequency band. The theoretical principle of detection method is a simple application of general relativity theory, the GWs which is far from the source can be treated as a time-dependent perturbation of the space-time metric, consequently causing length changing with respect to its polarizations. For example, Advanced LIGO measures linearly differential displacement along the arms which is proportional to the amplitude of the GWs. By means of the Michelson interferometer, the ground-based detectors have a much higher accuracy on detecting the small displacement. Any length changing in two arms of the interferometer will produce a phase shift, thus cause changes of interference pattern at the output of the detector^[17]. Therefore, the Advanced LIGO detectors translate strain into a measurable optical signal^[18].

In flat space-time, interferometers are widely used in science and industry for the measurement of small displacements. However, the principle for measuring proper distance in the background GWs is quite questionable. According to the general relativity, the local physical measurement of space-time can not detect the influence of the background metric, only global comparison can show the difference between areas of space-time with different local metric. This is because that the gravitational field affects space and time standards in exactly the same way as it affects the object being studied^[19]. This properties of space-time metric have been studied in many papers and textbooks, but the application of interferometers in curved space-time is hardly mentioned. Thus to confirm the existence of the GWs, the mechanism that using the laser interference needs to be subtly checked. In the

interference measurement, the laser is assumed as a flux of photon with a speed of light which is unaffected by the GWs. The measurement of the intensity variation can give the phase shift of the electromagnetic waves, then the displacements of the length of the arms in the interferometer. However, it is controversial for using “laser ruler” to measure length when it has been assumed that the ruler (wave length of the laser) is unaffected. The propagation of photons are in fact obey the classical electromagnetic theory. If we want to make sure about the phase shift, the solver of electromagnetic wave in the curved space time is needed. Only the electromagnetic waves equation under the background GWs can give a decisive judgement in revealing the characteristic of laser light.

In all, we will explore the electromagnetic wave equation and its interference in the interferometers in the curved space time with background GWs in this paper. The implications and comparisons with the current detection results of the GWs are also discussed. The paper is organized as following. In sec. 2, the coordinate distance and proper distance, and some basic concepts of the GWs are illustrated. We also discussed the method and the common understanding of the phase shift measured by the LIGO experiment. In sec. 3, we study the electromagnetic wave equation and the solution in a background gravitational wave, the interference of the electromagnetic waves and its relation with the GWs are also discussed. In sec. 4, we show our result of the phase shift and compare it with the LIGO's. We also try to show a new method of measurement of the GWs. The conclusion is in given the sec. 5.

1 Space-time Metric and the Gravitational Waves

In the framework of the general relativity, or any other metric based gravitational theories^[21], equivalence principle implies that the

phenomenon of gravitation can be considered as a specific nonlinearly coordinate transformations of space-time. Every physical theory can be generalized to a theory in the curved space by the general covariance principle which implies that the theory should be coordinate independent. According to general relativity, material fields will affect the measure of space-time via Einstein's equation. In four dimensional space-time, the metric tensor $g_{\mu\nu}$ is defined to measure space-time interval square ds^2 in a given frame x^μ

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad (1)$$

with symmetric indices $\mu\nu$. μ and ν equal to 0, 1, 2, 3 which correspond to the coordinates t, x, y, z as the usual convention in the literature. Generally, the space-time metric has 10 components, which define the measure of space-time. It is the basic dynamical variable in the gravitational theories, such as general relativity. There are many experimental tests of the variation of metric induced by the gravitational field, such as gravitational redshift, precession of planetary perihelion, gravitational deflection of light and time delay. The GWs is an important prediction of the theory general relativity. When the GWs pass through the earth, the metric will slightly deviate from flat space-time. The detection of the GWs also can be viewed as the direct measurements on the space components of metric^[22].

We find that understanding the relation between proper interval and coordinate interval is very important in the issue we are talking about in this paper. From equation (1), we can see that the proper distance differential interval dX along X direction is

$$dX = \sqrt{-g_{11}} dx \quad (2)$$

in which dx is the coordinate differential. In relativity theory, one should keep in mind that the proper interval dX and proper time interval are independent of frame of reference, they are actually the observable corresponding to a real physical process in the local place, such as a simple harmonic motion, stimulated radiation of atoms,

et. al. While the coordinate interval dx is globally established but it is in fact unmeasurable. Any local measurements will always indicate its proper distance for the measurements taken by the local ruler or clock. This circumstance is demonstrated by a sketch graph as shown in the Fig. 1 in which a clock is relative rest to harmonic oscillator. We can assume that the period of oscillator equals to the interval of the clock which may be recorded by other physical periodic processes. As talked in the introduction, a local measurement can not show the variation of the metric, the gravitational field affects space and time standards in exactly the same way as it affects the oscillator being studied and the internal periodic process happening in the clock. Whether they are rest in the gravitational field or falling freely in it. The proper time of the clock will always shows the period of the harmonic oscillator with exactly the same value, as long as the observation is disposed locally. This means that there is no effect of time dilation, change of length can be observed at this local place. To make this effect detectable, one must compare the clocks placed in different location. This can also be understood by virtue of the equivalence principle, the free falling observer will never experience a time dilation, such phenomenon can be detected by a relatively static observer. Note that, we often treat coordinate interval dx as being observed far away from the strong gravitational source, where the space-time is Minkowskian.

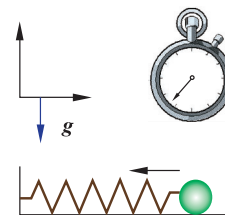


Fig. 1 The harmonic oscillator and a relatively static clock are always synchronous, no matter whether they are rest in the gravitational field or falling freely in it

This understanding of the proper interval

and the coordinate is crucial for LIGO detectors, since the experiment is taken place by using a local laser. In this case, the wavelength is in fact the local ruler, because the wavelength of laser determines the interference pattern in the interferometer. As talked in the above paragraph, it can not show the changing of the metric without a global comparison. However, things are different, this laser ruler is an electromagnetic wave, thus the details of wave equations which are influenced by GWs should be checked. And what we should ask is whether the wavelength change under the GWs? And what is detected by LIGO experiment exactly?

Before checking the wavelength of the laser, let's take a short review on the GWs in the framework of the general relativity. The metric of the weak GWs $h_{\mu\nu}$ (a small quantity) in vacuum is the solution of Einstein equation

$$\partial^2 \phi_{\mu\nu} = 0 \quad (3)$$

where^[19]

$$\phi_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h \quad (4)$$

$\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ is Minkowskian metric,

$$h = h_{\mu\nu} \eta^{\mu\nu} \quad (5)$$

is the trace of the GWs metric. The gauge condition of $\phi_{\mu\nu}$ is

$$\partial^\mu \phi_{\mu\nu} = 0 \quad (6)$$

The Eq. (3) is a wave equation, the plane wave solution can be expressed as

$$\phi_{\mu\nu} = A \epsilon_{\mu\nu} \cos k^a x_a \quad (7)$$

in which A is the amplitude of the GWs, $\epsilon_{\mu\nu}$ is polarization tensor, and k^a is gravitational wave vector. Wave vector k^a carries the information of the gravitational source, from which the frequency, wavelength and propagating direction of the GWs can be read. From Eq. (6) and Eq. (3), we have

$$k^2 = 0 \quad (8)$$

$$k^\mu \epsilon_{\mu\nu} = 0 \quad (9)$$

Suppose the GWs propagates along the Z axis direction, the gravitational wave vector k^a can be

written as

$$k^a = (\omega_g, 0, 0, \omega_g) \quad (10)$$

where ω_g is the circular frequency of the GWs. The polarization tensor is some linear combination of vector ϵ_1^μ , ϵ_2^μ and wave vector k^a . The vector ϵ_1^μ and ϵ_2^μ is denoted as

$$\epsilon_1^\mu = (0, 1, 0, 0) \quad (11)$$

$$\epsilon_2^\mu = (0, 0, 1, 0) \quad (12)$$

There are two physical polarization tensor $\epsilon_{\mu\nu}$, it can be proven that the rest of polarization tensors is just some gauge selection of $\phi_{\mu\nu}$. Two transverse polarization tensor is

$$\epsilon_{\oplus}^{\mu\nu} = \epsilon_1^\mu \epsilon_1^\nu - \epsilon_2^\mu \epsilon_2^\nu = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (13)$$

and

$$\epsilon_{\otimes}^{\mu\nu} = \epsilon_1^\mu \epsilon_2^\nu + \epsilon_2^\mu \epsilon_1^\nu = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (14)$$

The physical detection of these two kinds of polarization is similar.^[20] Thus in the following, we only study polarization tensor $\epsilon_{\oplus}^{\mu\nu}$. The corresponding metric $g_{\mu\nu}$ is

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & h-1 & 0 & 0 \\ 0 & 0 & -h-1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (15)$$

Where

$$h(x) = A \cos[\omega_g(t-z) + \eta] \quad (16)$$

η is the initial phase of the GWs.

From above formulations, we can see that the GWs propagating in Z axis direction does cause the expansion or contraction in X or Y direction periodically. Geometrically speaking, the displacements in the X , Y direction is dx , dy respectively, the metric (15) leads to the length change X , Y in the form of

$$dX = \sqrt{1-h} dx \simeq \left(1 - \frac{1}{2}h\right) dx \quad (17)$$

$$dY = \sqrt{1+h} dy \simeq \left(1 + \frac{1}{2}h\right) dy \quad (18)$$

Note that in the following section we will use the capital symbol X, Y to denote the proper and local length measured in the experiments. As the proper length is invariant in the general relativity, the expansion and the contraction here should take place in the coordinate. Or else, one can consider that the ruler expands or contracts accompanying with the objects it measures. The GWs in fact cause the oscillation of the coordinate. With the displacements above, the next question is, does the wavelength of laser in X and Y direction expands or contracts in the same way? If so, then the optical path will be invariant no matter what kinds of metric is considered. However, the wavelength of the laser is not an ordinary ruler, it is derived from electromagnetic wave equation of laser, namely, from the Maxwell equations. Only the solver of the wave equation in the curved space-time metric (15), can tell us how the electromagnetic waves change. Before that, let's take a quick review on the theoretical principle of the GWs' detection by the LIGO experiment.

In the flat Minkowskian space time, the wave equation of electromagnetic waves is^[23]

$$\eta^{\mu\nu} \partial_\mu \partial_\nu F_{\alpha\beta} = 0 \quad (19)$$

in which

$$F_{\alpha\beta} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix} \quad (20)$$

is the electromagnetic field strength tensor. The electric part of the plane wave solution is

$$E_x \sim \exp[i(\omega_{\text{em}} t - k_{\text{em}} y + \phi_1)] \quad (21)$$

$$E_y \sim \exp[i(\omega_{\text{em}} t - k_{\text{em}} x + \phi_2)] \quad (22)$$

where $\phi_{1,2}$ is the wave phase which can be adjusted by the laser. In the experiment of the Michelson interferometer, which sketch graph of the structure and measuring principle is shown in the Fig. 2, two polarized laser beams travel along two different path, arms along X axis and Y axis respectively, and recombine at beam-splitter. The electromagnetic fields will interfere into dif-

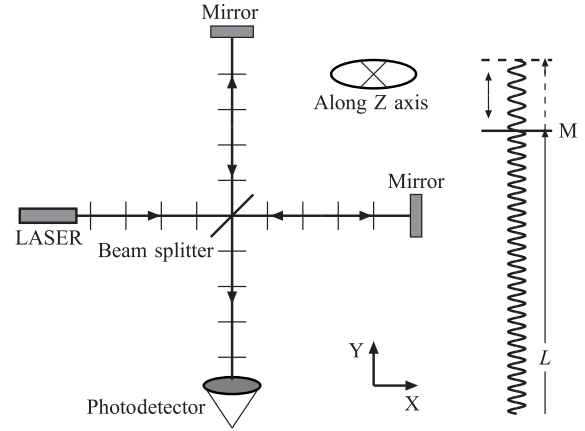


Fig. 2 The left plot shows the detection of LIGO experiment, the short thin line represents the polarization plane is parallel to XOY plane. Any small displacement in the two arms (shown in the left plot) will change of the interference pattern, output an optical signal to the detector

ferent patterns. By some adjustment in the wave phase of the initial beam, the total light intensity received by photodetector is

$$\begin{aligned} P &\simeq (\text{Re}[\mathbf{E}_x] + \text{Re}[\mathbf{E}_y])^2 \\ &= P_0 \sin^2[k_{\text{em}}(X - Y)] \\ &= \frac{P_0}{2} [1 - \cos(\Delta\varphi_{\text{em}})] \end{aligned} \quad (23)$$

In the above equation, X, Y are the lengths of the two arms, and $\Delta\varphi_{\text{em}}$ is the phase shift of electromagnetic wave. Therefore, by measuring the light intensity, we can get the phase shift $\Delta\varphi_{\text{em}}$ and thus get the the length differential between the two arms. Note that in case of $X = Y$, the phase shift is canceled out at beam-splitter.

The above theoretical principle of the Michelson interferometer is actually the detection principle of the most detection experiments of the GWs.^[24] Any change of optical path in Eq. (23) will cause a phase shift and create a signal. LIGO detects the small displacement of arms influenced by the GWs. By assuming that the GWs will influence the distance of the two arms, and also, the propagation of the photons are the same as in the flat space time. Using the Eq. (17) and Eq. (18), the phase shift $\Delta\varphi_{\text{em}}$ can be approximately expressed as

$$\Delta\varphi_{\text{em}} \simeq k_{\text{em}} hL \quad (24)$$

where the total travelling distance of laser light is L , and the amplitude of the GWs h can be obtained from the phase shift $\Delta\varphi_{\text{em}}$. There may be a misunderstanding here that hL is the difference between the proper length of the arms. However, as talked above, hL are the difference of the coordinates which are oscillating when the GWs passing by. Thus the phase shift in Eq. (24) is obtained under the assumption that the velocity of phase transition in the coordinate space is invariant. In case of the low frequency (kHz) GWs, the LIGO experiment measured that^[25]

$$h \sim 10^{-21} \quad (25)$$

Then the phase shift is proportional to the strength of the GWs. The amplitude A is a very small quantity which is indirectly observed by LIGO detectors, in deed. The shape change of two 4-kilometers arms is expected to have the form of cosine function as Eq. (23). Note that this is the case for the monochromatic GWs, the wave equation is a simple harmonic oscillator. However, if the GWs are consisted of many different frequency bands, meaning that actually the gravitational wave packet is travelling, then the wave equation of the GWs is the synthesis of many different harmonic oscillator. A gravitational wave packet could be possibly induced by its coupling with the energy-momentum tensor $T^{\mu\nu}$ of material fields when the GWs are travelling.

As talked above, what we concern is the influence of the GWs on the laser propagation. The shape change of the arms is not an observable quantity when being locally measured. To figure out the answer, we will focus on the laser under the background GWs, the Maxwell equations, wave equation and its solution, in the next section. We should also note that the metric Eq. (15) corresponds to a curved space-time. Since the amplitude A is a much more small quantity, any terms containing higher orders of A are always neglected in the following calculation.

Moreover, if the electromagnetic field is strong enough, according to the Einstein equation, the energy-momentum tensor of electromagnetic field $T_{\mu\nu}$ will also cause a corresponding curved metric, this case is complicated and less concerned here, and it is always neglected in the literature. In the following section, we just focus on how the metric Eq. (15) impacts on electromagnetic waves.

2 Laser Interference in Curved Space-time

According to the principle of general covariance, generalizing a theory in the flat space to the curved space is simple, what we should do is just to replace the ordinary derivative ∂_μ by the covariant derivative ∇_μ in ordinary Maxwell equations, i. e.

$$\nabla_\mu A^\nu = \partial_\mu A^\nu + \Gamma_{\mu\alpha}^\nu A^\alpha \quad (26)$$

where $\Gamma_{\mu\alpha}^\nu$ is the Christoffel symbol. The form of the Maxwell equations with no source in curved space-time are

$$\nabla^\mu F_{\mu\nu} = 0 \quad (27)$$

$$\nabla_a F_{\mu\nu} + \nabla_\nu F_{a\mu} + \nabla_\mu F_{\nu a} = 0 \quad (28)$$

in which $F^{\mu\nu}$ is electromagnetic field tensor.^[8] Once again with covariant derivative ∇^μ on Eq. (28), using Eq. (27) and relation

$$(\nabla_\beta \nabla_a - \nabla_a \nabla_\beta) F_{\mu\nu} = R_{\mu\alpha\beta}^\rho F_{\rho\nu} + R_{\nu\alpha\beta}^\rho F_{\mu\rho} \quad (29)$$

where $R_{\beta\mu\nu}^a$ is Riemann curvature tensor, we can obtain the wave equation of electromagnetic waves^[26]

$$\nabla^2 F_{\mu\nu} - R_{\mu}^\rho F_{\rho\nu} - R_{\nu}^\rho F_{\rho\mu} + 2R_{\mu\rho\nu}^\beta F^{\rho\beta} = 0 \quad (30)$$

$R_{\mu\nu}$ is Ricci tensor, defined as

$$R_{\mu\nu} = R_{\mu\alpha\nu}^\alpha \quad (31)$$

Note that here we use variable $F_{\mu\nu}$ rather than electromagnetic vector potential A^μ in wave equations, this is because that the polarization of laser can not be expressed explicitly when using A^μ . In the following we will find the polarization of laser light is significantly linked to the GWs' detection.

At first we assume that the GWs propagate

along the Z axis, and we check the electromagnetic wave equation on the X and Y axis. As the light is transverse wave, laser propagating in the X direction only has y, z component of electromagnetic field, and the one propagating in the Y direction only has x, z component, et. al. Taking $\mu\nu=01$, the wave equation (30) gives

$$g^{\mu\nu}\partial_\mu\partial_\nu F_{01} + A\omega_g^2 \cos[\omega_g(t-z)]F_{01} - A\omega_g \sin[\omega_g(t-z)](F_{21,2} + F_{01,0} + F_{03,1} - F_{01,3}) = 0 \quad (32)$$

Taking $\mu\nu=02$,

$$g^{\mu\nu}\partial_\mu\partial_\nu F_{02} + A\omega_g^2 \cos[\omega_g(t-z)]F_{02} + A\omega_g \sin[\omega_g(t-z)](F_{12,1} + F_{02,0} + F_{03,2} - F_{02,3}) = 0 \quad (33)$$

Taking $\mu\nu=03$,

$$g^{\mu\nu}\partial_\mu\partial_\nu F_{03} + A\omega_g \sin[\omega_g(t-z)](F_{13,1} - F_{23,2} + F_{01,1} - F_{02,2}) = 0 \quad (34)$$

Note that all the A^2 terms and higher orders of the metric in Christoffel symbols $\Gamma_{\mu\nu}^\alpha$ and Ricci tensor $R_{\mu\nu}$ are neglected.

From wave Eqs. (32) ~ (34), we can see that the electromagnetic wave has a strong dependency both on the amplitude and the frequency of the GWs. However, if we neglect the frequency ω_g terms at first, we can get the answer for the above section, that is whether the laser ruler expands or contracts in the same way as the length of the arms? The equations of the plane electromagnetic wave shown in the Fig. 1 should be

$$\left[\frac{\partial^2}{\partial t^2} - \frac{1}{1-h} \frac{\partial^2}{\partial x^2}\right] E_y = 0 \quad (35)$$

$$\left[\frac{\partial^2}{\partial t^2} - \frac{1}{1+h} \frac{\partial^2}{\partial y^2}\right] E_x = 0 \quad (36)$$

The first equation is the wave equation along the X arms, and another one is for the Y arms. One can see that the velocity of the wave phase can be faster or slower than the velocity of light in the vacuum in the coordinate space. However, this always happens in the general relativity theory in which the coordinates may be not physical. The circumstance is contrary to the assumption of universal velocity in the coordinate space which is

talked in the above section. Thus the phase shift should not be calculated by the difference between the length in the coordinates space. In fact, these equations show that the laser ruler, namely the wavelength, expands or contracts in the same way as the two arms of the interferometer. The local measurement can not tell the variation of the metric. To show it clearly, we can use the local physical proper distance X and Y

$$X = \sqrt{1-h} x \quad (37)$$

$$Y = \sqrt{1+h} y \quad (38)$$

Then the wave equation in the physical distance and time is

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial X^2}\right] E_y = 0 \quad (39)$$

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial Y^2}\right] E_x = 0 \quad (40)$$

Eqs. (39)(40) are exactly the same as Eq. (19), the electromagnetic wave equation derived by the Maxwell equations in the vacuum of the flat space. In fact, the local proper length of the arm is a physical quantity. The wave equations (39) (40) show the invariance of the physical velocity of light which is the principle of the relativity. This means that using the apparatus and the linearly polarized plane wave shown in the Fig. 1 can not measure the variation of the arms induced by the metric. It seems that only the local measurement of effects of the extra ω_g terms in the Eqs. (32) ~ (34) can show the difference between the arms and the ruler. This is a little subtle work, we need use some requirements on the variables appearing in the equations to find the solution of the wave equations and realize the real measurement.

The wave equations of electromagnetic waves consist of six partial differential equations of $F_{\mu\nu}$. The transverse laser implies that some components of $F_{\mu\nu}$ absent in specific propagation direction. For laser propagating in Y direction, we can chose F_{01} and F_{12} for consideration. Thus we require that

$$F_{02} = F_{31} = F_{03} = 0 \quad (41)$$

$$F_{21,2} = F_{01,0} - F_{31,3} + \mathcal{O}(A) \quad (42)$$

$$F_{01,3} = F_{03,1} + F_{31,0} + \mathcal{O}(A) \quad (43)$$

In which we omit all the higher orders terms in Eqs. (32) ~ (34). The resulted two Eqs. (42) (43) are in fact the components of the Maxwell equations. These requirement are based on two reasons: first, linear polarization of the plane wave, second, A^2 terms and higher terms in Eqs. (32) ~ (34) are subleading terms. Under these requirements, the wave equations of F_{01} , which describe the linear polarization laser propagating in Y direction, now has the form of

$$(\partial^2 - 2c_1 \partial_t + c_2)E_x = 0 \quad (44)$$

Similarly, the wave equation propagation along X axis is

$$(\partial^2 + 2c_1 \partial_t + c_2)E_y = 0 \quad (45)$$

For E_z along X or Y axis, the equation is

$$(\partial^2 - c_1 \partial_t)E_z = 0 \quad (46)$$

In the above equations, we denote

$$c_1 \equiv A\omega_g \sin(\omega_g t + \eta) \quad (47)$$

$$c_2 \equiv A\omega_g^2 \cos(\omega_g t + \eta) \quad (48)$$

We also assumed that $z = 0$. These two terms c_1, c_2 are variables which depends on the amplitude A and circular frequency ω_g of the GWs. They are much smaller compared with the frequency of electromagnetic waves in the experiment, thus they can be considered as a constant in the wave equations. We can see that there is no c_2 term and c_1 term lacks a factor 2 in Eq. (46).

This is because that the polarization is along the direction of the GWs, the special direction in the circumstance.

Wave equations (44) and (45) are typical telegrapher's equations, in which the c_1 term is called the dissipation term, and the c_2 term is the dispersion term. Usually the telegrapher's equation describes the voltage and current on an electrical transmission line with distance and time^[27], as shown in the Fig. 3 in which we show the TE₀₁ mode in the waveguide. Note that the phase velocity in the waveguide can always exceed the velocity of light in the vacuum in the steady state. Wave equations (44) and (45) describe the influence of a passing GWs on the propagation of electromagnetic waves. Like electromagnetic waves in a waveguide with resistance, the phase is propagating along Z axis, forming the standing waves are along X and Y direction. The width of the Y boundary of the waveguide is a , which determines the the wavelength of the standing wave along Y axis. In the case of the real waveguide, dispersion coefficient c_2 is determined by the widths of the waveguide, i. e. TE_{*mn*} mode

$$c_2 = \frac{m^2 \pi^2}{a^2} + \frac{n^2 \pi^2}{b^2} \quad (49)$$

in which b is the width of the X boundary. In the background of the GWs, the phase of the GWs

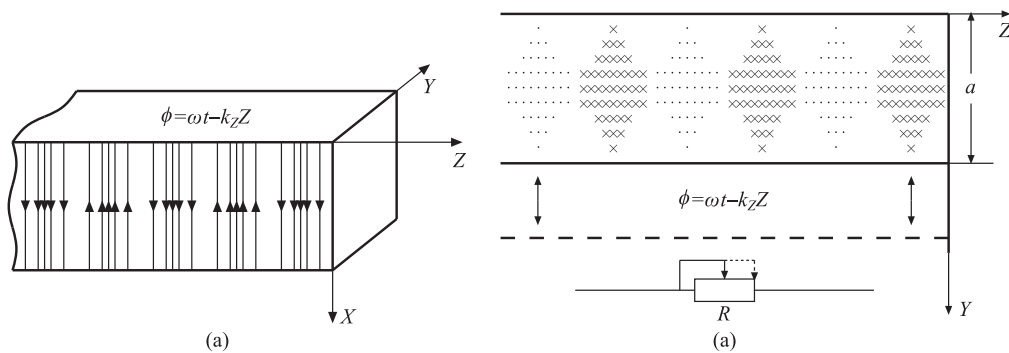


Fig. 3 The TE₀₁ wave mode in the waveguide. The background of the GWs leads to dissipation and dispersion in the propagation of electromagnetic wave. Like electromagnetic waves propagating in a waveguide with resistance, as shown in the figure, the phase is propagating along Z axis, forming the standing waves are along X and Y direction. The phase of the GWs control the dissipation c_1 term and dispersion c_2 term

(a) the sketch of waveguide; (b) Profile of TE₀₁ mode

control the dissipation c_1 term and dispersion c_2 term. However, as there is no boundary of the laser, c_2 can not be considered as coming from the widths of a waveguide. There should be some additional parameters which is determined by the laser. We will talk about them in the following. If there is no GWs, wave equations will return to the ordinary electromagnetic wave equation. The dissipation term, arising from the Christoffel symbols $\Gamma_{\mu\nu}^a$, causing an effective GWs-frequency-dependent resistivity which acts like a sliding resistance as shown in the right plot of the Fig. 3. The dispersion term, arising from the second derivative of metric, provides an effective square of wave number k_{em}^i which determine the phase transition along Z axis.

As there are dissipation and dispersion in the wave propagation, we can simply propose a steady-state solution in the TE_{01} mode for the waves along different arms, i. e. $E_x(t, Y)$ along the Y arm is

$$E_x(t, Y) = E \exp(-\delta_1 Y) \sin(p_1 Z) \times \exp(i(\omega_{em} t - \beta_1 Y + \phi_1)) \quad (50)$$

$E_y(t, X)$ along the X arm is

$$E_y(t, X) = E \exp(-\delta_1 X) \sin(p_2 Z) \times \exp(i(\omega_{em} t - \beta_2 X + \phi_2)) \quad (51)$$

In above equations $p_{1,2}$ are the additional parameters which is determined by the laser. In the ordinary telegrapher's equation

$$p_i^2 = c_2 \quad (52)$$

As talked above, this condition is not necessarily hold in the background of the GWs. For the simplicity, In the following, we consider the laser is plane wave which implies that the variation of the field along Z direction can be neglected. In this case, Eqs. (50) (51) can be rewritten as

$$E_x(t, Y) = E \exp(-\delta_1 Y) \exp(i(\omega_{em} t - \beta_1 Y + \phi_1)) \quad (53)$$

$$E_y(t, X) = E \exp(-\delta_2 X) \exp(i(\omega_{em} t - \beta_2 X + \phi_2)) \quad (54)$$

Similarly $E_z(t, X_\perp^i)$ can be written as

$$E_z(t, X_\perp^i) = E \exp(-\delta' X_\perp^i) \cdot \exp(i(\omega_{em} t - \beta' X_\perp^i + \phi_3)) \quad (55)$$

In above equation, $X_\perp^i$ means the directions perpendicular to the propagation direction. $X_\perp^i$ can be X or Y . $\phi_{1,2,3}$ are the phase parameters, $\beta_{1,2}$, β' are the wave numbers. In the language of waveguide physics, $1/\delta_{1,2}$, $1/\delta'$ is the skin depth, they cause an exponential depression of the amplitude E along propagating axis. Eqs. (53)~(55) can also have an increasing form of the amplitude which we will not discuss in the context. We assume that the intensity of the photon flux is dominantly controlled by the phase of the electromagnetic waves. Solving the Eqs. (44)~(46), we can get

$$\delta_1 = \delta_2 = \frac{\omega_{em} c_1}{\beta} \quad (56)$$

$$\beta_1^2 = \beta_2^2 = \beta^2 = \frac{\omega_{em}^2 - c_2 + \sqrt{(\omega_{em}^2 - c_2)^2 + 4\omega_{em}^2 c_1^2}}{2} \quad (57)$$

$$\delta' = \frac{\omega_{em} c_1}{2\beta'} \quad (58)$$

$$\beta'^2 = \frac{\omega_{em}^2 + \sqrt{\omega_{em}^4 + \omega_{em}^2 c_1^2}}{2} \quad (59)$$

We can see that c_1 and c_2 which are from the parameter set of the GWs determine the wave vector k_{em}^i . Note that c_1 terms in β and β' always appear in quadratic form, which can be ignored in the following estimation of the phase propagation. This means that the differences between the phase velocity dominantly comes from the c_2 term. While wave equation of E_z of Eq. (55) can be distinguished from E_x and E_y , through a different wave number β' and a factor 2 in c_1 term. The phase velocity of E_x , E_y and E_z is

$$v_x = \frac{\omega_{em}}{\beta} \quad (60)$$

$$v_y = v_x \quad (61)$$

$$v_\perp = \frac{\omega_{em}}{\beta'} \quad (62)$$

The plane wave solution of Eq. (53) and Eq. (54) has the same wave phase velocity. As the interference pattern dominantly determined by the phase, we can ignoring the dissipation term including δ and δ' . The total light intensity P for the plane wave E_x , E_y is

$$P \simeq (E_x + E_y)^2 = P_0 \sin^2[\beta(X - Y)] \quad (63)$$

Since the physical length of two arms are invariant, and the phase velocity is the same too, if the propagation wave along X and Y arms obey these two equations, there will be no changing of interference pattern. This case is similar to the case that ignoring the linear A terms of the wave equation, as talked above. Two linearly polarized laser beams interfere mutually. The same pattern of expansion and contraction leads to an invariant proper distance, eliminating any signal of the GWs. The same form of electromagnetic wave number is another factor leading to this situation. Neither the dissipation term nor the dispersion term contribute to the total light intensity formula. In the case of the invariant proper distance, only different wave number of two laser light can cause a phase shift. Such circumstance could happen in the consideration of a specific linearly polarized laser. From the Eqs. (60) ~ (62), we can see that interference of E_z with E_x or E_y can give a local measurement on the metric.

Take E_x and E_z interference as an example. Despite of the invariant proper distance, different wave number will generate a phase shift, leading to an ideal optical signal. However, the polarization directions of E_x and E_z are perpendicular to each other, we need additional apparatus to realize the interference. In fact, in the interference there is a special direction: the direction of the GWs. Thus, any change of the interference pattern should involve in this special direction. For example, we can use a specific polaroid to change the polarization direction of the laser before arriving at photodetector. An optical signal could possibly be detected, indicating a passing GWs propagating in Z axis direction. Or else, We need one linearly polarized laser beam propagating in the direction of the incoming GWs to give a different wave number compared with another a paralleled polarized plane wave. This could be realized by disposing an arm along Z axis with another one unchanged. In this situation, an interference pattern could be formed at photodetec-

tor without a crystal. The detail will be discussed in the following section.

3 Inference on the Gravitational Waves and Comparison with the Current Detection Results

In this section, we show the phase shift of our result and the implication on the GWs. Also we will discuss and compare the result with the detection results in the market. First we should make clear about the difference between the detection of the GWs in the above two section. All the detection are based on the change of the interference pattern which gives the phase shift of electromagnetic wave in the detector of interferometer. In the sec. 2, the phase shift comes from the changing of the difference between the lengths of the two arms in the coordinate space. The velocity in the coordinate space is invariant. In the sec. 3, we calculate the electromagnetic waves in the curved space, finding that the length of the wave expands and contracts accompanying with the arms. It's hard to detect the difference of length of the arms. Only special interference mode can give proper phase shift. We think only these solutions can give a more robust principle of the detection of the GWs. Thus in the following we show the detail to infer the amplitude and frequency of the GWs in the interference of the laser.

As talked in the above section, only the interference between the laser along the direction of the GWs and along other direction can give proper local measurement of the GWs. Thus the modification of interference is illustrated in the left plot of Fig. 4, in which two polaroids A and B generate two orthogonal linearly polarized plane laser beams. A special crystal C makes the two beams polarized to a direction with included angle equals to 45° between them, forming an interference pattern. However, there is no design of these apparatus equipped in the detectors of the GWs in the market. Thus, the only left possibility of the signal of the GWs detected on

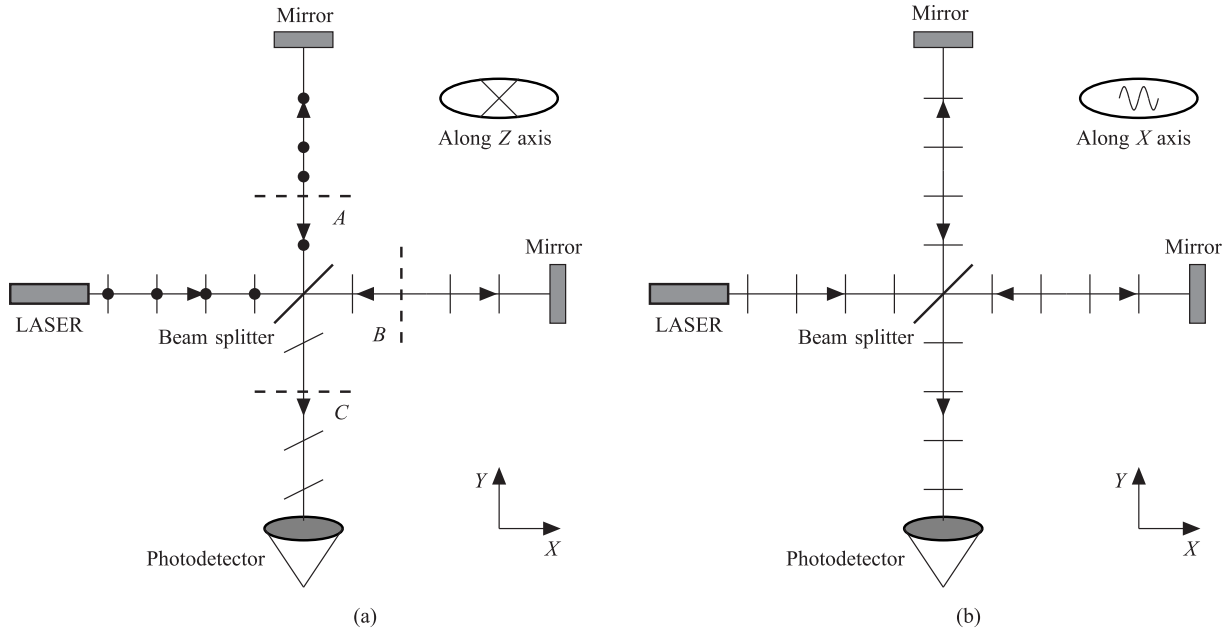


Fig. 4 Interference of electromagnetic waves in the Michelson interferometer

(a) the GWs propagate along Z axis, wave which emits from the laser is unpolarized, and A, B and C are Polaroids. A (B) makes the laser along Y (X) axis polarized along Z (Y) axis. C makes the two mutually interference of the two beams polarized to a direction with included angle equals to 45° between the two beams; (b) the GWs propagate along X axis, wave which emits from the laser is polarized along the XOY plane can interfere. The change of the interference pattern give a signal of the GWs

the interferometer is that the GWs must propagate along one arm of the interferometer. Say, along the X axis, which is shown in the right panel of the Fig. 4. In this case, a phase shift could be generated without additional polaroids. Thus we will take this situation to check the phase shift measured by the LIGO collaboration.

As for GWs propagating in X axis direction, the laser propagating in Y axis direction only has E_x component, laser propagating in X axis direction only has E_y component. In this circumstance, the wave equation along Y axis will change to the Eq. (55) with corresponding change of the E_z to E_x . However, the wave equation along the X axis is not exactly the telegrapher's equation. It has to change to

$$\left[\partial^2 + c_1 \partial_t - c_1 \frac{\partial}{\partial X} + c_2 \right] E_y = 0 \quad (64)$$

according Eq. (33). The solution of the equation is similar, we can assume that

$$E_x(t, Y) = E \exp(-\delta' Y) \exp(i(\omega_{em} t - \beta' Y + \phi_1)) \quad (65)$$

$$E_y(t, X) = E \exp(-\delta X) \exp(i(\omega_{em} t - \beta X + \phi_2)) \quad (66)$$

The result of δ' and β' are the same as Eqs. (58) (59). δ and β become much more complicated, i. e.

$$\delta = \frac{(\omega_{em} + \beta)c_1}{2\beta} \quad (67)$$

To get the dispersion relation one needs to solve the following equation

$$\omega_{em}^2 + \frac{(\omega_{em} + \beta)^2 c_1^2}{4\beta^2} - \beta^2 - \frac{(\omega_{em} + \beta)c_1^2}{2\beta} - c_2 = 0 \quad (68)$$

The analytical expression of β is complicated, However, as talked above, the c_1^2 term can be neglected, the phase difference dominantly comes from c_2 terms. Thus the analytical β is not shown here for its little influence on the phase shift. The total light intensity formula will be

$$P \simeq P_0 \sin^2(\beta' X - \beta Y) = \frac{P_0}{2} [1 - \cos(\Delta\beta L)] \quad (69)$$

The differences in wave number $\Delta\beta = \beta' - \beta$ could

cause a phase shift and generate an optical signal. Note that, in the ordinary detection principle talked in the sec. 2, the phase shift comes from ΔL , Here the phase shift comes from the change of the wave number. In SI unit, use the formulas (57) and (59) up to the first order of A ,

$$\Delta\beta(A) \simeq aA \quad (70)$$

in which

$$a = \frac{\omega_g^2 \cos(\omega_g t + \eta)}{2\omega_{em} c_g} \quad (71)$$

The dimension of a is L^{-1} , and c_g is the speed of GWs and we assume c_g equals to the velocity of light in the vacuum.^[7] We can see that there is a suppression factor ω_g/ω_{em} , which implies that if the frequency of the electromagnetic wave is much greater than the frequency of the GWs, a will be highly suppressed by the factor. This is different from the ordinary detection principle in which the frequency of the GWs is absent. The total phase shift can be obtained

$$\Delta\varphi_{em} = \Delta\beta L \quad (72)$$

We use the phase shift detected by LIGO detectors, (Which we calculated from Eq. (24) by using the h measured by LIGO.) we can obtain the amplitude A by

$$A = \frac{\Delta\varphi_{em}}{aL} \quad (73)$$

If we chose the frequency of the GWs at 100Hz and the frequency of the laser is 10^{14} Hz, then we can get the amplitude A will be order 1 which is much greater than the amplitude assumed in the community.^[20] Is our consideration and calculation wrong? We don't think so. The case is much complicate than the usual one. Detection of the amplitude A depends on the frequency of the GWs and the propagation of the electromagnetic waves. If the GWs contain some high frequency modes, say $\nu_g \sim 10^9$ Hz, the corresponding amplitude will be

$$A \sim 6.3 \times 10^{-12} \quad (74)$$

which is much comfortable for the audience. In fact, such high frequency modes are absolutely

needed in the detection of the GWs, which we will show in the following.

Here we give a summary about what we are doing now. In sec. 3 we find that a local measurement of the metric is impossible if we ignore the differential of the metric. In this section, we chose the special direction of the GWs and make the measurement possible. We emphasize that the key point in the theoretical side is second derivative of metric in Eq. (30) gives the dominant c_2 terms. c_1 terms which are the first derivative of metric are subleading terms. While the absolute value of metric are in fact irrelevant in local measurement.

All the discussions in above section are in assumption that a plane gravitational wave passing through the interferometer. However, the signals of the GWs are all in fact pulsed signals which last less than one second. If we are serious about this, at first, the GWs seems to be a pulsed wave which is shown in the left panel of Fig. 5. If we use the Eqs. (44)~(46) to research the interference, we need to Fourier expansion of $h(t)$ to get amplitude of every plane wave modes $H(\omega_g)$ and calculate the corresponding phase shift $\Delta\varphi(\omega_g)$. The convolution of $\Delta\varphi(\omega_g)$ in the ω_g space can give the final phase shift to be measured. Nevertheless, every interference pattern are in fact a steady mode which results from an assumption that the interference relaxation time of electromagnetic wave is much shorter than the period of the GWs. This condition can not hold in case of the appropriately high frequency of the GWs.

For simplicity, we consider the GWs is a wave packet composed of a monochromatic wave

$$h(t) = A \sin \omega_0 t \quad (75)$$

in which ω_0 is the monochromatic frequency, and there are $2n$ period during the pulse which implies that the duration time t is

$$-n \frac{2\pi}{\omega_0} < t < n \frac{2\pi}{\omega_0} \quad (76)$$

As talked in above paragraph, we need the corre-

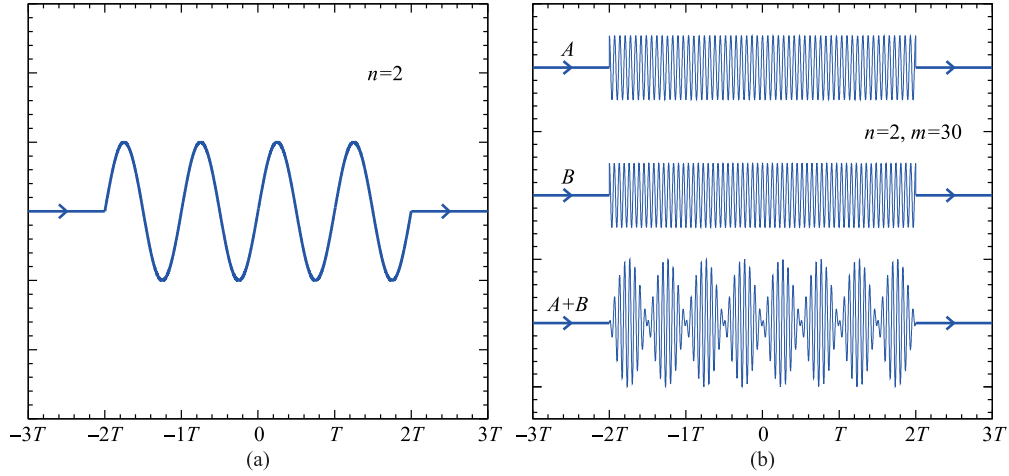


Fig. 5 A pulsed monochromatic wave and a pulsed beats of the GWs

the left plot is pulsed monochromatic wave with 2×2 periods and the right one is a pulsed beats which frequency equals to the monochromatic frequency of the left plot. There are $m = 30$ periods during one beat

sponding Fourier expansion $h(t)$ for the calculation of the phase shift

$$\begin{aligned} H(\omega_g/\omega_0) &= \frac{A}{2\pi} \int_{-\frac{2n\pi}{\omega_0}}^{\frac{2n\pi}{\omega_0}} \sin \omega_0 t \sin \omega_g t dt \\ &= H_0 f_1(\omega_g/\omega_0) \end{aligned} \quad (77)$$

in which

$$H_0 = \frac{nA}{\omega_0} \quad (78)$$

and

$$\begin{aligned} f_1(x) &= \frac{1}{2n\pi} \left[\frac{\sin[(1-x)2n\pi]}{1-x} - \frac{\sin[(1+x)2n\pi]}{1+x} \right] \end{aligned} \quad (79)$$

The amplitude in the frequency space is shown in

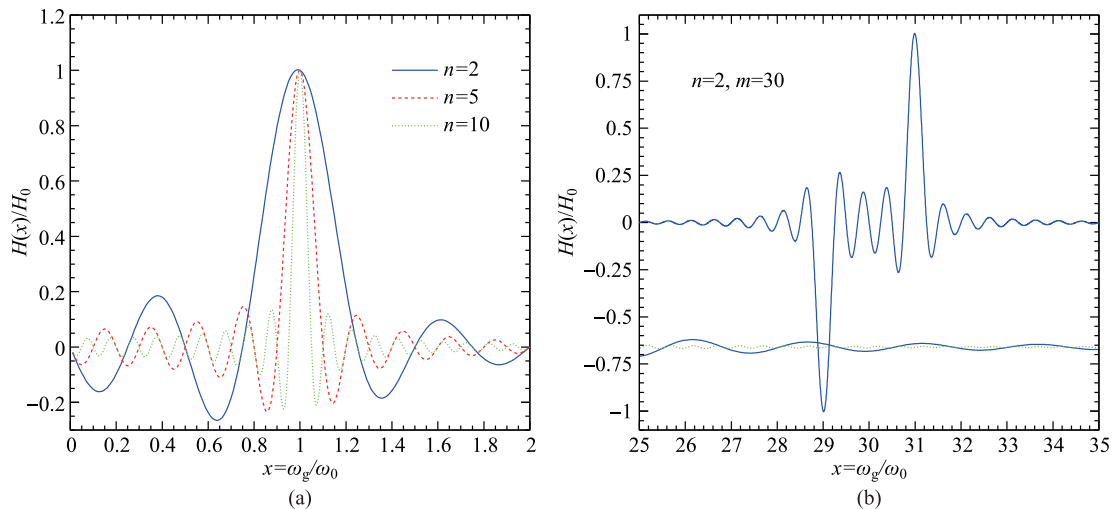


Fig. 6 Same as Fig. 5, but shows the Fourier expansion of the pulsed monochromatic wave (left) and pulsed beats (right)

the in left panel of Fig. 6 in which we show $n = 2, 5, 10$, respectively. We can see that though the ω_g has the largest amplitude at ω_0 , there are sizeable amplitudes at other frequencies, especially when n is smaller. However, they decay quickly with the growth of the deviation from the monochromatic frequency ω_0 . We can imagine that the convolution of the $\Delta\varphi$ is still depends on ω_0 , thus, low frequency of the GWs still implies that the amplitude A is incredibly large.

What we will do now is to propose a pulsed beats which is a special phenomenon in the waves to escape from the dilemma. The synthesis of two harmonic oscillations with different but al-

most equal frequency gives the beats which shown in the right panel of the Fig. 5.^[28] The amplitude of the beats is not a constant but a modulation of almost harmonic oscillation. Frequency of beats is much smaller than the frequency of the harmonic oscillation. For example, we chose a beats as

$$h(t) = A \sin \omega_0 t \cos m \omega_0 t \quad (80)$$

in which $A \sin \omega_0 t$ is the amplitude of the modulation. For simplicity, m is an integer which can be much greater than n . If the beats is also pulsed as shown in the right panel of Fig. 5, in which we take $n=2, m=30$ for example. Fortunately, we can get a much larger frequency amplitude in the frequency space. The Fourier expansion $h(t)$ is similar

$$\begin{aligned} H(\omega_g/\omega_0) &= \frac{A}{2\pi} \int_{-\frac{2n\pi}{\omega_0}}^{\frac{2n\pi}{\omega_0}} \sin \omega_0 t \cos m \omega_0 t \sin \omega_g t dt \\ &= H'_0 f_2(\omega_g/\omega_0) \end{aligned} \quad (81)$$

in which

$$H'_0 = \frac{nA}{2\omega_0} \quad (82)$$

and

$$\begin{aligned} f_2(x) &= \frac{1}{2n\pi} \times \\ &\left[\frac{\sin[(m+1)-x]2n\pi}{[(m+1)-x]} - \frac{\sin[(m+1)+x]2n\pi}{[(m+1)+x]} - \right. \\ &\left. \frac{\sin[(m-1)-x]2n\pi}{[(m-1)-x]} + \frac{\sin[(m-1)+x]2n\pi}{[(m-1)+x]} \right] \end{aligned} \quad (83)$$

Numerical results of $f_2(\omega_g/\omega_0)$ are shown in the right panel of the Fig. 6. We can see that frequency with maximum amplitude can be much greater than the ω_0 . This means that we can choose a pulsed beats with a much higher frequency of the almost harmonic oscillation and very low frequency of beats. This may be a similar signal obtained by the LIGO. Note that, the simple gravitational wave equation Eq. (3) are obtained under the low frequency ω_g assumption. In case of a high frequency, gravitational wave equation is not a simple linear wave equation. The non-linear wave equation and the solution

are beyond this work^[29]. The high frequency ω_g implies a small wavelength of the GWs, thus the calculation of the phase shift along the direction of the GWs may be not a local quantity.

However, one may suspect on the assumption above that the detector only shows a pulsed beats. As the oscillation of the metric is so fast, the interference pattern may be able to be detected. We should argue that in case of the much faster harmonic oscillation, the relaxation of the interference must be considered. This can be understood as following: the laser beams need a relaxation time to form a steady-state, which can be evaluated by time interval of the laser propagating along the arms of the interferometer. For LIGO experiment the relaxation time Δt_r

$$\Delta t_r \simeq \frac{4\text{km}}{c} \simeq 10^{-5} \text{s} \quad (84)$$

If the frequency of GWs is much higher, i. e. $\nu_g = 10^9$ Hz which has been mentioned above, the period of the GWs will be about of 10^{-9} s which is much smaller than the Δt_r , a signal will be disturbed before it forms into a steady-state. Thus a clean signal in fact is not able to be detected. Only the modulation of the pulsed beats is detected. The detail of the detection of the relaxation and the pattern is beyond this work. Note that, $\nu_g = 10^9$ Hz implies that the amplitude is still much greater than the amplitude predicted by the numerical simulation of coalescence of two black holes. In this work, we concentrate on the detection of the GWs, as for the source of the GWs, we argue that, coalescence of two black holes may be much complicate than the ordinary simulation.^[30]

In all, the key point of our paper is that it is very difficult to detect the variation of the metric in a local measurement. Even we solve the Maxwell equation in the curved space, we can see that, only introducing a much complicated pulsed beats can give an appropriate explanation of the detection result of the GWs in the market. Similar to that the phenomenon of redshift verifies

the theory of the general relativity, a globally comparison of length of time can easily verify the variation of the metric. Thus we propose a non-local interference which is shown in the Fig. 7 to detect the high-frequency GWs. The setting of the apparatuses is show in the figure. In the experiment, the initial laser beam is divided into two beams by the beam-splitter and mirror, the divided beams propagate in an opposite direction along the Z axis. After that they are reflected to propagate along X axis (horizon line in the Fig. 7) at different coordinate of Z axis. The distance between them is d . The two beams meets each other at the detectors by reflections. Assume a GWs is propagating along the Z axis, the upper arm and the lower arm have different wave phase. The changing of interference pattern in the detector will give a global measurement of small difference between the lengths of the upper and lower arms. This is similar to the measurement of redshift caused by gravitation.

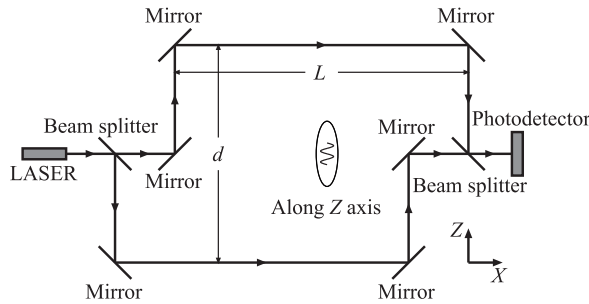


Fig. 7 A proposal of a detection of the GWs with high frequency of beats, the GWs propagating along Z axis, and the length between the two neighbour beats is assumed to be d . The upper and the lower horizon lines expand or contract when the GWs passing by. The no local interference can give change of the interference pattern at the detector

The best event of the GWs will be that the distance d happens to be equal to one half of the GWs' length. Then the difference of the two arms will be largest in the duration of the GWs. There is always a phase difference of the GWs between two arms. Suppose the gravitational phase of upper arm is η_1 , and η_2 for lower arm.

To generate the phase shift of laser light, we consider the wave equation of E_y and E'_y , for laser travelling along the upper and the lower arms respectively

$$[\partial^2 + 2c_1(\eta_1)\partial_t + c_2(\eta_1)]E_y = 0 \quad (85)$$

$$[\partial^2 + 2c_1(\eta_2)\partial_t + c_2(\eta_2)]E'_y = 0 \quad (86)$$

Where $c_1(\eta_i)$ and $c_2(\eta_i)$ refer to

$$c_1(\eta_i) = A\omega_g \sin(\omega_g t + \eta_i) \quad (87)$$

$$c_2(\eta_i) = A\omega_g^2 \cos(\omega_g t + \eta_i) \quad (88)$$

These wave equations lead to the same expression of wave number but there is a phase difference between them

$$\Delta\beta = \beta(\eta_2) - \beta(\eta_1) \quad (89)$$

Two beams of laser will interfere, the corresponding phase shift can be obtained by the same expressions above. Up to the first order of amplitude A

$$\begin{aligned} \Delta\varphi_{em} &= \Delta\beta L \\ &= \frac{A\omega_g^2 L}{2\omega_{em} c_g} [\cos(\omega_g t + \eta_1) - \cos(\omega_g t + \eta_2)] \end{aligned} \quad (90)$$

Similarly, this phase shift leads to an optical signal for photodetector.

We know that, the experiment we proposed above always suffers the space it occupies. d can not be too large, say about 3km, implying that a high frequency of the GWs. Thus the respondent time and the sensitivity of the detector will be a problem in the real experiment. However, similar circumstance can be detected too, that the experiment can detect pulsed beats, which can give a global measurement of the beats. Together with the ordinary detector as shown in the Fig. 4, they will give us a further and mutually measurements of the GWs.

4 Conclusion

In the work, we checked laser interference in the background of the GWs. In the ordinary principle of the interference measurement, the laser is assumed as a flux of photon with a speed of light which is unaffected by the GWs. The

measurement is the variation of intensity which can give the phase shift of the electromagnetic waves, and then the length displacements of the arms in the interferometer. However, we find that the expansion and contraction of wave lengths are always synchronous with the arms. We must be very careful on the local measurement of the variation of metric. Like electromagnetic waves propagating in a waveguide with resistance, the background of the GWs leads to dissipation and dispersion in the propagation of electromagnetic wave. The phase of the GWs controls the dissipation term and dispersion term in the telegrapher's equation. In fact, in the interference of laser in the local measurement, there is the special direction which is the direction of the GWs. Any change of the interference pattern should involve in this special direction.

In order to detect the GWs, we can use a specific polaroid to change the polarization direction of the laser before it arriving at the photodetector. An optical signal could possibly be detected, indicating a passing GWs. Or else, we need one linearly polarized laser beam propagating in the direction of the incoming GWs to give a different wave number compared with the linearly polarized plane wave in another path. We use the phase shift measured by the LIGO to check the case of pulsed plane wave, finding that, the amplitude is incredibly large. However, we propose a pulsed beats to solve the problem. Our analysis show that, in case of the pulsed beats passing by, the relaxation time is greater than the period of the GWs, the signal will be disturbed before it forms a steady-state, thus a clean signal in fact is not able to be detected. The detector may only show a signal of the modulation of the beats.

We should comment that, in this paper, we emphasized that the local measurement can not show the variation of metric. This is because a local distance and time interval must be defined by the invariance of velocity of light in the vacuum. This is a fundamental principle of relativity. Only global comparison can show the difference

of the metric. Nevertheless, Einstein equation of the gravitation is the dynamics of the metric, Any physical quantity derived from Einstein equation must involve differential of the metric, not only the absolute value. Thus what we should concern in the expansion or the approximation of the gravitation are in fact the derivative of the metric. As shown in the paper, its effects are really what we can measure in the local experiment.

Finally we proposed a non-local interference experiment to detect the high-frequency GWs. It is similar to the measurement of redshift caused by gravitation. Together with the ordinary detector, it will give us further and mutually measurements of the GWs. Of course the detail of the relaxation of the interference pattern, et. al. needs our further studies.

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