

Predictive Proportional-Integral Control for Current Tracking of LCL-Type Grid-Connected Inverters Leveraging a FAS Approach

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Abstract—This study considers the current tracking control of the LCL-type grid-connected inverters by using a fully actuated system (FAS) approach. To tackle this problem, a FAS approach-based predictive proportional-integral (PI) control is constructed with simple design and better performance. Firstly, an equivalent transformation is presented to convert the model of the LCL-type grid-connected inverters and its general form into the input-delay FASs, so that the proposed work can be regarded as the tracking control of the input-delay FASs. Secondly, a predictive PI control is developed by means of the FAS approach to realize the desired tracking control for eliminating the original nonlinearities and compensating for the input delays. Then, a sufficient criterion is proposed for the bounded stability and tracking performance of the closed-loop systems. Finally, a compared simulation is shown to demonstrate the feasibility.

Index Terms—LCL-type grid-connected inverters, current tracking, FAS approach, predictive PI control, bounded stability and tracking performance.

ABBREVIATION

FAS	fully actuated system
PI	proportional-integral
CMV	common mode voltage
PCC	point of common coupling

I. INTRODUCTION

BECAUSE of large-scale grid connection of distributed renewable energy, e.g., photovoltaic [1] and wind power [2], the performances for the grid-connected inverters, including the power quality, capacity, and operation stability, have been required to raise to a higher rank. Considering the pro-

posed challenges, the LCL-type grid-connected inverter has been the mainstream choice because it can overcome the disadvantages of the traditional L-type and LC-type ones, e.g., insufficient harmonic attenuation, large size, and weak anti-disturbance ability. Hence, there exist numerous studies on the LCL-type grid-connected inverters to improve the adaptability of renewable energy grid connection and to guarantee the safe and stable operation of power systems [3], [4], [5]. Among them, the current tracking control can be regarded as a technical core to directly determine the quality of grid-connected current and the operational stability, which has attracted the great attention of numerous scholars.

Recently, [6] constructed an integrated division-summation control scheme to achieve the grid-connected current tracking, neutral-point potential balance control, resonance suppression, and low-frequency common mode voltage (CMV), the advantages of which are that the unobservability of sensorless CMV can be avoided and the low-frequency CMV can be compensated. Reference [7] adopted a harmonic linearization method to derive a third-order admittance matrix model with the frequency coupling of both AC-side and AC-DC for the LCL-type grid-connected inverters, and proposed a generalized Nyquist condition for the stability analysis. [8] established an observer-based control with the help of the impedance modeling and Nyquist stability condition to implement the current tracking control, and then discussed the closed-loop system stability by developing two interval stability conditions. [9] designed a strong adaptability control by using a dual-division-summation control to realize the current tracking and resonance suppression when existing the variation of grid impedance and filter inductance. In [10], an adaptive integral backstepping control method was derived to provide the maximum active power and adjust the reactive power under external disturbances. [11] offered an admittance shaping method to realize the current control of both inverter-side and grid-side, and then utilized a nested black-widow optimization algorithm to cope with the design of controller parameters. [12] provided a design idea of generalized multi-resonant controller for the current control, in which a new multi-optimized

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parameter tuning method was proposed to guarantee the control performances.

Although the results of [6], [7], [8], [9], [10], [11], [12] and their related studies make the positive contributions to the current tracking control of the LCL-type grid-connected inverters, a disadvantage is that the control design becomes more difficult because of the coupled and complex dynamics, as well as the sensitivity to parameter perturbation, containing the grid impedance variation, the shift of resonance point, etc. It is noted that the above problem can be expanded to a general one in the control design of complex nonlinear systems, instead of only a specific one in the certain scenario.

Towards this dilemma, a fully actuated system (FAS) approach, presented by Prof. Duan in [13], is regarded as a novel framework of the analysis and control for nonlinear systems. Its original idea, periodical progress, and future directions have been discussed in [14]. A technical core of the FAS approach is to develop a FAS model of the considered nonlinear systems for guaranteeing the full actuation feature, so that the original nonlinearities can be eliminated to obtain a linear constant system with the desired closed-loop performances. In this way, the control design can be effectively simplified and the control performances can be obviously improved. Because of the above advantages, a large number of scholars focus on the combination research on the FAS approach with other advanced control strategies, and have achieved several significant results in theoretical studies [15], [16], [17] and practical applications [18], [19], [20]. Among them, [21] designed a FAS approach-based controller containing a model transformation for the current tracking of the LCL-type grid-connected inverters with strict-feedback dynamics, and then took an experiment to verify the effectiveness and practicality. However, [21] achieved the above result in the continuous-time domain. Due to rapid development and wide application of digital control, discrete-time system is considered as a more suitable model in practice. It possesses more advantages in the flexibility of controller implementation, the ease of storage, transmission and processing, the simplicity in the realization of complex control algorithms, the reusability of hardware resources, etc. Combining the above with the improvement of digital standards for renewable energy power systems, it is important to further consider the FAS approach-based control

for the current tracking of the LCL-type grid-connected inverters in the discrete-time domain.

To continue to investigate the proposed problem, this study is concerned with a current tracking problem of the LCL-type grid-connected inverters in the discrete-time domain by using a FAS approach, where the major works can be drawn in the below. Firstly, a discrete-time strict-feedback dynamic model is adopted to describe the LCL-type grid-connected inverters. Secondly, an equivalent transformation is presented to convert a kind of general discrete-time strict-feedback systems into the input-delay FASs. Then, a predictive proportional-integral (PI) control is proposed to implement the tracking control with the elimination of original nonlinearities and the compensation of input delays. Further, a sufficient criterion is developed for the bounded stability and tracking performance of the closed-loop systems. To the end, a simulation is shown to demonstrate the feasibility. Compared to the existing studies, the highlights are concluded as follows.

(1) A new transformation is presented to construct a FAS model for discrete-time strict-feedback systems, which gives a model foundation of the FAS approach-based digital control and its associated application example in the current tracking of the LCL-type grid-connected inverters.

(2) The negative impacts of input delays can be actively removed by applying the proposed predictive control strategy, so that the closed-loop control performance is exactly the same as the case without input delays and the stability criterion is delay-independent with less conservatism.

II. PROBLEM FORMULATION

A. Strict-feedback model of LCL-type grid-connected inverters

The sketch of the LCL-type grid-connected inverter is given in Fig. 1, where u_{dc} is the capacitor voltage of DC bus and the LCL filter is adopted to connect the inverter and the grid. u_j and i_j ($j \in \{a, b, c\}$) are the three-phase voltage and current of the grid-side inverter, respectively. L_{fj} and R_{fj} ($j \in \{a, b, c\}$) are the three-phase inductance and parasitic resistance of the LCL filter approaching the inverter side, respectively. C_j and u_{cj} ($j \in \{a, b, c\}$) are the three-phase filter capacitor and the

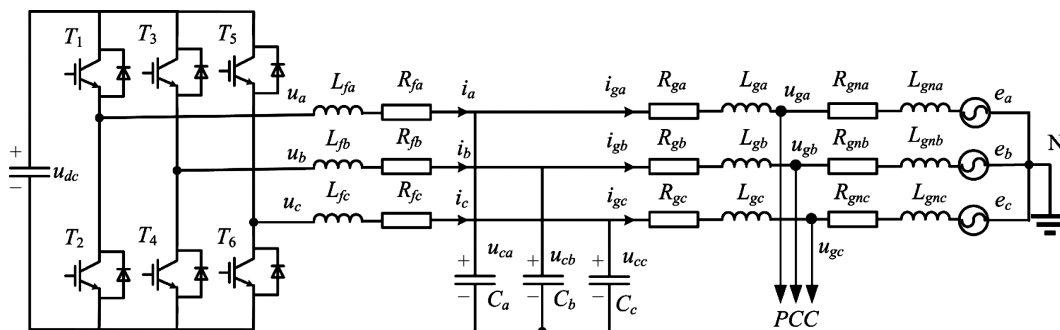


Fig. 1. The schematic diagram of the LCL-type grid-connected inverter.

corresponding voltage, respectively. i_{gj} and u_{gj} ($j \in \{a, b, c\}$) are the three-phase current and the three-phase PCC voltage of the grid side, respectively. L_{gj} and R_{gj} ($j \in \{a, b, c\}$) are the three-phase inductance and parasitic resistance of the LCL filter approaching the grid side, respectively. L_{gnj} and R_{gnj} ($j \in \{a, b, c\}$) are the three-phase inductance and parasitic resistance of the grid side line, respectively. N is the neutral point of the power grid.

Because of the Kirchhoff's Law, the mathematical model of the LCL-type grid-connected inverter can be established as

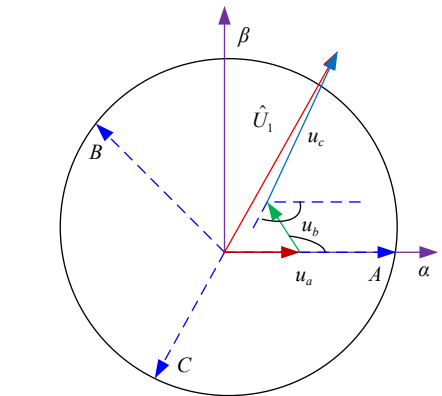
$$\begin{aligned} L_f \dot{i}_j &= u_j - u_{cj} - R_{fj} i_j, \\ L_{gj} \dot{i}_{gj} &= u_{cj} - R_{gj} i_{gj} - u_{gj}, \\ C_j \dot{u}_{cj} &= i_j - i_{gj} \end{aligned} \quad (1)$$

with $j \in \{a, b, c\}$. From [21], it is generally assumed that the three-phase parameters of the LCL filter are consistent, so that system (1) is balanced to ignore the zero-sequence component.

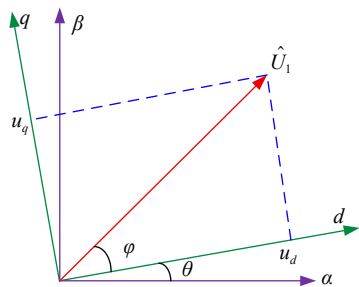
By taking the Clarke and Park transformations on system (1), i.e.,

$$\begin{aligned} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} &= \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}, \\ \begin{bmatrix} i_d \\ i_q \end{bmatrix} &= \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \end{aligned}$$

where the related coordinate systems are shown in Fig. 2, and



(a) The coordinate system of Clarke transformation



(b) The coordinate system of Park transformation

Fig. 2. The coordinate system of Clarke and Park transformations.

then the model of the LCL-type grid-connected inverter under the d - q coordinate system can be derived as

$$\begin{aligned} L_g \dot{i}_{gd} &= -R_g i_{gd} + \omega L_g i_{gq} + u_{cd} - u_{gd}, \\ L_g \dot{i}_{gq} &= -R_g i_{gq} + \omega L_g i_{gd} + u_{cq} - u_{gq}, \\ C \dot{u}_{cd} &= i_d - i_{gd} + \omega C u_{cq}, \\ C \dot{u}_{cq} &= i_q - i_{gq} + \omega C u_{cd}, \\ L_f \dot{i}_d &= -R_f i_d + \omega L_f i_q + u_d - u_{cd}, \\ L_f \dot{i}_q &= -R_f i_q + \omega L_f i_d + u_q - u_{cq} \end{aligned} \quad (2)$$

where i_d and i_q are the d -axis and the q -axis components of the inverter current i_j ($j \in \{a, b, c\}$), respectively, u_d and u_q are the d -axis and the q -axis components of the inverter voltage u_j ($j \in \{a, b, c\}$) respectively. u_{cd} and u_{cq} are the d -axis and the q -axis components of the filter capacitor voltage u_{cj} ($j \in \{a, b, c\}$) respectively. i_{gd} and i_{gq} are the d -axis and the q -axis components of the grid-connected inverter current i_{gj} ($j \in \{a, b, c\}$) respectively, and u_{gq} are the d -axis and the q -axis components of the point of common coupling (PCC) voltage u_{gj} ($j \in \{a, b, c\}$) respectively. ω is the angular frequency of the grid voltage. Based on [6], [7], all variables can be directly measured.

Let $x_1 = \begin{bmatrix} i_{gd} \\ i_{gq} \end{bmatrix}$, $x_2 = \begin{bmatrix} u_{cd} \\ u_{cq} \end{bmatrix}$, $x_3 = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$, and $u = \begin{bmatrix} u_d \\ u_q \end{bmatrix}$, system (2) can be compactly written as

$$\begin{aligned} \dot{x}_1 &= L_g^{-1} \left(\begin{bmatrix} -R_g & \omega L_g \\ \omega L_g & -R_g \end{bmatrix} x_1 - \begin{bmatrix} u_{gd} \\ u_{gq} \end{bmatrix} + x_2 \right), \\ \dot{x}_2 &= C^{-1} (-x_1 + x_3) + \begin{bmatrix} 0 & \omega \\ \omega & 0 \end{bmatrix} x_2, \\ \dot{x}_3 &= L_f^{-1} \left(-x_2 + \begin{bmatrix} -R_f & \omega L_f \\ \omega L_f & -R_f \end{bmatrix} x_3 + u \right), \\ y &= x_1 \end{aligned} \quad (3)$$

By utilizing a forward difference $\dot{x}_i = (x_i(t+1) - x_i(t))/T_p$ to discretize system (3), a discrete-time model of the LCL-type grid-connected inverter can be provided as

$$\begin{aligned} x_1(t+1) &= f_1(x_1(t)) + T_p L_g^{-1} x_2(t), \\ x_2(t+1) &= f_2(x_1(t), x_2(t)) + T_p C^{-1} x_3(t), \\ x_3(t+1) &= f_3(x_2(t), x_3(t)) + T_p L_f^{-1} u(t), \\ y(t) &= x_1(t) \end{aligned} \quad (4)$$

with

$$\begin{aligned} f_1(x_1(t)) &= T_p L_g^{-1} \left(- \begin{bmatrix} u_{gd}(t) \\ u_{gq}(t) \end{bmatrix} + \begin{bmatrix} T_p^{-1} L_g - R_g & \omega L_g \\ \omega L_g & T_p^{-1} L_g - R_g \end{bmatrix} x_1(t) \right), \\ f_2(x_1(t), x_2(t)) &= -T_p C^{-1} x_1(t) + \begin{bmatrix} 1 & T_p \omega \\ T_p \omega & 1 \end{bmatrix} x_2(t), \\ f_3(x_2(t), x_3(t)) &= T_p L_f^{-1} \left(-x_2(t) + \begin{bmatrix} T_p^{-1} L_f - R_f & \omega L_f \\ \omega L_f & T_p^{-1} L_f - R_f \end{bmatrix} x_3(t) \right) \end{aligned}$$

where T_p is the sampling period. On the basis of [22], system (4) can be considered as a kind of discrete-time strict-feedback systems, and then this study will further investigate the output tracking control problem of system (4) and its more general form.

Actually, the current tracking control for the LCL-type grid-connected inverters in the proposed work can be achieved if

$$y(t) = \begin{bmatrix} i_{gd}(t) \\ i_{gq}(t) \end{bmatrix} \rightarrow y^*(t) = \begin{bmatrix} i_{gd}^*(t) \\ i_{gq}^*(t) \end{bmatrix}, t \rightarrow \infty$$

where $y^*(t)$ is the desired reference signal. Hence, the objective of the proposed work is to design a control law $u(t)$ for system (4) to ensure that $y(t) \rightarrow y^*(t)$ with $t \rightarrow \infty$.

B. General Problem

Combining the extension of system (4) with the result of [22], a general class of discrete-time strict-feedback systems is given as

$$x_1(t+1) = f_1(x_1(t)) + g_1(x_1(t))x_2(t) \quad (5a)$$

$$x_i(t+1) = f_i(x_{1 \sim i}(t)) + g_i(x_{1 \sim i}(t))x_{i+1}(t), \\ i = 2, 3, \dots, n-1 \quad (5b)$$

$$x_n(t+1) = f_n(x_{1 \sim n}(t)) + g_n(x_{1 \sim n}(t))u(t) \quad (5c)$$

$$y(t) = x_1(t) \quad (5d)$$

with $x_{1 \sim i}(t) = \begin{bmatrix} x_1^T(t) & \dots & x_i^T(t) \end{bmatrix}^T$ ($i = 2, 3, \dots, n$), where $x_i(t) \in \mathbb{R}^m$ ($i = 1, 2, \dots, n$) are the state vectors, $u(t) \in \mathbb{R}^m$ is the control input, and $y(t) \in \mathbb{R}^m$ denotes the measured output. $f_i(\cdot, t) \triangleq f_i(x_{1 \sim i}(t)) \in \mathbb{R}^m$ and $g_i(\cdot, t) \triangleq g_i(x_{1 \sim i}(t)) \in \mathbb{R}^{m \times m}$ ($i = 1, 2, \dots, n$) indicate the known nonlinear vector and matrix functions, respectively.

Assumption 1 ([23]). $\forall i = 1, 2, \dots, n$, $x_i(t)$ can be directly available in the control design.

Assumption 2 ([24]). $\forall t \geq 0$, $\forall i = 1, 2, \dots, n$, $\forall x_{1 \sim i}(t)$, $\det(g_i(\cdot, t)) \neq 0$ or ∞ .

Problem 1. For system (2) under assumptions 1 and 2, this study is to construct an equivalent transformation $\mathcal{T}$, so that system (5a) with the output in system (5d) can be converted into a kind of input-delay FASs, i.e., the following FAS in system (6)

$$y(t+1) = \mathcal{F}(f_{1 \sim n}(x_{1 \sim n}^{[n_x, 0]}(t)), g_{1 \sim n}(x_{1 \sim n}^{[n_x, 0]}(t))) \\ + \mathcal{G}(g_{1 \sim n}(x_{1 \sim n}^{[n_x, 0]}(t)))\kappa(t) \quad (6)$$

with $\kappa(t) = u(t - \varsigma)$, $n_x = \varsigma = n - 1$, and

$$x_{1 \sim n}(t) = \begin{bmatrix} x_1(t) & \dots & x_n(t) \end{bmatrix}^T,$$

$$x_{1 \sim n}^{[n_x, 0]}(t) \triangleq \{x_{1 \sim n}(t-i) | i = 0, 1, \dots, n_x\},$$

$$f_{1 \sim n}(x_{1 \sim n}^{[n_x, 0]}(t)) \triangleq \{f_i(x_{1 \sim n}^{[n_x, 0]}(t)) | i = 1, 2, \dots, n\},$$

$$g_{1 \sim n}(x_{1 \sim n}^{[n_x, 0]}(t)) \triangleq \{g_i(x_{1 \sim n}^{[n_x, 0]}(t)) | i = 1, 2, \dots, n\}$$

In the FAS in system (6), $\mathcal{G}(\cdot, t) \triangleq \mathcal{G}(g_{1 \sim n}(x_{1 \sim n}^{[n_x, 0]}(t))) \in \mathbb{R}^{m \times m}$ and $\mathcal{F}(\cdot, t) \triangleq \mathcal{F}(f_{1 \sim n}(x_{1 \sim n}^{[n_x, 0]}(t)), g_{1 \sim n}(x_{1 \sim n}^{[n_x, 0]}(t))) \in \mathbb{R}^m$ represent the known nonlinear matrix and vector functions to be the equivalent dynamics during transforming system (5) to the FAS in system (6).

Because of assumptions 1 and 2, the following assumption 3 can be satisfied for the FAS in system (6).

Assumption 3. $\forall t \geq 0$, $\forall x_{1 \sim n}(t)$, $\det(\mathcal{G}(\cdot, t)) \neq 0$ or ∞ .

For a desired reference signal $r(t) \in \mathbb{R}^m$, the problem of output tracking control for system (5) can be equivalent to that of the FAS in system (6). To deal with this problem with compensating for the input delay ς , a FAS approach-based predictive tracking control is designed as

$$u(t) = \hat{\kappa}(t + \varsigma | t), \\ \hat{\kappa}(t + \varsigma | t) = \mathcal{G}^{-1}(\cdot, t) (\hat{u}_{pi}(t + \varsigma | t) - \mathcal{F}(\cdot, t)) \quad (7)$$

where the reason of using the real values of $\mathcal{F}(\cdot, t)$ and $\mathcal{G}(\cdot, t)$ instead of their prediction values is that $\mathcal{F}(\cdot, t)$ and $\mathcal{G}(\cdot, t)$ only include the historical available information of $x_i(t)$ because of Assumption 1. In system (7), $-\mathcal{F}(\cdot, t)$ is used to eliminate the original nonlinearities, $\hat{u}_{pi}(t + \varsigma | t) \in \mathbb{R}^m$ is a predictive PI control to achieve the desired tracking control. Then, $\kappa(t)$ is developed as $\kappa(t) = u(t - \varsigma) = \hat{\kappa}(t | t - \varsigma)$ based on system (7), i.e.,

$$\kappa(t) = \mathcal{G}^{-1}(\cdot, t) (\hat{u}_{pi}(t | t - \varsigma) - \mathcal{F}(\cdot, t)) \quad (8)$$

By taking system (8) into system (6), the closed-loop FAS can be realized as

$$y(t+1) = \hat{u}_{pi}(t | t - \varsigma) \quad (9)$$

Problem 2. For the FAS in system (6) under assumptions 1 and 3, this study is to propose a FAS approach-based predictive tracking control $\kappa(t)$ in system (8) to guarantee the bounded stability and tracking performance of the closed-loop FAS in system (9), so that the following conditions can be satisfied.

- (1) $\forall t \geq 0$, $\|y(t)\| < \infty$ when $\|r(t)\| < \infty$,
- (2) $\lim_{t \rightarrow \infty} \|y(t) - r(t)\| = 0$.

III. MAIN RESULTS

A. Construction of FAS Model

To address problem 1, theorem 1 is proposed here.

Theorem 1. Let assumptions 1 and 2 be satisfied, an equivalent transformation $\mathcal{T}$ can be constructed in system (10), i.e.,

$$x_i(t) = \sum_{j=i}^n \left(\prod_{l=i}^{j-1} g_l(\cdot, t-l+i) \right) f_j(\cdot, t-j+i) \\ + \left(\prod_{l=i}^n g_l(\cdot, t-l+i) \right) u(t-n+i) \quad (10)$$

where $i = 2, 3, \dots, n$, and $\prod_{l=i}^{j-1} (\cdot) = I$ when $j = i$, system (5a)

with the output in system (5d) can be converted into the FAS in system (6) with

$$\begin{aligned}\mathcal{F}(\cdot, t) &= \sum_{j=1}^n \left(\prod_{l=1}^{j-1} g_l(\cdot, t-l+1) \right) f_j(\cdot, t-j+1), \\ \mathcal{G}(\cdot, t) &= \prod_{l=1}^n g_l(\cdot, t-l+1)\end{aligned}\quad (11)$$

where $\prod_{l=1}^0 (\cdot) = I$.

Proof. The step-backward form of system (5c) is provided as

$$x_n(t) = f_n(\cdot, t-1) + g_n(\cdot, t-1)u(t-1)$$

which is matched with the case of $i = n$ in system (10). By taking the above into system (5b) with $i = n-1$, it is obtained that

$$\begin{aligned}x_{n-1}(t+1) &= f_{n-1}(\cdot, t) + g_{n-1}(\cdot, t)x_n(t) \\ &= f_{n-1}(\cdot, t) + g_{n-1}(\cdot, t)(f_n(\cdot, t-1) + g_n(\cdot, t-1)u(t-1))\end{aligned}$$

its step-backward form is provided as

$$\begin{aligned}x_{n-1}(t) &= f_{n-1}(\cdot, t-1) + g_{n-1}(\cdot, t-1)f_n(\cdot, t-2) \\ &\quad + g_{n-1}(\cdot, t-1)g_n(\cdot, t-2)u(t-2)\end{aligned}$$

It clearly implies that the above is matched with the case of $i = n-1$ in system (10). Repeating the above process with the iteration from $i = n-2$ to $i = 2$, the step-backward form of system (5b) can be finally summarized as

$$\begin{aligned}x_2(t) &= \sum_{j=2}^n \left(\prod_{l=2}^{j-1} g_l(\cdot, t-l+1) \right) f_j(\cdot, t-j+1) \\ &\quad + \left(\prod_{l=2}^n g_l(\cdot, t-l+1) \right) u(t-n+1)\end{aligned}$$

which is matched with the case of $i = 2$ in system (10). By taking the above into system (5a) with the output in system (5d), the FAS in system (6) can be obtained, where $\mathcal{F}(\cdot, t)$ and $\mathcal{G}(\cdot, t)$ can be obtained in system (11).

The proof is completed. $\square$

B. Design of Predictive PI Control

(1) *Without input delay:* When $\varsigma = 0$, a basic PI control is designed as

$$\begin{aligned}u_{pi}(t) &= u_{pi}(t-1) + \Delta u_{pi}(t), \\ \Delta u_{pi}(t) &= \sum_{l=0}^{n_x^*} K_{P,l} \Delta e(t-l) + K_I e(t)\end{aligned}\quad (12)$$

where $e(t) = y(t) - r(t)$ is the tracking error, $K_{P,l} \in \mathbb{R}^{m \times m}$ ($l = 0, 1, \dots, n_x^*$) and $K_I \in \mathbb{R}^{m \times m}$ denote the PI control gains to be calculated, and $n_x^* \in [0, n_x]$ is the known integer. In this case, $\hat{u}_{pi}(t|t-\varsigma) = u_{pi}(t)$, so the closed-loop FAS (system (9)) can be degenerated as

$$y(t+1) = u_{pi}(t) \quad (13)$$

Because $e(t)$ is the tracking error, the bounded stability and tracking performance of the closed-loop system (13) can be equivalent to that $e(t) \rightarrow 0$ with $t \rightarrow \infty$. Hence, the following theorem 2 is proposed as the previous result for considering problem 2.

Theorem 2. Let assumptions 1 and 3 be satisfied for the FAS in system (6). By utilizing the controllers in systems (7) and (8) with $\varsigma = 0$ as well as system (12), $e(t) \rightarrow 0$ with $t \rightarrow \infty$ can be achieved if the PI control gains $K_{P,l}$, $l = 0, 1, \dots, n_x^*$, and K_I can be solved as

$$\begin{aligned}K_{P,n_x^*} &= -K_{n_x^*+1}, \\ K_{P,l} &= K_{P,l+1} - K_{l+1}, l = 0, 1, \dots, n_x^* - 1, \\ K_I &= K_0 - K_{P,0}\end{aligned}\quad (14)$$

with

$$\begin{bmatrix} K_0 & \cdots & K_{n_x^*+1} \end{bmatrix} = \left(\mathcal{Z} \Lambda^{n_x^*+2} - \mathcal{Z} \Lambda^{n_x^*+1} \right) \begin{bmatrix} \mathcal{Z} \Lambda^{n_x^*+1} \\ \vdots \\ \mathcal{Z} \Lambda \\ \mathcal{Z} \end{bmatrix}^{-1} \quad (15)$$

where Λ is a Schur matrix and $\mathcal{Z}$ is an arbitrary parameter matrix with the degrees of design freedom satisfying

$$\det \begin{bmatrix} \mathcal{Z} \Lambda^{n_x^*+1} \\ \vdots \\ \mathcal{Z} \Lambda \\ \mathcal{Z} \end{bmatrix} \neq 0 \quad (16)$$

Proof. By using a difference operator, the incremental form of system (13) is shown as $\Delta y(t+1) = \Delta u_{pi}(t)$, where $\Delta u_{pi}(t)$ is given in system (12). For the $e(t)$, it can be derived that

$$e(t+1) = y(t+1) - r(t+1) = e(t) + \Delta y(t+1), \quad (17)$$

with a general assumption that $r(\cdot) = r$ in [25] and its related literature. By taking $\Delta y(t+1) = \Delta u_{pi}(t)$ with system (12) into system (17), it can be yielded that

$$\begin{aligned}e(t+1) &= e(t) + \Delta u_{pi}(t) \\ &= e(t) + \sum_{l=0}^{n_x^*} K_{P,l} \Delta e(t-l) + K_I e(t) \\ &= e(t) + (K_I + K_{P,0})e(t) + (K_{P,1} - K_{P,0})e(t-1) \\ &\quad + \cdots + (K_{P,n_x^*} - K_{P,n_x^*-1})e(t-n_x^*) - K_{P,n_x^*}e(t-n_x^*-1) \\ &= e(t) + K_0 e(t) + K_1 e(t-1) \\ &\quad + \cdots + K_{n_x^*} e(t-n_x^*) + K_{n_x^*+1} e(t-n_x^*-1)\end{aligned}\quad (18)$$

so that the relationship in system (14) can be held. Hence, the proposed problem can be equivalently converted into how to determine the coefficients K_l , to guarantee the stability of (18), i.e., $e(t) \rightarrow 0$ with $t \rightarrow \infty$.

Denote $E(t) = \begin{bmatrix} e^T(t) & \cdots & e^T(t-n_x^*-1) \end{bmatrix}^T$, system (18) can be equivalently rewritten as

$$E(t+1) = \mathcal{K}_{0 \sim n_x^*+1} E(t) \quad (19)$$

with

$$\mathcal{K}_{0 \sim n_x^*+1} = \begin{bmatrix} I+K_0 & K_1 & \cdots & K_{n_x^*+1} \\ I & & & 0 \\ & \ddots & & \vdots \\ & & I & 0 \end{bmatrix}$$

so that system (19) can achieve the stability if and only if there exist an arbitrary Schur matrix Λ and its corresponding eigenvector matrix $\mathcal{V}_c = \begin{bmatrix} V_0^T & V_1^T & \cdots & V_{n_x^*+1}^T \end{bmatrix}^T$ to ensure that $\mathcal{K}_{0 \sim n_x^*+1} \mathcal{V}_c = \mathcal{V}_c \Lambda$, which can be expanded as

$$\begin{aligned} (I+K_0)V_0 + \sum_{l=1}^{n_x^*+1} K_l V_l &= V_0 \Lambda, \\ V_0 &= V_1 \Lambda, \\ &\vdots \\ V_{n_x^*} &= V_{n_x^*+1} \Lambda \end{aligned} \quad (20)$$

Let $V_{n_x^*+1} = \mathcal{V}$, the eigenvector matrix $\mathcal{V}_c$ can be represented as $\mathcal{V}_c = \begin{bmatrix} (\mathcal{V} \Lambda^{n_x^*+1})^T & \cdots & (\mathcal{V} \Lambda)^T & \mathcal{V}^T \end{bmatrix}^T$ based on the iteration from the second equation to the last one in system (20). Further, let $K = \begin{bmatrix} K_0 & \cdots & K_{n_x^*+1} \end{bmatrix}$ and $\mathcal{W} = K \mathcal{V}_c$, the first equation of system (20) can be converted into the following generalized Sylvester equation, i.e.,

$$\mathcal{V} \Lambda^{n_x^*+2} - \mathcal{V} \Lambda^{n_x^*+1} = \mathcal{W}$$

By applying the solving algorithm of generalized Sylvester equation in [26], the general solutions of $\mathcal{V}$ and $\mathcal{W}$ are provided as

$$\mathcal{V} = \mathcal{Z}, \mathcal{W} = \mathcal{Z} \Lambda^{n_x^*+2} - \mathcal{Z} \Lambda^{n_x^*+1}$$

where $\mathcal{Z}$ is an arbitrary parameter matrix with the degrees of design freedom, so the eigenvector matrix $\mathcal{V}_c$ can be further written as $\mathcal{V}_c = \begin{bmatrix} (\mathcal{Z} \Lambda^{n_x^*+1})^T & \cdots & (\mathcal{Z} \Lambda)^T & \mathcal{Z}^T \end{bmatrix}^T$. When $\det(\mathcal{V}_c) \neq 0$, i.e., the condition (16) can be satisfied, $K = \begin{bmatrix} K_0 & \cdots & K_{n_x^*+1} \end{bmatrix}$ can be solved by $K = \mathcal{W} \mathcal{V}_c^{-1}$, i.e., Eq. (15) can be held.

The proof is completed. $\square$

(2) *With input delay*: When $\varsigma \neq 0$, an idea from predictive control is adopted to compensate for the input delay ς . Firstly, the predictions of $e(t)$ by using system (18) can be derived as

$$\begin{aligned} \hat{e}(t+1|t) &= e(t) + \Delta u_{pi}(t), \\ \hat{e}(t+2|t) &= \hat{e}(t+1|t) + \Delta \hat{u}_{pi}(t+1|t), \\ &\vdots \\ \hat{e}(t+\varsigma|t) &= \hat{e}(t+\varsigma-1|t) + \Delta \hat{u}_{pi}(t+\varsigma-1|t) \end{aligned} \quad (21)$$

where

$$\Delta \hat{u}_{pi}(t+i|t) = \sum_{l=0}^{n_x^*} K_{P,l} \Delta \hat{e}(t-l+i|t) + K_I \hat{e}(t+i|t) \quad (22a)$$

$$= \sum_{l=0}^{n_x^*+1} K_l \hat{e}(t-l+i|t) \quad (22b)$$

with $\hat{e}(t-l+i|t) = e(t-l+i)$ if $i \leq l$.

Following systems (12) and (22b), a predictive PI control $\hat{u}_{pi}(t+\varsigma|t)$ in system (7) is designed as

$$\begin{aligned} \hat{u}_{pi}(t+\varsigma|t) &= \hat{u}_{pi}(t+\varsigma-1|t) + \Delta \hat{u}_{pi}(t+\varsigma|t), \\ \Delta \hat{u}_{pi}(t+\varsigma|t) &= \sum_{l=0}^{n_x^*+1} K_l \hat{e}(t-l+\varsigma|t) \end{aligned} \quad (23)$$

where $\hat{e}(t-l+\varsigma|t)$ ($l=0,1,\dots,n_x^*+1$) is the $(\varsigma-l)^{\text{th}}$ ahead prediction of $e(t)$, and $\hat{e}(t-l+\varsigma|t) = e(t-l+\varsigma)$ when $\varsigma \leq l$ with $l=0,1,\dots,n_x^*+1$. On the basis of system (23), $\hat{u}_{pi}(t|t-\varsigma)$ in system (8) can be developed as

$$\begin{aligned} \hat{u}_{pi}(t|t-\varsigma) &= \hat{u}_{pi}(t-1|t-\varsigma) + \Delta \hat{u}_{pi}(t|t-\varsigma), \\ \Delta \hat{u}_{pi}(t|t-\varsigma) &= \sum_{l=0}^{n_x^*+1} K_l \hat{e}(t-l|t-\varsigma) \end{aligned} \quad (24)$$

Theorem 3. The closed-loop FAS in system (9) with the predictive PI control in system (24) can be simplified to be that system (13) with the PI control in system (12).

Proof. By replacing the time t in systems (21) and (22b) by the time $t-\varsigma$, it can be yielded that

$$\begin{aligned} \hat{e}(t-\varsigma+i|t-\varsigma) &= \hat{e}(t-\varsigma+i-1|t-\varsigma) \\ &+ \Delta \hat{u}_{pi}(t-\varsigma+i-1|t-\varsigma) \end{aligned} \quad (25a)$$

$$\Delta \hat{u}_{pi}(t-\varsigma+i-1|t-\varsigma) = \sum_{l=0}^{n_x^*+1} K_l \hat{e}(t-\varsigma-l+i-1|t-\varsigma) \quad (25b)$$

with $i=1,2,\dots,\varsigma$, which can derive the following causality because of available data up to the time $t-\varsigma$, i.e.,

$$\Delta \hat{u}_{pi}(t-\varsigma|t-\varsigma) = \Delta u_{pi}(t-\varsigma), \hat{e}(t-\varsigma|t-\varsigma) = e(t-\varsigma)$$

so it can be obtained that

$$e(t-\varsigma+1) = e(t-\varsigma) + \Delta u_{pi}(t-\varsigma) = \hat{e}(t-\varsigma+1|t-\varsigma).$$

By taking $\Delta \hat{u}_{pi}(t-\varsigma|t-\varsigma) = \Delta u_{pi}(t-\varsigma)$ and $e(t-\varsigma+1) = \hat{e}(t-\varsigma+1|t-\varsigma)$ into (25b), i.e., $i=2$, it can be obtained that

$$\Delta \hat{u}_{pi}(t-\varsigma+1|t-\varsigma) = \sum_{l=0}^{n_x^*+1} K_l \hat{e}(t-\varsigma-l+1|t-\varsigma) = \Delta u_{pi}(t-\varsigma+1)$$

and then taking the above into system (25a), it can be derived that

$$e(t-\varsigma+2) = e(t-\varsigma+1) + \Delta u_{pi}(t-\varsigma+1) = \hat{e}(t-\varsigma+2|t-\varsigma)$$

Similarly, repeating the above process based on system (25b) from $i=2$ to $i=\varsigma$, it can be yielded that

$$\Delta \hat{u}_{pi}(t-1|t-\varsigma) = \sum_{l=0}^{n_x^*+1} K_l \hat{e}(t-l+1|t-\varsigma) = \Delta u_{pi}(t-1)$$

and then taking the above into system (25a), it can be derived that

$$e(t) = e(t-1) + \Delta u_{pi}(t-1) = \hat{e}(t|t-\varsigma)$$

Summarizing the above shows that

$$\begin{aligned} \hat{e}(t-\varsigma+i|t-\varsigma) &= e(t-\varsigma+i), \\ \Delta \hat{u}_{pi}(t-\varsigma+i-1|t-\varsigma) &= \Delta u_{pi}(t-\varsigma+i-1) \end{aligned}$$

with $i = 0, 1, \dots, \varsigma$. Combining the above with system (25), it can be derived that

$$\hat{e}(t|t-\varsigma) = e(t), \Delta \hat{u}_{pi}(t|t-\varsigma) = \Delta u_{pi}(t)$$

Hence, the closed-loop FAS in system (9) with the predictive PI control in system (24) can be completely represented as

$$\begin{aligned} y(t+1) &= \hat{u}_{pi}(t|t-\varsigma), \\ \hat{u}_{pi}(t|t-\varsigma) &= \hat{u}_{pi}(t-1|t-\varsigma) + \Delta \hat{u}_{pi}(t|t-\varsigma), \\ \Delta \hat{u}_{pi}(t|t-\varsigma) &= \sum_{l=0}^{n_x^*+1} K_l \hat{e}(t-l|t-\varsigma) \end{aligned}$$

which can be simplified as

$$\begin{aligned} y(t+1) &= u_{pi}(t), \\ u_{pi}(t) &= u_{pi}(t-1) + \Delta u_{pi}(t), \\ \Delta u_{pi}(t) &= \sum_{l=0}^{n_x^*} K_{P,l} \Delta e(t-l) + K_I e(t) = \sum_{l=0}^{n_x^*+1} K_{I,l} e(t-l) \end{aligned}$$

so the proposed conclusion can be held.

The proof is completed. $\square$

Based on theorem 3, the bounded stability and tracking performance of the closed-loop FAS in system (9) can be equivalent to that of system (13). By means of theorem 2, corollary 1 is proposed in the below to consider problem 2.

Corollary 1. *Let assumptions 1 and 3 be satisfied for the FAS in system (6), the closed-loop FAS in system (9) can achieve the bounded stability and tracking performance if the PI control gains can be solved in systems (14) and (15) when satisfying system (16).*

IV. SIMULATION VERIFICATION

A numerical simulation is given to illustrate the feasibility of the proposed FAS approach-based predictive PI control, where the parameters of system (4) are given in Table 1.

Based on theorem 1, a transformation $\mathcal{T}$ for system (4) can be constructed as

$$\begin{aligned} x_3(t) &= f_3(\cdot, t-1) + T_p L_f^{-1} u(t-1), \\ x_2(t) &= f_2(\cdot, t-1) + T_p C^{-1} f_3(\cdot, t-2) + T_p^2 C^{-1} L_f^{-1} u(t-2) \end{aligned}$$

TABLE 1
THE PARAMETERS OF SYSTEM (4)

Symbol	Value	Symbol	Value
R_g	0.05 Ω	R_f	0.05 Ω
L_g	0.0005 H	L_f	0.0002 H
C	40 μF	ω	100 π rad/s
u_{gd}	220 $\sqrt{2}$ V	u_{gq}	0 V
T_p	100 μs		

so the first equation of system (4) can be equivalently converted into the following FAS in system (26), i.e.,

$$y(t+1) = \mathcal{F}(\cdot, t) + \mathcal{G}(\cdot, t) \kappa(t) \quad (26)$$

with $\kappa(t) = u(t-\varsigma)$, $\varsigma = 2$, and

$$\begin{aligned} \mathcal{F}(\cdot, t) &= f_1(\cdot, t) + T_p L_g^{-1} f_2(\cdot, t-1) + T_p^2 L_g^{-1} C^{-1} f_3(\cdot, t-2), \\ \mathcal{G}(\cdot, t) &= T_p^3 L_g^{-1} C^{-1} L_f^{-1} \end{aligned}$$

Let $n_x^* = 1$ in (12), a FAS approach-based basic PI control for system (26) is designed as

$$\begin{aligned} \kappa(t) &= u(t-2), \\ u(t) &= \mathcal{G}^{-1}(\cdot, t) (u_{pi}(t) - \mathcal{F}(\cdot, t)), \\ u_{pi}(t) &= u_{pi}(t-1) + \Delta u_{pi}(t), \\ \Delta u_{pi}(t) &= K_{P,0} \Delta e(t) + K_{P,1} \Delta e(t-1) + K_I e(t) \end{aligned} \quad (27)$$

Based on system (24) with $n_x^* = 1$, a FAS approach-based predictive PI control for system (26) can be developed as

$$\begin{aligned} u(t) &= \hat{\kappa}(t+2|t), \\ \hat{\kappa}(t+2|t) &= \mathcal{G}^{-1}(\cdot, t) (\hat{u}_{pi}(t+2|t) - \mathcal{F}(\cdot, t)), \\ \hat{u}_{pi}(t|t-2) &= \hat{u}_{pi}(t-1|t-2) + \Delta \hat{u}_{pi}(t|t-2), \\ \Delta \hat{u}_{pi}(t|t-2) &= \sum_{l=0}^2 K_{I,l} \hat{e}(t-l|t-2) \end{aligned} \quad (28)$$

By using theorem 2 and corollary 1, the Schur matrix Λ is

chosen as $\Lambda = \begin{bmatrix} 0.1 & 1 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.4 \end{bmatrix} I$ and the parameter matrix $\mathcal{Z}$ is chosen as $\mathcal{Z} = \begin{bmatrix} I & 2I & 3I \end{bmatrix}$, the PI control gains can be solved as $K_{P,0} = \text{diag}\{0.086, 0.086\}$, $K_{P,1} = \text{diag}\{-0.004, -0.004\}$, and $K_I = \text{diag}\{-0.486, -0.486\}$.

For the reference signal, it is specified that the initial value of the active power command P^* is 4 kW and is changed as 8 kW at 0.1s, and the reactive power command Q^* is always zero, such that the $r(t)$ is set as

$$\begin{bmatrix} i_{gd}^* \\ i_{gq}^* \end{bmatrix} = \begin{cases} \begin{bmatrix} 8.57 & 0 \end{bmatrix}^T A, & t \in [0, 0.1] \text{ s} \\ \begin{bmatrix} 17.15 & 0 \end{bmatrix}^T A, & t \in [0.1, 0.2] \text{ s} \end{cases}$$

By adopting systems (27) and (28), the compared simulation can be completed on our self-designed web platform

Mathematics To Practice Laboratory^①, where the proposed platform can realize the control systems simulation based on the M Language. The results are shown in Figs. 3 and 4.

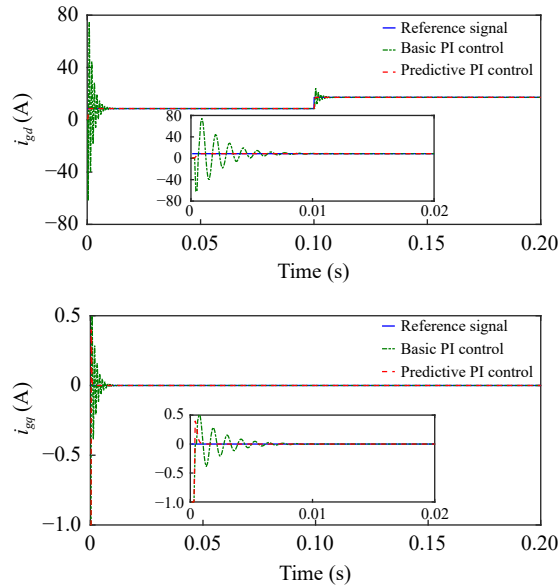


Fig. 3. The current tracking control for the LCL-type grid-connected inverter in system (4) by using systems (27) and (28).

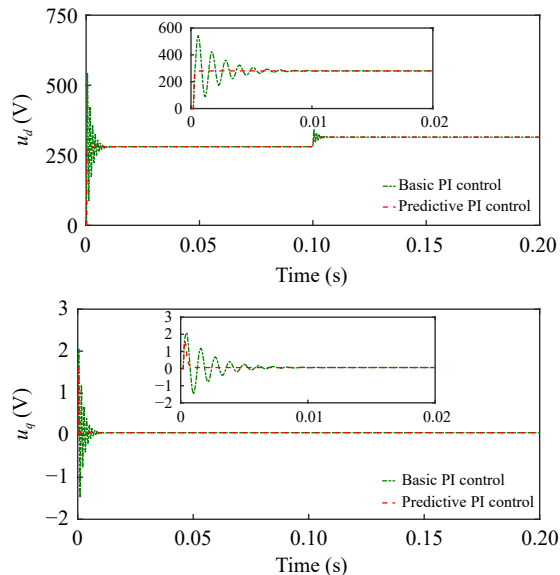


Fig. 4. The control inputs generated by systems (27) and (28).

Fig. 3 means that the proposed FAS approach-based predictive PI control in system (28) can implement the desired tracking control during the dynamical and steady processes because of active compensation of input delays. Relatively, the basic PI control in system (27) results in multiple oscillations and large overshoot in the dynamical process because it cannot

deal with the input delays. Fig. 4 shows the corresponding control inputs, which implies that the cost of predictive PI control in system (28) is less than that of basic PI control in system (27). Further, the 2-norm of the tracking error is given in Fig. 5 to perform the quantitative analysis between systems (27) and (28), which shows that the predictive PI control can lead to the better tracking performance with a fast convergence rate and a smaller absolute overshoot.

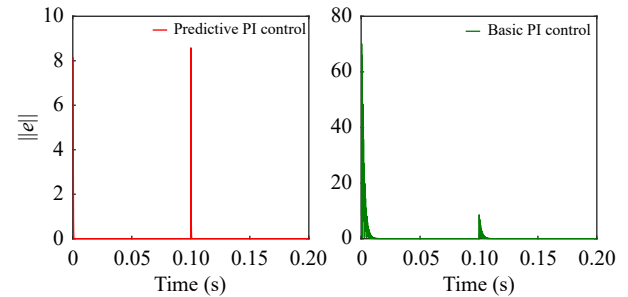


Fig. 5. The 2-norm of the tracking error generated by systems (27) and (28).

According to the results of the Figs. 3–5, the feasibility and superiority of the FAS approach-based predictive PI control in the current tracking control for the LCL-type grid-connected inverters can be fully demonstrated, which provides a better foundation for the experimental verification and the associated large-scale industrial applications.

V. CONCLUSIONS

This study has investigated the current tracking control for the LCL-type grid-connected inverters. A predictive PI control using a FAS approach, including an equivalent transformation for constructing the input-delay FASs of discrete-time strict-feedback systems and a predictive PI control for achieving the desired tracking control with the compensation of input delays, has been developed to solve this problem and its general ones. The further analysis has presented a sufficient criterion for the bounded stability and tracking performance of the closed-loop systems. The future directions will pay close attention to the FAS approach-based grid-connected operation and control of renewable energy power systems.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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^① <https://ncslab.sdim.sustech.edu.cn/m2pcode/?lang=zh-CN>

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AI USAGE STATEMENT

“None.” OR “During the preparation of this manuscript, the authors used [NAME TOOL/SERVICE] in order to [REASON].” The authors have reviewed and edited the content and take full responsibility for the final version of the manuscript.

REFERENCES

- [1] P. Tuo, Z. Gong, Z. Y. Meng, and X. J. Wu, “An adaptive virtual impedance-based closed-loop voltage control strategy for smart transformer-interfacing distribution grid with high PV penetration,” *IEEE Trans. Smart Grid*, vol. 17, no. 2, pp. 1331–1344, Mar. 2026.
- [2] Y. P. Zhang, Y. Y. Xie, S. Cai, Q. W. Wu, H. B. Zhu, and Z. R. Xiang, “Coordinated control of grid-forming wind turbines and grid-forming energy storage systems for power system restoration,” *IEEE Trans. Sustain. Energy*, vol. 16, no. 4, pp. 2812–2827, Oct. 2025.
- [3] J. H. Liu, X. B. Zhang, and B. H. Wang, “An improved grid-voltage full-feedforward strategy for the grid-voltage-harmonics rejection of grid-connected LCL-type inverter based on Padé approximant,” *IEEE J. Emerg. Sel. Top. Power Electron.*, to be published.
- [4] D. Woo and S. Kim, “Capacitor current differential feedback active damping to enhance robustness of LCL-type inverters under weak grid conditions,” *IEEE Trans. Ind. Appl.*, vol. 62, no. 3, pp. 4151–4163, May/Jun. 2026.
- [5] X. Y. Zhang, J. C. Sun, Z. Z. Zhou, Z. Kang, W. L. Yao, T. Kerekes, X. B. Zhang, and D. Zhou, “Evaluating the reliability degradation caused by resonant current in photovoltaic inverters with LCL filters,” *IEEE Trans. Ind. Appl.*, vol. 62, no. 2, pp. 2869–2880, Mar./Apr. 2026.
- [6] J. Liu, G. B. Song, M. J. Deng, Y. M. Ren, F. B. Guo, W. Z. Song, Z. C. Chen, and P. Wheeler, “Multiobjectives integrated division-summation method for LCL-type three-level grid-connected inverter considering low-frequency common mode voltage,” *IEEE J. Emerg. Sel. Top. Power Electron.*, vol. 13, no. 2, pp. 1905–1915, Apr. 2025.
- [7] L. C. Hong, R. Y. Tang, Q. R. Jiang, X. R. Xie, and Y. Y. Zhu, “Admittance-based stability analysis of LCL-type grid-connected inverter considering AC-Side and AC-DC frequency coupling effects,” *IEEE Trans. Power Delivery*, vol. 39, no. 3, pp. 1351–1363, Jun. 2024.
- [8] C. Cheng, S. J. Xie, Q. Qian, J. Q. Lv, and J. M. Xu, “Observer-based single-sensor control schemes for LCL-filtered grid-following inverters,” *IEEE Trans. Ind. Electron.*, vol. 70, no. 5, pp. 4887–4900, May 2023.
- [9] J. Liu, X. D. Sun, B. Y. Ren, W. Z. Song, and P. Wheeler, “Strong adaptability control based on dual-division-summation current control for an LCL-type grid-connected inverter,” *IEEE Trans. Power Electron.*, vol. 37, no. 12, pp. 14157–14172, Dec. 2022.
- [10] M. Rahman, S. P. Biswas, R. Islam, A. Rahman, and K. M. Muttaqi, “An advanced nonlinear controller for the LCL-type three-phase grid-connected solar photovoltaic system with a DC-DC converter,” *IEEE Syst. J.*, vol. 16, no. 2, pp. 3203–3214, Jun. 2022.
- [11] L. G. Liu, J. B. Xu, J. Ye, Z. J. Zheng, and A. W. Shen, “Controller parameter optimization of LCL-type grid-connected inverters for passivity margin enhancement,” *Int. J. Electr. Power Energy Syst.*, vol. 159, pp. 110046, Aug. 2024.
- [12] J. Ye, S. T. Huang, Y. K. Huang, Y. H. Hu, C. Zhang, J. B. Xu, and A. W. Shen, “A new parameter tuning method for generalized multiresonant current controllers in grid-connected inverters,” *IEEE Trans. Power Electron.*, vol. 39, no. 9, pp. 10962–10975, Sep. 2024.
- [13] G. R. Duan, “High-order fully actuated system approaches: Part I. Models and basic procedure,” *Int. J. Syst. Sci.*, vol. 52, no. 2, pp. 422–435, Jan. 2021.
- [14] G. R. Duan, “Fully actuated system approach for control: an overview,” *IEEE Trans. Cybern.*, vol. 54, no. 12, pp. 7285–7306, Dec. 2024.
- [15] D. W. Zhang and G. P. Liu, “GPIO-based predictive control for nonlinear fully actuated systems under lumped disturbances,” *IEEE Trans. Cybern.*, vol. 56, no. 7, pp. 3715–3724, Jul. 2026.
- [16] X. B. Wang and G. R. Duan, “Predictive control for Euler-Lagrange systems with input saturation and dead-zone: a FAS approach,” *IEEE Trans. Syst., Man, Cybern. : Syst.*, vol. 55, no. 9, pp. 5935–5945, Sep. 2025.
- [17] D. W. Zhang and G. P. Liu, “Secure tracking control of cyber-physical systems against hybrid attacks via FAS terminal sliding-mode predictive control,” *IEEE Trans. Cybern.*, vol. 55, no. 11, pp. 5177–5188, Nov. 2025.
- [18] K. X. Cui, D. W. Zhang, and X. Y. Xu, “Discrete-time robust adaptive stabilization control for a type of combined spacecraft: a fully actuated system approach,” *IEEE Trans. Ind. Electron.*, to be published.
- [19] L. X. Xu, X. B. Wang, and G. R. Duan, “Bi-level predictive safety control for multi-axis CNC systems based on FAS approaches,” *Control Eng. Pract.*, vol. 169, pp. 106798, Apr. 2026.
- [20] K. X. Cui, G. R. Duan, D. W. Zhang, and D. K. Gu, “Discrete-time high-order fully actuated adaptive stabilization control for a type of combined spacecraft with unknown parameters,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 60, no. 3, pp. 3379–3389, Jun. 2024.
- [21] X. M. Zheng, Y. Y. Zhao, Q. Li, and P. Mattavelli, “Current control of grid-forming LCL-type inverter with a fully actuated control strategy,” *Int. J. Syst. Sci.*, to be published.
- [22] S. S. Ge, C. G. Yang, S. L. Dai, Z. X. Jiao, and T. H. Lee, “Robust adaptive control of a class of nonlinear strict-feedback discrete-time systems with exact output tracking,” *Automatica*, vol. 45, no. 11, pp. 2537–2545, Nov. 2009.
- [23] G. R. Duan, “High-order fully actuated system approaches: Part II. Generalized strict-feedback systems,” *Int. J. Syst. Sci.*, vol. 52, no. 3, pp. 437–454, Feb. 2021.
- [24] D. K. Gu, Q. L. Wang, and Y. D. Liu, “Robust dynamic surface control for high-order strict-feedback systems with output constraints based on fully actuated system approach,” *IEEE Trans. Cybern.*, vol. 56, no. 7, pp. 4159–4171, Jul. 2026.
- [25] D. W. Zhang, G. P. Liu, and L. M. Wang, “Anti-disturbance predictive tracking control for disturbed nonlinear fully actuated systems,” *Control Theory Appl.*, vol. 42, no. 12, pp. 2439–2448, Dec. 2025.
- [26] G. R. Duan, *Generalized Sylvester Equations—Unified Parametric Solutions*, Boca Raton: CRC Press, 2015.



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