

Nonlocal optomechanics: Hybrid anapole opens a route to unconventional optical tweezing

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Nano Res., **Just Accepted Manuscript** • <https://doi.org/10.26599/NR.2026.94909195>

<https://www.sciopen.com/journal/1998-0124> on Sep. 17, 2026

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Research Article

Nonlocal optomechanics: Hybrid anapole opens a route to unconventional optical tweezing

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Received: 2 June 2026; **Revised:** 16 August 2026; **Accepted:** 17 September 2026

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Cite this article: *Nano Research*, 2026, 19, 94909195 <https://doi.org/10.26599/NR.2026.94909195>

Abstract: In conventional optical tweezers, trapping forces are governed by the local multipolar response of a particle to the incident electromagnetic field. Under this paradigm, suppression of far-field scattering, such as in hybrid anapole states, is generally expected to weaken the corresponding optical forces. Here, we show that tailoring the resonant response of a nanoparticle to a hybrid anapole leads instead to a qualitative reshaping of the optical-force landscape under Gaussian-beam illumination. By introducing generalized gradient-dependent polarizability tensors, we demonstrate that higher-order spatial derivatives of the electromagnetic field can make significant contributions to the optomechanical response when dominant radiative scattering channels are suppressed. As a result, several distinct trapping regimes emerge, including saddle-like force distributions, extended near-zero-force plateaus, asymmetric trapping, and stable off-axis equilibrium positions inaccessible in conventional optical tweezers. These findings establish resonant scattering engineering as an effective route toward gradient-driven optomechanical manipulation in dielectric nanostructures.

1 Introduction.

Optomechanical manipulation, pioneered by A. Ashkin [1–3], has evolved into a versatile and powerful tool for trapping and guiding micro- and nanoscale objects across biophysics [4–7], quantum physics [8–10] and lab-on-a-chip-microfluidics [11–13]. The conventional description of optical tweezers for subwavelength nanoparticles treats the optomechanical response as effectively local: the induced electric and magnetic dipoles are determined by the incident fields evaluated at the particle center, while higher multipoles are related to the corresponding lowest-order spatial derivatives of these fields [14–16]. Consequently, in most conventional trapping scenarios, Gaussian beam generates a simple

trapping landscape governed predominantly by the local field intensity, with a single stable equilibrium position (Fig. 1 a,b).

This conventional picture can change substantially for resonant particles, whose optical response involves the excitation of multiple electric and magnetic multipoles. High-index dielectric nanoparticles provide a particularly suitable platform for exploring such effects, as they support pronounced electric and magnetic Mie-type resonances while exhibiting relatively low dissipative losses [16–18]. The spectral positions and interference of these multipolar resonances can be efficiently controlled by the particle size and geometry, providing a powerful means of tailoring its scattering response [19,20]. The interference of these resonantly excited multipolar channels gives rise to a variety of unconventional scattering regimes, including Kerker effect [21,22], superscattering [23], and anapole states [24,25].

These resonant scattering phenomena also strongly affect electromagnetic momentum transfer and therefore the optical forces acting on resonant nanoparticles. In the Mie scattering regime, where electric and magnetic multipoles of multiple orders contribute simultaneously, decomposing the optical force into their individual and interference contributions provides direct insight into its physical origin [26,27]. The multipole moments, in turn, can be related to the incident electromagnetic field through the corresponding polarizability tensors. In the conventional formulation, these relations involve the local fields and their lowest-order spatial derivatives evaluated at the particle center. Even within this local approximation, resonant variations of the multipolar response can produce qualitatively different optical forces: tuning through Mie resonances can switch the transverse force between trapping and antitrapping regimes [28–30]. As the particle size increases, however, the response can become sensitive to spatial variations of the incident field across the particle volume, making the conventional low-order relations insufficient. This spatially nonlocal response can be described by including higher-order field derivatives and the corresponding generalized polarizabilities [31]. The importance of higher-order contributions has been demonstrated for resonant dielectric nanocylinders [32], and their relative contribution can become particularly significant near an anapole state [33].

Anapole state represents a particularly interesting regime of resonant light scattering. It originates from destructive interference between a primitive Cartesian multipole and corresponding toroidal term, which have identical far-field radiation patterns, resulting in suppression of scattering from the corresponding channel [24,25]. Since other multipolar channels generally remain radiative, this does not result in a pronounced minimum of the total scattering. A more complete suppression of the total scattering can be achieved in the hybrid anapole state (HAS), in which several first-order electric and magnetic anapoles spectrally overlap [34,35]. Such a regime can be realized by breaking the spherical symmetry of the particle, which provides additional geometric degrees of freedom for independently tuning the relevant multipolar channels [35].

In this work, we use the hybrid anapole state as a platform for the systematic investigation of nonlocal effects in the multipole response and optical forces. To this end, we introduce generalized polarizabilities associated with field derivatives of different orders, quantify their contributions to electric and magnetic multipoles up to the octupole order, and subsequently analyze the corresponding contributions to optical force. By comparing two silicon nanocylinders operating in the conventional scattering and HAS regimes, respectively, we show that the local approximation accurately reproduces the full-wave optical force only in the conventional case (Fig. 1b), whereas in the HAS higher-order contributions become significant and give rise to a qualitatively different force landscape (Fig. 1d). Our results demonstrate that generalized polarizabilities provide a systematic framework for describing optical forces beyond the conventional low-order approximation and highlight the importance of higher-order spatial response in the optomechanics of resonant dielectric nanoparticles.

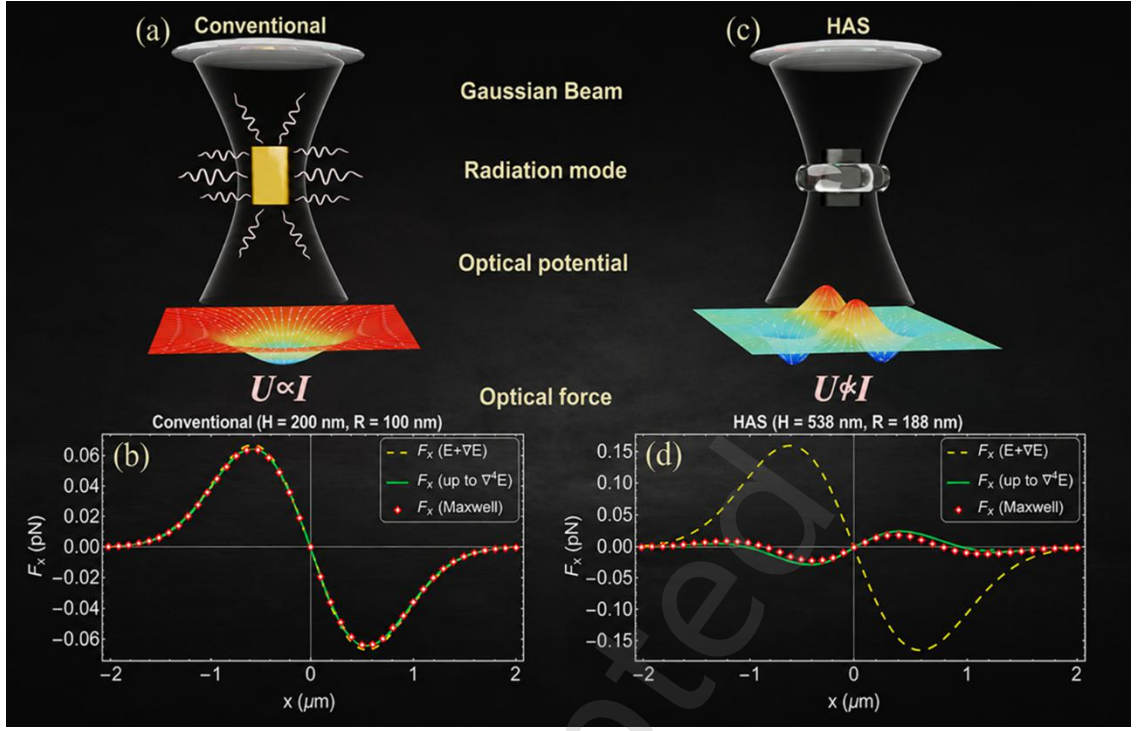


Fig. 1. From local to nonlocal optomechanics: (a) Classical optical tweezers: the interaction of a Gaussian beam with a particle exhibiting a local response creates an effective potential well with $U \propto I$, into which the particle is drawn and trapped. (b) Transverse optical force profile in conventional local models, which implicitly assume that the particle responds only to the field at its center. (c) In the hybrid anapole state, far-field scattering is strongly reduced, effectively disabling radiative channels and allowing nonlocal contributions driven by incident field gradients to dominate the optical force. (d) Transverse force profile in the nonlocal regime: higher-order spatial derivatives of the field (up to fourth order) reshape the force landscape, giving rise to nontrivial force distributions and new types of stable equilibria. Panels (b) and (d) compare three levels of theory: the yellow dashed curves show the local model obtained from Eq. (1) and (2); the green solid curves show our nonlocal theory based on Eq. (3), including higher-order gradient terms; and the red markers give the full-wave reference result from numerical integration of the Maxwell stress tensor.

2 Results and Discussion

2.1 Hybrid Anapole State

While the conventional electrodynamic anapole corresponds to a scattering minimum in a single multipole (typically dipole) channel [24,36], the contributions from other multipoles prevent the complete suppression of far-field radiation. By engineering the particle geometry, however, the scattering minima for all multipolar channels can be spectrally overlapped, achieving an almost vanishing far-field response [35,37]. This regime is known as the hybrid anapole state (HAS). In what follows, we examine the peculiar optomechanics of a particle supporting such a state within a standard optical trap.

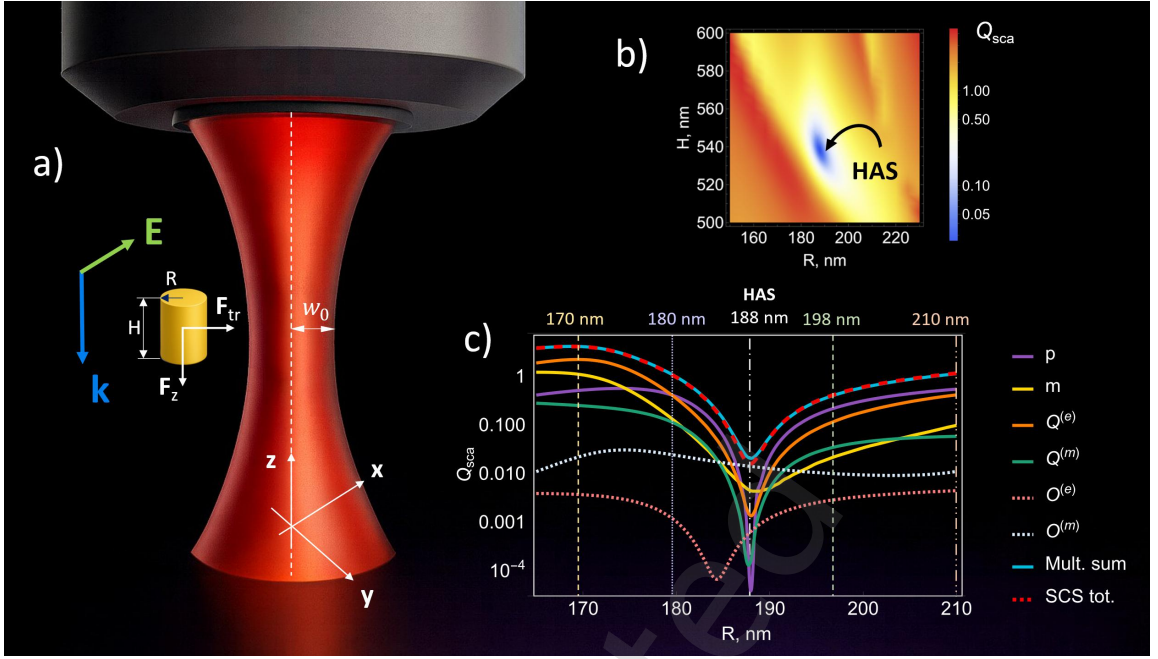


Fig. 2. a) A schematic of the investigated setup: cylindrical nanoparticle placed in the focal plane of the laser beam and displaced laterally from its axis. Therefore, both optical pressure F_z and a transverse optical force F_{tr} govern its dynamics; b) The scattering cross section of the nanocylinder normalized to its geometric cross section ($Q_{sca} = \frac{\sigma_{sca}}{\pi R^2}$) shown as a function of height H and radius R ; c) The spherical multipole decomposition for the nanoparticle with fixed height $H = 538$ nm as a function of its radius (p – electric dipole, m – magnetic dipole, $Q^{(e)}$ – electric quadrupole, $Q^{(m)}$ – magnetic quadrupole, $O^{(e)}$ – electric octupole, $O^{(m)}$ – magnetic octupole). The vertical lines correspond to different radii of the nanoparticles, which exhibit different optomechanical patterns, including the hybrid anapole state ($R = 188$ nm).

We consider a silicon nanocylinder in air positioned in the focal plane of a linearly polarized Gaussian beam (Fig. 2(a)). The wavelength is fixed at $\lambda = 1064$ nm (Yb:YAG), while the nanocylinder's radius R and height H serve as tuning parameters. The beam waist is equal to the wavelength $w_0 = \lambda$; light propagates along the z -axis and is polarized along the x -axis. The field amplitude at the beam center is $E_0 = 10^6$ V/m, which is typical for conventional optical tweezers.

Fig. 2b reveals a region in (H, R) space, where the total scattering is almost completely suppressed with the absolute minimum occurring at $H=538$ nm and $R=188$ nm. Fig. 2c shows the normalized scattering cross section, Q_{sca} , and its multipole decomposition up to the octupole near this minimum as functions of the radius [38,39]. As R approaches 188 nm, all dipole and quadrupole contributions are seen to decrease substantially due to the destructive interference of Cartesian and toroidal multipoles (see Supplementary Materials, Section 2.2). This reduces the total scattering by nearly two orders of magnitude and, as the result, leading to the hybrid anapole state. Moreover, Fig. 2c also shows an excellent agreement between the scattering cross sections calculated by integrating the Poynting vector over a surface enclosing the nanocylinder and by summing all multipole contributions.

2.2 Transverse Optical Forces

We computed the average optical force on the particle via full-wave simulation and integration of the Maxwell stress tensor over a surface enclosing the particle [40] (Supplementary Materials, Eqs. (7-8)).

Our analysis focuses on the transverse force components, which determine trapping stability and in-plane motion. The longitudinal (radiation pressure) component F_z is analyzed in Supplementary Materials, Section 3.2. To map out the force landscape, we swept the

nanocylinder radius R from 150 nm to 250 nm and its transverse position over $\pm 1.5 \mu\text{m}$ in both the x - and y -directions while keeping the height fixed at $H = 538$ nm. Additionally, we evaluated force profiles for 5 different radii, namely 170 nm, 180 nm, 188 nm (HAS), 198 nm and 250 nm – Fig. 3.

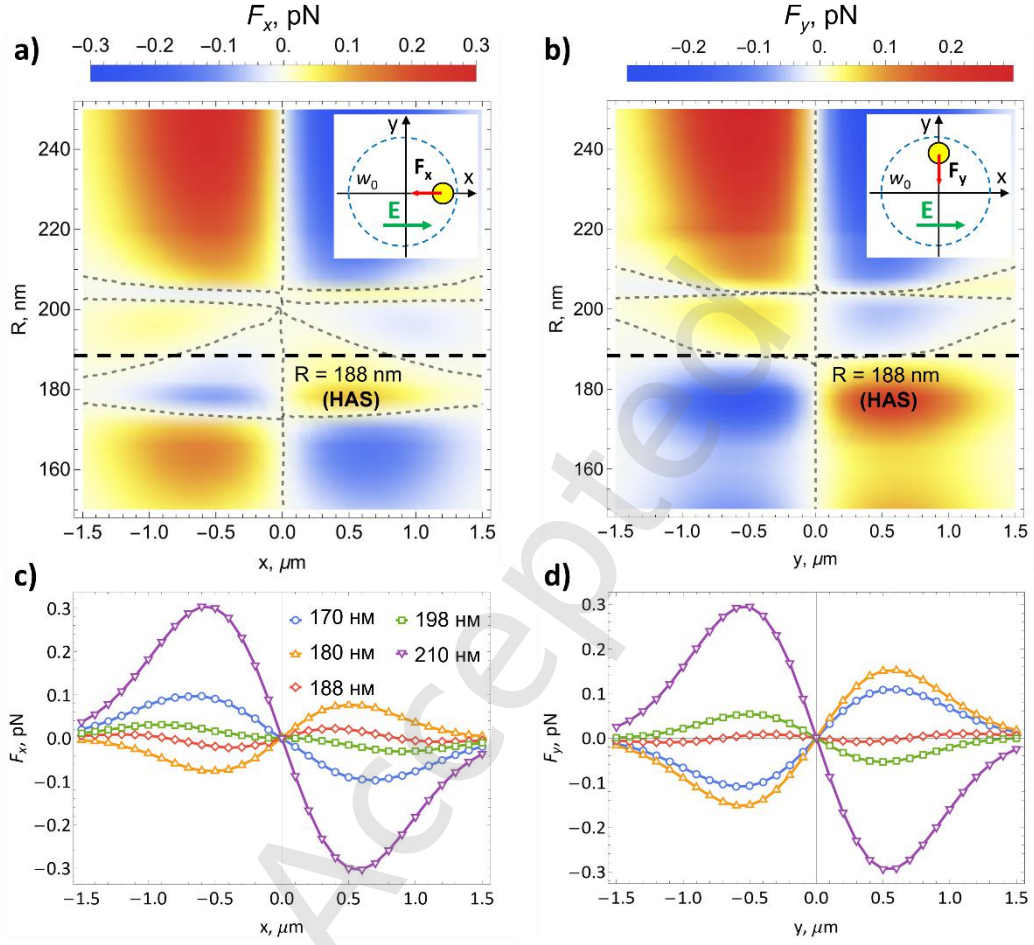


Fig. 3. Transverse optical forces in the radius-coordinate space: a) F_x in (x, R) space, b) F_y in (y, R) space (dotted lines indicate the areas of zero forces, black dashed line indicates the radius of the HAS). c) F_x as a function of x -coordinate, d) F_y as a function of y -coordinate. F_x and F_y on panels c)-d) are plotted for different radii: 170 nm (blue), 180 nm (yellow), 188 nm (red, HAS radius), 198 nm (green), 210 nm (purple).

Fig. 3 (a,b) shows that the forces F_x and F_y exhibit distinct sign reversals and magnitude variations depending on the nanocylinder radius. These behaviors generate nontrivial optomechanical landscapes, which we examine in detail in Section 2.5.

2.3 Optical forces in the HAS

The unusual force profiles observed in the HAS motivate a detailed decomposition of the optical force into its multipolar contributions. To elucidate their origin, we decompose the optical force into interception F^I and recoil F^R contributions, arising from the interference between the incident field and the induced multipoles and among the induced multipoles themselves, respectively. Accordingly, the i^{th} Cartesian component of the total optical force is written as [26]:

$$\left\{ \begin{aligned} F_i &= F_i^I + F_i^R \\ F_i^I &= \frac{1}{2} \text{Re}[p_j \nabla_i E_j^*] + \frac{1}{12} \text{Re}[Q_{jk}^e \nabla_i \nabla_k E_j^*] + \frac{1}{12} \text{Re}[O_{jkl}^e \nabla_i \nabla_l \nabla_k E_j^*] + \\ &\quad + \frac{1}{2} \text{Re}[m_j \nabla_i B_j^*] + \frac{1}{12} \text{Re}[Q_{jk}^m \nabla_i \nabla_k B_j^*] + \frac{1}{12} \text{Re}[O_{jkl}^m \nabla_i \nabla_l \nabla_k B_j^*] \\ F_i^R &= -\frac{k^5}{120\pi\epsilon_0} \text{Im}[Q_{ij}^e p_j^*] - \frac{k^7}{1890\pi\epsilon_0} \text{Im}[O_{ijk}^e (Q_{jk}^e)^*] - \\ &\quad - \frac{k^5}{120\pi\epsilon_0 c^2} \text{Im}[Q_{ij}^m m_j^*] - \frac{k^7}{1890\pi\epsilon_0 c} \text{Im}[O_{ijk}^m (Q_{jk}^m)^*] \\ &\quad - \frac{k^4}{12\pi\epsilon_0 c} \text{Re}[\epsilon_{ijk} p_j m_k^*] - \frac{k^6}{2160\pi\epsilon_0 c^2} \text{Re}[\epsilon_{ijk} Q_{lj}^e (Q_{lk}^m)^*], \end{aligned} \right.$$

* MERGEFORMAT (1)

Here and throughout, Latin indices $i, j, k, l, n \in \{1, 2, 3\}$ denote Cartesian components, and Einstein summation convention over repeated indices is assumed.

To validate the multipole decomposition of the optical force, we compare the total force reconstructed from the sum of the interception and recoil contributions with the total force obtained by direct integration of the Maxwell stress tensor (MST). The force distributions are evaluated along x- and y-axis. In addition, we further analyze the individual contributions to the optical force by separately examining recoil and interception-force components, thereby revealing the role of each multipole in shaping the overall force landscape - Fig. 4.

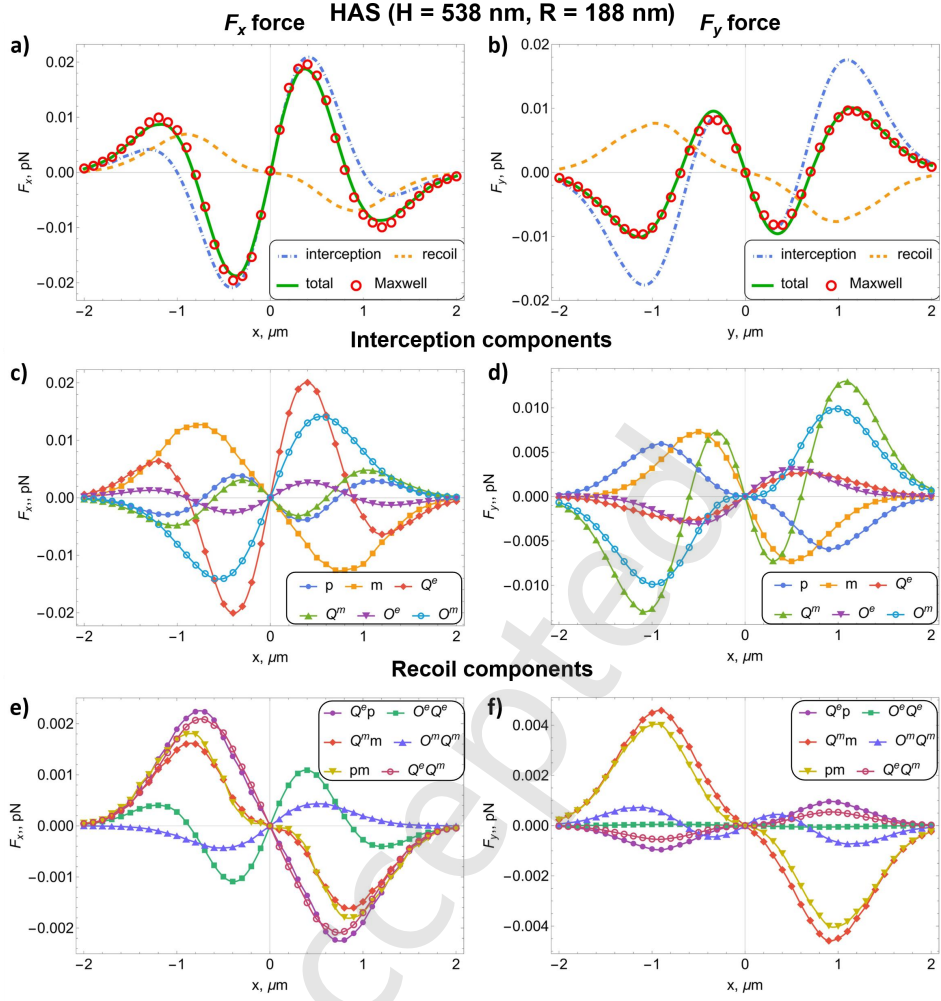


Fig. 4. (a,b) Decomposition of Cartesian components of transverse optical force (F_x and F_y) into interception (blue dot-dashed curves) and recoil (yellow dashed curves) contributions along the x- and y-directions. Green line is a sum of two contributions, red markers denote the Maxwell stress tensor calculations; (c,d) interception components, (e,f) recoil components

Fig. 4 (a, b) show that, unlike the conventional trapping scenario with a single equilibrium point on the optical axis, in the HAS there are two additional off-axis zero-force points along the x- and y-directions. Force decomposition reveals that these unconventional force profiles are determined primarily by the interception contribution, whereas the recoil force, although typically one to two orders of magnitude smaller, in total also provide a non-negligible quantitative correction to the force. Most interception-force components exhibit spatial dependence with three zero crossings, the same as for the total force (Figs. 4 (c,d)). Their combined contribution largely determines the additional off-axis zero-force points of the total force, corresponding to stable equilibrium along the x-axis and unstable equilibrium along the y-axis, as confirmed by the particle trajectories in Fig. 6(e). Such force profiles differ qualitatively from the conventional dipolar gradient force in a Gaussian beam, which has a single zero crossing at the beam center and drives the particle either toward or away from the optical axis [41].

Notably, the transverse interception force receives substantial contributions from several multipolar orders rather than being dominated only by octupoles. To examine the multipolar response underlying these force profiles, we next analyze the induced multipole moments and the corresponding polarizability tensors that relate them to incident field.

2.4 Generalized Nonlocal Polarizability Calculation

In the conventional approximation, the induced multipoles are assumed to be determined solely by the value of the incident electromagnetic field and its first derivatives at the particle center multiplied by the corresponding polarizability tensors $\alpha^p, \alpha^m, \alpha^{Q^e}, \alpha^{Q^m}$ [15,16,42]:

$$\begin{aligned} \mathbf{p} &= \alpha^p \mathbf{E}; \quad \mathbf{m} = \alpha^m \mathbf{H} \\ \mathbf{Q}^e &= \alpha^{Q^e} \frac{\nabla \mathbf{E} + \mathbf{E} \nabla}{2}; \quad \mathbf{Q}^m = \alpha^{Q^m} \frac{\nabla \mathbf{H} + \mathbf{H} \nabla}{2}, \end{aligned}$$

* MERGEFORMAT (2)

where $\mathbf{E}$ and $\mathbf{H}$ are incident (background) electric and magnetic fields.

These relations can be viewed as the lowest-order terms of a more general expansion describing the response of a finite particle to an arbitrary incident field. The field can be expanded into a Taylor series about the particle center, allowing the particle response to spatial derivatives of different orders to be treated systematically [33]. Within the conventional approximation, only the field itself and its first spatial derivatives at the particle center are assumed to contribute significantly to the multipole moments, while higher-order terms are neglected. As we will demonstrate below, however, this approximation becomes insufficient for describing the hybrid anapole state.

To establish the role of higher-order terms, we consider the general relation between the induced electric multipole moments and the spatial distribution of the background field. A convenient framework for this analysis is provided by the multipolar polarizability tensors, which relate the induced multipole moments of a particle to the background electromagnetic field. Specifically, consider a nanoparticle located at $\mathbf{r}_0$ with respect to the origin. The induced electric dipole moment can then be written as a linear functional of the background electric field distribution within the particle:

$$\mathbf{p}(\mathbf{r}_0) = \mathbf{P} [\mathbf{E}(\mathbf{r}_0 + \mathbf{r})] = \mathbf{P} [\mathbf{E}(\mathbf{r})], \quad \mathbf{r} = \mathbf{r}_0 + \mathbf{r}, \quad \mathbf{r} \in V_0,$$

* MERGEFORMAT (3)

where $\mathbf{r}$ denotes the position relative to the particle center and takes all values within the particle volume.

The background field can be expanded in a Taylor series around particle center as [15]:

$$\mathbf{E}(\mathbf{r}_0 + \boldsymbol{\rho}) = \mathbf{E}(\mathbf{r}_0) + \boldsymbol{\rho} \cdot \nabla \mathbf{E}(\mathbf{r}_0) + \frac{1}{2!} [\boldsymbol{\rho} \cdot \nabla]^2 \mathbf{E}(\mathbf{r}_0) +$$

* MERGEFORMAT (4)

Expanding the field in Cartesian coordinates gives:

$$E_j(\mathbf{r}_0 + \boldsymbol{\rho}) = E_j(\mathbf{r}_0) + \rho_k \frac{\partial E_j}{\partial r_k}(\mathbf{r}_0) + \frac{1}{2!} \rho_k \rho_l \frac{\partial^2 E_j}{\partial r_k \partial r_l}(\mathbf{r}_0) +$$

* MERGEFORMAT (5)

Here and throughout the following derivation, Latin indices $i, j, k, l, n \in \{1, 2, 3\}$ denote Cartesian components, and Einstein summation convention over repeated indices is assumed.

The i^{th} component of vector-valued functional $\mathbf{P}$ can be represented in coordinate form as:

$$P_i[\mathbf{E}] = P_i[\mathbf{e}_j E_j],$$

* MERGEFORMAT (6)

There $\mathbf{e}_j$ is a unit vector along the j -axis in the Cartesian basis.

Combining Eqs. (3), (5), (6) and taking into account the linearity of each P_i , we can take constant values of the field and its derivatives out of the functional. Thus, the i^{th} component of the electric dipole can be expressed as a hierarchy of responses to successive spatial derivatives of the field:

$$\begin{aligned} p_i(\mathbf{r}_0) &= P_i[\mathbf{e}_j E_j(\mathbf{r})] = P_i[\mathbf{e}_j] E_j(\mathbf{r}_0) + P_i[\rho_k \mathbf{e}_j] \frac{\partial E_j}{\partial r_k}(\mathbf{r}_0) + P_i[\rho_k \rho_l \mathbf{e}_j] \frac{1}{2!} \frac{\partial^2 E_j}{\partial r_k \partial r_l}(\mathbf{r}_0) + \dots = \\ &= \alpha_{ij}^p E_j(\mathbf{r}_0) + \alpha_{ijk}^p \frac{\partial E_j}{\partial r_k}(\mathbf{r}_0) + \alpha_{ijkl}^p \frac{1}{2!} \frac{\partial^2 E_j}{\partial r_k \partial r_l}(\mathbf{r}_0) + \dots \end{aligned}$$

* MERGEFORMAT (7)

Where all the derivatives of the field are considered at the particle center. The coefficients of proportionality between dipole and electric field $\alpha_{ij}^p, \alpha_{ijk}^p, \dots$ are the generalized polarizability tensors [31].

Thus, the corresponding polarizabilities can be calculated as the response to a uniform field, a linearly growing field with a unit gradient, and so on. Although these basis fields provide a convenient mathematical definition of generalized polarizabilities, they do not satisfy Maxwell's equations and therefore cannot be directly realized in numerical simulations. To address this, we employ the widely used scattered-field formulation [43,44], which is described in more detail in Supplementary, Section 4.1.

So far, the discussion has focused on the electric dipole moment. In general, the same formalism applies to any induced electric or magnetic multipole. In general, an arbitrary electric or magnetic multipole moment M can be represented as:

$$M_i[\mathbf{E}(\mathbf{r})] = \alpha_{ij}^M E_j(\mathbf{r}_0) + \sum_{n=1}^N \sum_{k_1 \dots k_n} \frac{1}{n!} \alpha_{ijk_1 \dots k_n}^M \frac{\partial^n E_j}{\partial r_{k_1} \dots \partial r_{k_n}}(\mathbf{r}_0)$$

* MERGEFORMAT (6)

where indices $i = (i_1 \dots i_s)$ denote the set of Cartesian components of the multipole ($s = 1$ – dipole, $s = 2$ – quadrupole, etc.), $N \geq 1$ is the order of the last significant term in the series, $k_1, \dots, k_n \in \{1, 2, 3\}$, $\alpha_{ijk_1 \dots k_n}^M$ is a component of a polarizability tensor relating the i th component of the multipole M to the spatial derivatives of a j th component of an incident field.

Throughout this work, we adopt the electric-field representation, in which all generalized polarizabilities are defined with respect to the electric field and its spatial derivatives. This choice does not restrict the generality of the formalism, since for monochromatic fields Maxwell's equations provide an equivalent representation in terms of only magnetic field of both electric and magnetic fields [32,33,45].

The conventional dipole and quadrupole relations are recovered by truncating the general expansion after the first few terms. This approximation is valid, for example, for small nonresonant particles or for incident fields that vary only weakly over the particle volume. In this

limit, the induced multipoles are determined solely by the field and its lowest-order spatial derivatives evaluated at the response can therefore be regarded as effectively local.

As the particle size increases, however, the incident field becomes increasingly nonuniform across the particle volume, making higher-order spatial derivatives of the field essential for accurately describing the induced multipoles. We refer to this behavior as a **nonlocal response**, emphasizing that it is distinct from the intrinsic microscopic nonlocality of the material. Instead, it originates from the effective dependence of the induced multipolar response on higher-order spatial derivatives of the incident field. Consequently, the multipolar contributions to the optical force, which are proportional to the induced multipole moments according to Eq. (1), also acquire a nonlocal character and exhibit profiles that cannot arise within the conventional local approximation for small nanoparticles. In the following sections, we demonstrate that, in the hybrid anapole state, these higher-order gradient contributions become dominant for all multipoles up to the electric octupole.

2.5 Conventional and Hybrid Anapole Regimes

Having established the general formalism, we now quantify the role of higher-order terms in the conventional and hybrid-anapole regimes. The symmetry of the system automatically eliminates a subset of terms in the expansion [32,46]. In particular, for the electric dipole, magnetic quadrupole, and electric octupole nonzero contributions arise from even-order derivatives (i.e. $\mathbf{E}$, $\nabla^2 \mathbf{E}$, and $\nabla^4 \mathbf{E}$), whereas the magnetic dipole, electric quadrupole, and magnetic octupole interact with odd-order derivatives only (i.e., $\nabla \mathbf{E}$ and $\nabla^3 \mathbf{E}$). In addition, the convergence analysis of the series and comparison with full-wave modeling showed that it is sufficient to limit the series to fourth derivatives for even multipoles and third derivatives for odd multipoles.

Now, to compare the contribution of the linear terms (which correspond to the conventional approximation) with that of higher-order terms, we examined two cases: nanocylinder in the HAS ($H = 538$ nm and $R = 188$ nm) and small nanocylinder in the conventional scattering regime ($H = 200$ nm, $R = 100$ nm). Fig. 5 compares the components of the electric dipole obtained from the full-wave numerical calculations based on the Maxwell stress tensor with those calculated using formula (7) in three approximations: considering only the linear term, considering derivatives up to 2nd order, and considering derivatives up to 4th order. The components $\text{Re}(p_x)$ and $\text{Im}(p_z)$ are compared because they solely contribute to the dipole force, as shown in Supplementary Materials, Section 4.4.

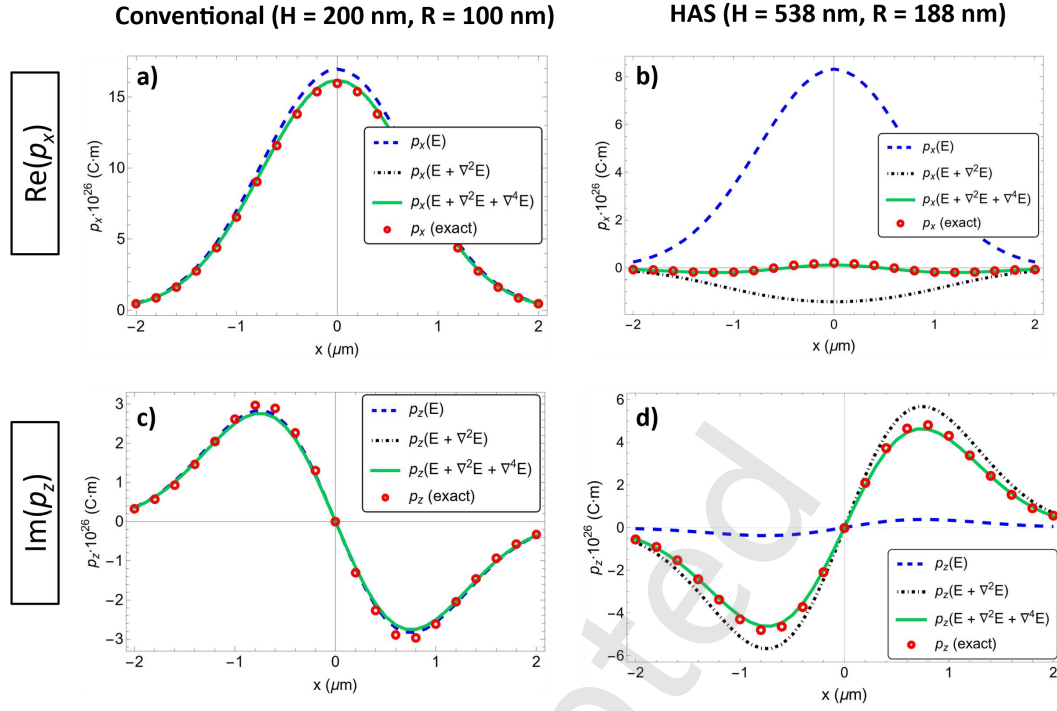


Fig. 5. Electric dipole components $\text{Re}(p_x)$ and $\text{Im}(p_z)$ induced in the nanoparticle in the conventional scattering regime (a,c) and in the HAS (b,d). The blue dashed curves show the result of calculation including response only to the incident field in Eq. ; the black dotdashed curves – up to the 2nd derivatives; the green solid curves – up to the 4th derivatives; the red markers give the full-wave simulation results.

The behaviour differs markedly across regimes. For the conventional regime, the linear term alone suffices (blue dashed curve), as higher-order contributions are negligible. In contrast, for the HAS, convergence requires including up to fourth-order derivatives (green solid line), but neither the linear term nor second-order derivatives suffice. This trend is maintained when calculating all the multipole force components (see Supplementary Materials, Section 4.5.) which clearly demonstrates the limitations of the conventional approximation in the HAS.

The total force (sum of all multipole components) is shown in Fig. 1 for two cases: HAS and the conventional regime. In the conventional regime (Fig. 1b), the local approximation (yellow dashed) agrees well with the full-wave solution (red markers). In the HAS (Fig. 1d), the local model (yellow dashed) deviates strongly from full-wave results (red markers) and including non-local terms up to fourth order restores excellent agreement (green solid). This result further demonstrates the contribution of higher-order derivatives to the optomechanical properties of particles in the HAS, thereby confirming our hypothesis.

2.6 Lateral Dynamics

To connect the spatial variations of the force near the HAS shown in Fig. 3 to observable particle dynamics, we simulate the in-plane motion by integrating the Langevin equations (see Methods, Section 1.2 for details). In air, viscous damping and stochastic forces are weak, so the motion of the nanocylinder is chiefly governed by the optical forces. The Langevin dynamics simulations presented here are intended solely to illustrate the qualitative particle dynamics associated with the calculated optical-force landscapes. A quantitative analysis of trapping stability in liquid environments would require recalculating the optical response and force distributions in the modified dielectric environment, since the surrounding medium substantially affects the formation of the hybrid anapole state.

To visualize the optical force landscape, we also evaluate the line integral of the force along a prescribed path, obtaining a scalar quantity with the dimensions of energy. Given that optical forces in structured fields generally possess non-conservative components ($\nabla \times \mathbf{F} \neq 0$), a true scalar potential cannot be strictly defined. Nevertheless, we obtained the quantity U via the numerical integration of the transverse optical force $\mathbf{F}_{tr} = \mathbf{F}_x + \mathbf{F}_y$ over the focal plane (more detailed information and the integration scheme are given in Supplementary, Section 1.3):

$$U(x_0, y_0) = \int_{\infty}^{(x_0, y_0)} \mathbf{F}_{tr} \cdot d\mathbf{r}$$

* MERGEFORMAT (12)

While U does not represent the true potential energy due to possible nonconservative components of optical forces, it retains the dimension of energy and serves as a convenient indicator of the effective “motion landscape” and locations of fixed points (see Fig. 6). For each radius, we initialize an ensemble of identical particles at random positions with zero initial velocity and identify their trajectories in the focal plane. The path-integrated energy calculation and Langevin trajectories simulation for different nanoparticle radii are shown in Fig. 6.

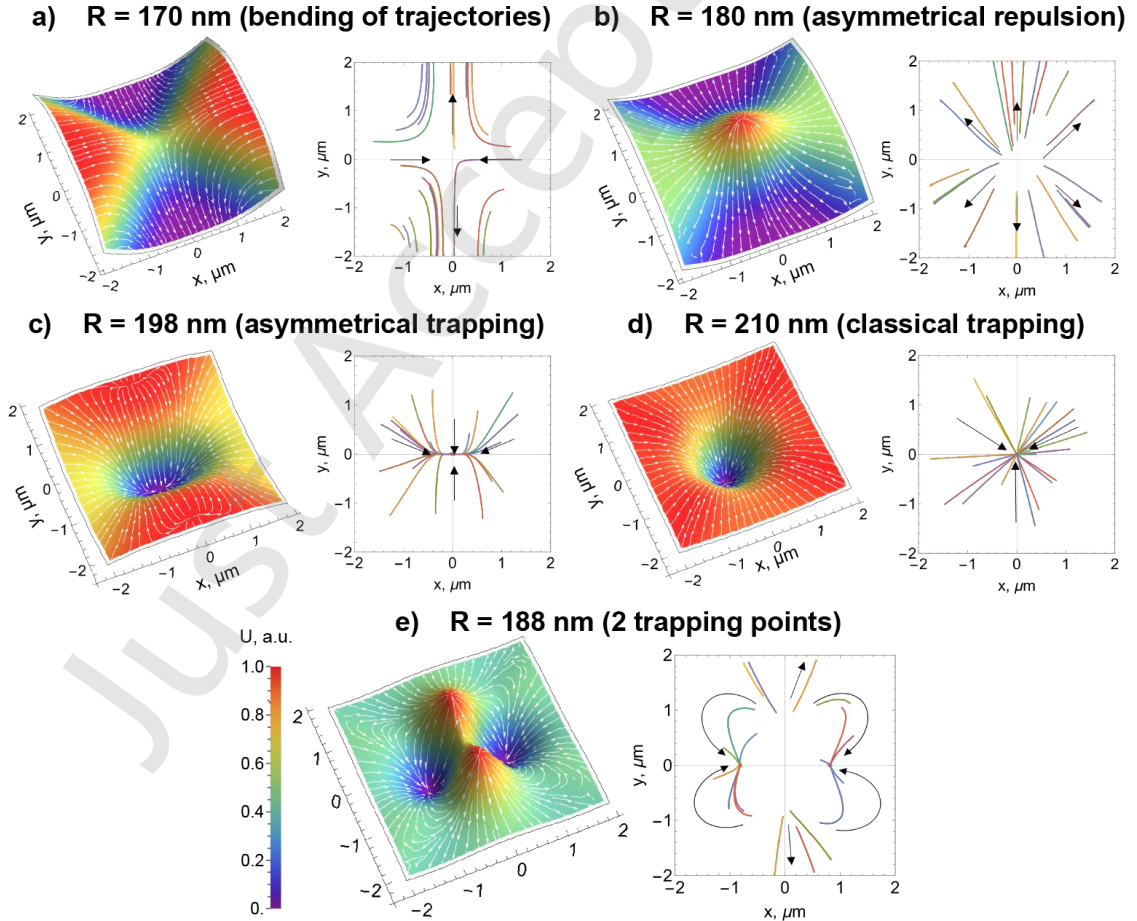


Fig. 6. Left panels (a–e): focal-plane distributions of the quantity U (an energy-dimensional path integral of the transverse force, according to Eq. (4)), serving as an effective particle “motion landscape”. The maps are normalized to their maxima, with the minima set to zero. Right panels: the trajectories of 20 nanocylinders of the same radius (for each panel) in the corresponding force fields. All the nanocylinders are suspended in air with randomly chosen starting positions and zero initial speed. The particle’s movement time is 20 ms, the calculation step is 0.02 ms

Fig. 5 shows that particle dynamics is highly sensitive to its cylinder radius, especially for parameters close to the HAS. For the particle with $R = 210$ nm, which is far from the HAS force exhibit conventional trapping behavior: for small lateral displacements from the beam center the restoring force drives the particle back toward the optical axis. At $R = 170$ nm we observe a mixed regime: trapping persists along x axis, whereas anti-trapping occurs along y axis, resulting from a saddle-like variation of the force in the plane. At $R = 180$ nm both transverse components turn repulsive, corresponding to asymmetrical repulsion. The transition from trapping to antitrapping is governed by the sign of the polarizability tensors of individual multipoles. This scenario is also achievable in other resonant particles, as shown in [28–30].

Near the HAS, at $R = 198$ nm, trapping re-emerges, but exhibits a marked asymmetry: while a conventional trap with an equilibrium position at the beam center is formed along the y -axis the force along the x -axis has a pronounced flattening near the beam center (green curve in Fig. 5a and b). Consequently, a spatially extended near-zero-force plateau develops around the optical axis, resulting in weak confinement of the particle. Finally, in the HAS ($R = 188$ nm), both F_x and F_y change sign at finite submicrometer distances from the beam center, yielding stable (on x axis) and unstable (on y axis) equilibrium positions. This is an inherently unusual situation for conventional optomechanics driven by a Gaussian beam.

3 Conclusion

In summary, we developed a generalized theoretical framework describing optical forces beyond the conventional approximation. By introducing a hierarchy of generalized polarizability tensors, we demonstrated that, near special scattering regimes such as the hybrid anapole state, the optomechanical response of resonant high-index nanoparticles becomes strongly influenced by spatial derivatives of the incident electromagnetic field. In this regime, suppression of dominant radiative scattering channels enhances the relative contribution of gradient-driven force components, leading to a qualitative reshaping of the optical-force landscape.

In addition, we showed that geometric tuning near the hybrid anapole condition gives rise to several distinct optomechanical regimes: classical trapping, saddle-shaped trajectory bending, asymmetric repulsion, the emergence of extended zero-force plateaus and multi-point off-axis equilibria. Crucially, these spatially complex trapping behaviours emerge under conventional Gaussian illumination, highlighting their practical accessibility.

These results indicate that resonant scattering engineering provides an effective route for accessing gradient-driven and spatially dispersive optomechanical effects in dielectric nanoparticles. The proposed framework may be useful for the development of advanced optical manipulation and sorting techniques, resonant optomechanical platforms, and integrated lab-on-a-chip systems.

4 Methods

For numerical simulations we utilized COMSOL Multiphysics®, module ‘Electromagnetic Waves, Frequency Domain’. A cylindrical silicon nanoparticle with varying radius R and height H was placed into the spherical shaped computational domain. Perfectly Matched Layers (PML) with the thickness of $\lambda/2$ were used to simulate the escape of a scattered light to infinity. A virtual cylindrical surface with radius of $R_1 = 1.5R$ and a height of $H_1 = 1.5H$, surrounding the nanoparticle, was incorporated for integrating the Maxwell stress tensor in the calculation of optical forces. The refractive index of silicon is taken from the article [47].

Acknowledgements. This work was supported by the Russian Science Foundation № 25-22-00266.

Disclosures. The authors declare that there are no conflicts of interest related to this article.

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Electronic Supplementary Material

Nonlocal optomechanics: Hybrid anapole opens a route to unconventional optical tweezing

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Supporting information to <https://doi.org/10.26599/NR.2026.94909195>

1 Methods

1.1 Gaussian beam simulation

To calculate the field of a Gaussian beam linearly-polarized along the x-axis and propagating along the z-axis, we use the paraxial approximation, which is given by [1]:

$$\begin{cases} E_{gauss}^x \approx E_0 \frac{e^{-\rho/(1+z^2/z_0^2)}}{\sqrt{1+z^2/z_0^2}} e^{i\{kz[1+z_0\rho^2/k(z^2+z_0^2)]-\arctg(z/z_0)\}}, \\ E_{gauss}^y = 0, \\ E_{gauss}^z \approx -\frac{ix}{z_0} E_x \frac{e^{-i\arctg(z/z_0)}}{\sqrt{1+z^2/z_0^2}}, \\ k = \frac{\omega}{c}; \rho = \frac{\sqrt{x^2+y^2}}{w_0}; z_0 = \frac{kw_0^2}{2}, \end{cases}$$

* MERGEFORMAT (1)

Where c is the speed of light in vacuum, k is the wavenumber of light, w_0 is the characteristic radius of the beam at its waist (focus) and z_0 is the Rayleigh range.

To simulate laser radiation, we used a Gaussian beam with the radius equal to the wavelength ($w_0 = \lambda$). It is important to note that linearly polarized Gaussian beam has a purely real x-component and a purely imaginary z-component. This is a fundamental property of the Gaussian beam (which is described in more detail, for example, in [2]).

1.2 Langevin Dynamics

To study the planar dynamics of nanoparticles in optical tweezers, we utilize the Langevin equation, which, in general, has the form:

$$m \frac{d^2 \mathbf{r}(t)}{dt^2} = \mathbf{F}_{tr}(\mathbf{r}, t) - \gamma \frac{d\mathbf{r}(t)}{dt} + \mathbf{f}_{rand}(t),$$

* MERGEFORMAT (2)

where m is the mass of the nanocylinder, $\mathbf{r}$ is the location of the mass center, $\mathbf{F}_{tr} = \mathbf{F}_x + \mathbf{F}_y$ is the total transverse optical force, γ is a damping factor due to viscous friction of the surrounding medium, and $\mathbf{f}_{rand}$ is a stochastic force. The medium is assumed to be air, which allows us to neglect the impact of viscous and stochastic forces and consider only the effects of optical forces.

We note that Eq. (2) employs a scalar viscous drag coefficient, which is formally exact only for spherical particles. A more rigorous description of the

dynamics of cylindrical particles in a viscous medium would require a resistance matrix accounting for anisotropic translational and rotational hydrodynamic drag. In the present work, however, the Langevin dynamics simulations are intended solely as a qualitative illustration of the particle dynamics associated with the calculated optical-force landscapes. Furthermore, the simulations are performed for a particle suspended in air, where viscous damping is weak, and the simplified drag model is sufficient for the purposes of the present study.

The results of applying this approach to nanoparticles of various radii are shown in the main text, Fig. 6, right panels a)-e).

1.3 Optical energy landscapes

An optomechanical interaction can also be characterized using a potential energy approach; for spherical nanoparticles, this is described in detail in [3]. For a cylindrical nanoparticle, however, the optical force may include non-conservative components. Nevertheless, we can define an energy-dimension quantity U representing the work needed to move the particle from infinity to a given point against the optical forces. In this study, we are interested in the transverse optical energy, defined as the work against the transverse components F_x and F_y . As the force distribution is computed numerically on a discrete grid, the integral is approximated as follows:

$$U(x_0, y_0) = - \int_{\infty}^{(x_0, y_0)} \mathbf{F}_{opt} \cdot d\mathbf{r} \approx - \iint_S F_{opt}(x, y) dx dy$$

* MERGEFORMAT (3)

Where $S = [0, L/2] \times [0, L/2]$ is a square integration domain with side $L = 2\mu m$ length. The grid was uniform with a step $\Delta x = \Delta y = 100nm$, $N = 20$ is the number of grid nodes along one axis.

In the numerical calculation, we used the previously calculated matrices of optical force values along x and y axis $\{F_{i,j}^x\}$ and $\{F_{i,j}^y\}$. The energy at the origin was taken to be zero. Then, the energy values were calculated along the x and y axes. The values in the entire domain were then calculated iteratively by averaging the values calculated along the two paths (along x and y). The final computational scheme for energy:

$$\begin{cases} U_{1,1} = U(0,0) = 0 \\ U_{i,1} = U_{i-1,1} - F_{i,j}^x \Delta x, \quad i = 2 \dots N \\ U_{1,j} = U_{1,j-1} - F_{i,j}^y \Delta y, \quad j = 2 \dots N \\ U_{i,j} = \frac{1}{2} [(U_{i-1,j} - F_{i,j}^x \Delta x) + (U_{i,j-1} - F_{i,j}^y \Delta y)], \quad i, j = 2 \dots N \end{cases}$$

* MERGEFORMAT (4)

The grid was uniform with a step $\Delta x = \Delta y = 100nm$, $N = 20$ is the number of grid nodes along one axis, $\{F_{i,j}^x\}$ and $\{F_{i,j}^y\}$ are matrices of optical force values along x and y axes.

2 Hybrid Anapole State

2.1 Multipole decomposition

To describe the electromagnetic fields scattered by nanoparticles, we used two types of multipole decomposition: exact spherical multipoles [4,5] (were used in the scattering cross-section and optical force calculations) and Cartesian multipoles with toroidal terms [6,7] (were used to demonstrate the fulfillment of the HAS condition).

Table 1. Spherical multipoles in Cartesian coordinates

Multipole type	Expression
Electric dipole	$p_i = \frac{i}{\omega} \left\{ \int J_i^\omega j_0(kr) d^3\mathbf{r} + \frac{k^2}{2} \int [3(\mathbf{r} \cdot \mathbf{J}^\omega) r_i - r^2 J_i^\omega] \frac{j_2(kr)}{(kr)^2} d^3\mathbf{r} \right\}$
Magnetic dipole	$m_i = \frac{3}{2} \int [\mathbf{r} \times \mathbf{J}^\omega]_i \frac{j_1(kr)}{kr} d^3\mathbf{r}$
Electric quadrupole	$EQ_{ij} = \frac{3i}{\omega} \left\{ \int [3(r_j J_i^\omega + r_i J_j^\omega) - 2(\mathbf{r} \cdot \mathbf{J}^\omega) \delta_{ij}] \frac{j_2(kr)}{kr} d^3\mathbf{r} + \right. \\ \left. + 2k^2 \int [5r_i r_j (\mathbf{r} \cdot \mathbf{J}^\omega) - (r_j J_i^\omega + r_i J_j^\omega) r^2 - r^2 (\mathbf{r} \cdot \mathbf{J}^\omega) \delta_{ij}] \frac{j_3(kr)}{(kr)^3} d^3\mathbf{r} \right\}$
Magnetic quadrupole	$MQ_{ij} = 15 \int \left\{ r_i [\mathbf{r} \times \mathbf{J}^\omega]_j + r_j [\mathbf{r} \times \mathbf{J}^\omega]_i \right\} \frac{j_2(kr)}{(kr)^2} d^3\mathbf{r}$
Electric octupole	$EO_{ijk} = \frac{15i}{\omega} \left\{ \int r_i r_j J_k + r_i r_k J_j + r_j r_k J_i - \frac{1}{5} (\delta_{jk} (2r_i (\mathbf{r} \cdot \mathbf{J}) + r^2 J_i) + \right. \\ \left. + \delta_{ik} (2r_j (\mathbf{r} \cdot \mathbf{J}) + r^2 J_j) + \delta_{ij} (2r_k (\mathbf{r} \cdot \mathbf{J}) + r^2 J_k)) \frac{j_2(kr)}{(kr)^2} d^3\mathbf{r} \right\}$
Magnetic octupole	$MO_{ijk} = \frac{105}{4} \left\{ \int r_i r_j [\mathbf{r} \times \mathbf{J}]_k + r_i r_k [\mathbf{r} \times \mathbf{J}]_j + r_j r_k [\mathbf{r} \times \mathbf{J}]_i - \right. \\ \left. - \frac{r^2}{5} [\delta_{ij} [\mathbf{r} \times \mathbf{J}]_k + \delta_{ik} [\mathbf{r} \times \mathbf{J}]_j + \delta_{jk} [\mathbf{r} \times \mathbf{J}]_i] \frac{j_3(kr)}{(kr)^3} d^3\mathbf{r} \right\}$

Where all integrals are taken over the volume of the particle, Latin indices $i, j, k, l, n \in \{1, 2, 3\}$ denote Cartesian components, $\mathbf{J}^\omega = \mathbf{J}^\omega(\mathbf{r})$ is the spatially localized electric current density distribution corresponding to light with angular frequency ω , $k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$ is the wavenumber of incident light, $j_n(kr)$ is a spherical Bessel function of the n^{th} order.

We can also use the decomposition of fields into Cartesian moments (also called irreducible) and toroidal terms, which are higher order summands in the Taylor series expansion of scalar and vector field potentials [8]. Under the toroidal

terms we will understand both toroidal multipoles and mean square radii of various orders, a more detailed classification of which is given in [9]:

Table 2. First three orders of irreducible multipoles

Multipole type	Expression
Electric dipole	$p_i^c = \frac{i}{\omega} \int J_i^\omega d^3\mathbf{r}$
Magnetic dipole	$m_i^c = \frac{1}{2} \int [\mathbf{r} \times \mathbf{J}^\omega]_i d^3\mathbf{r}$
Electric quadrupole	$EQ_{ij}^c = \frac{i}{\omega} \left\{ \int \left[3(r_j J_i^\omega + r_i J_j^\omega) - 2(\mathbf{r} \cdot \mathbf{J}^\omega) \delta_{ij} \right] d^3\mathbf{r} \right.$
Magnetic quadrupole	$MQ_{ij}^c = \int \left\{ r_i [\mathbf{r} \times \mathbf{J}^\omega]_j + r_j [\mathbf{r} \times \mathbf{J}^\omega]_i \right\} d^3\mathbf{r}$

Table 3. Toroidal terms corresponding to irreducible multipoles

Multipole type	Expression
Electric dipole	$T_i^p = \frac{1}{10} \int [(\mathbf{J}^\omega \cdot \mathbf{r}) r_i - 2r^2 J_i^\omega] d^3\mathbf{r};$ $T_i^{p(2)} = \frac{1}{280} \int [3r^4 J_i^\omega - 2r^2 (\mathbf{J}^\omega \cdot \mathbf{r}) J_i^\omega] d^3\mathbf{r}$
Magnetic dipole	$T_i^m = \frac{i\omega}{20} \int r^2 [\mathbf{r} \times \mathbf{J}^\omega]_i d^3\mathbf{r}$
Electric quadrupole	$T_{ij}^{EQ} = \frac{1}{14} \int \left[4(\mathbf{J}^\omega \cdot \mathbf{r}) r_i r_j + 2\delta_{ij} r^2 (\mathbf{J}^\omega \cdot \mathbf{r}) - 5r^2 (r_j J_i^\omega + r_i J_j^\omega) \right] d^3\mathbf{r}$
Magnetic quadrupole	$T_{ij}^{MQ} = \frac{i\omega}{42} \int r^2 \left\{ r_i [\mathbf{r} \times \mathbf{J}^\omega]_j + r_j [\mathbf{r} \times \mathbf{J}^\omega]_i \right\} d^3\mathbf{r}$

2.2 HAS condition

In light of this formalism, an anapole state can be defined as a destructive interference of the irreducible multipole with the corresponding toroidal multipole, which results in the cancellation of the corresponding contribution to the scattering cross-section. For example, the scattering cross section of electric dipole is proportional to the sum of cartesian multipole and toroidal terms:

$$\sigma_{sca}^p = \frac{k^2}{6\pi\epsilon_0^2} \left| \mathbf{p}_{car} + \frac{ik}{c} \mathbf{T}^p + \frac{ik^3}{c} \mathbf{T}^{p(2)} \right|^2 = 0$$

* MERGEFORMAT (5)

Therefore, the condition of electric dipole anapole can be formulated as [10]:

$$\mathbf{p}_{car} = -\left(\frac{ik}{c}\mathbf{T}^p + \frac{ik^3}{c}\mathbf{T}^{p(2)}\right)$$

* MERGEFORMAT (6)

An anapole condition requires the irreducible dipole and its toroidal counterpart to possess equal magnitudes and a phase difference of $\Delta\phi = \pm\pi$. We confirm these destructive interference conditions for the first two multipole orders in Figure S1, which plots the magnitudes and phase differences between Cartesian multipoles and corresponding toroidal terms. The spectral overlap first four multipoles is called a "hybrid anapole" and poses a near-zero scattering.

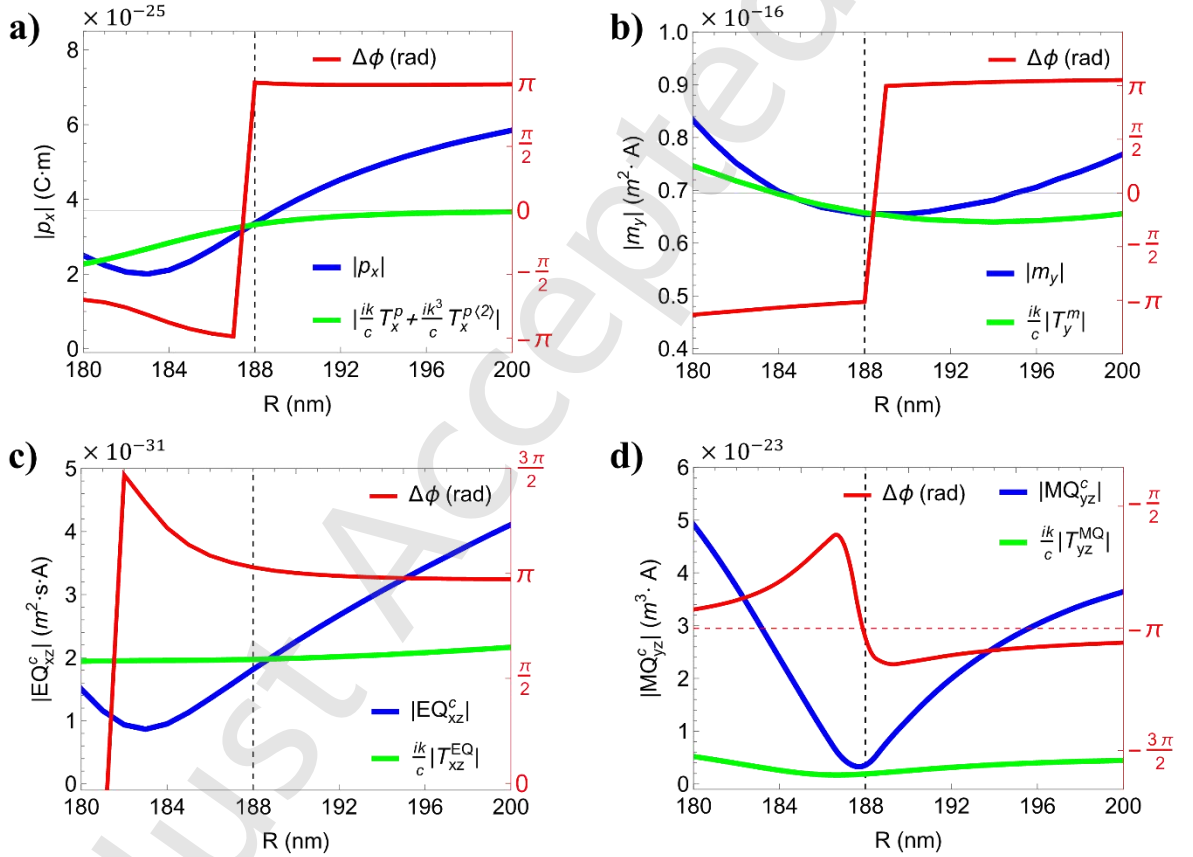


Figure S1. Amplitudes and phase differences between the cartesian multipoles and their toroidal counterparts. a) the basic electric and electric toroidal dipoles; b) the basic magnetic and magnetic toroidal dipoles; c) the basic electric and electric toroidal quadrupoles; d) the basic magnetic and magnetic toroidal quadrupoles. Amplitudes correspond to the left ordinate-axis, and phase differences are read from the right ordinate-axis. The black dashed line is drawn through the minimum of the total scattering cross section (corresponding to $R = 188$ nm).

Figure S1 shows, that for a nanoparticle with $R = 188$ nm the Cartesian multipoles and their toroidal part have the same magnitudes, and the phase difference between them $\pm\pi$ for both electric and magnetic dipoles and quadrupoles. This means that the anapole condition is satisfied for these multipoles simultaneously, resulting in a hybrid anapole state.

2.3 Effect of the surrounding medium on the hybrid anapole state

The results presented in the main text are obtained for a silicon nanocylinder in vacuum. To assess the influence of the surrounding refractive index, we additionally calculated the scattering response of the same particle in water. Figure S2 compares the parametric maps of the total scattering cross section normalized to geometric cross section as a function of the cylinder height H and incident light wavelength λ in vacuum and water.

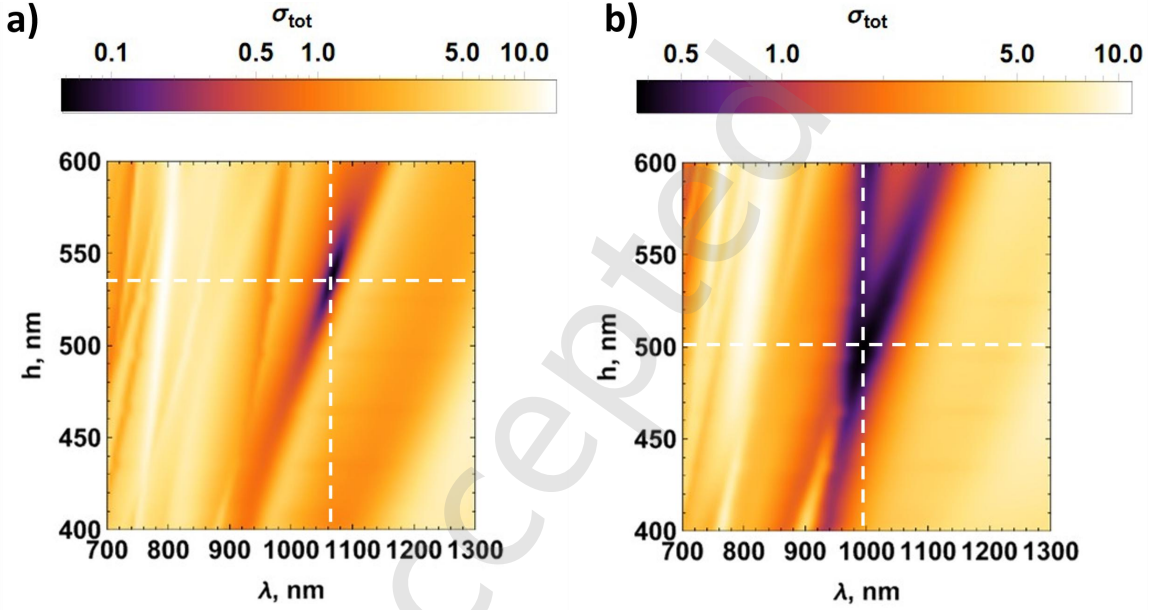


Figure S2. Parametric maps of the total scattering cross section normalized to scattering cross section as a function of the incident wavelength λ and cylinder height H for a silicon cylinder in (a) vacuum and (b) water. The cylinder radius is $R = 188$ nm. The dashed lines indicate the vicinity of the scattering minimum associated with the hybrid anapole state (HAS).

As shown in Figure S2, changing the surrounding medium substantially modifies the hybrid anapole regime. In vacuum, the overlap of the individual anapole conditions produces a well-localized minimum of the total scattering cross section in the (H, λ) parameter space. In water, the low-scattering region becomes considerably broader and elongated, and the distinct localized minimum characteristic of the HAS in vacuum becomes less pronounced. Thus, although a low-scattering regime persists in water, the simultaneous suppression of the dominant scattering channels is less sharply localized in the particle-parameter space.

This comparison also motivates our choice of vacuum as the surrounding medium for the present study. Our aim is to investigate the fundamental relation between the multipolar response, its higher-order spatial contributions, and the resulting optical forces, rather than to optimize a particle for a particular experimental environment. The pronounced and well-localized HAS obtained in vacuum provides a convenient model system in which these effects can be clearly

distinguished. In a higher-index surrounding medium such as water, the broadening of the low-scattering region makes the HAS less distinct and is therefore expected to make the associated effects investigated here less pronounced.

3 Optical Forces

3.1 General Formulation

The average optical force acting on the particle can be calculated by integrating Maxwell stress tensor on a closed surface surrounding the object:

$$\langle \mathbf{F} \rangle_t = \oint_{\partial V} \langle \vec{\mathbf{T}}(\mathbf{r}, t) \rangle \cdot \mathbf{n}(\mathbf{r}) da = \frac{1}{2} \text{Re} \oint_{\partial V} \vec{\mathbf{T}}(\mathbf{r}, t) \cdot \mathbf{n}(\mathbf{r}) da$$

* MERGEFORMAT (7)

Where $\langle \cdot \rangle_t$ means the time average over an oscillation period of the electromagnetic field, $\mathbf{n}$ is the normal unit vector to the surrounding surface ∂V , is da is an element of the surface, $\vec{\mathbf{T}}$ is the average Maxwell's stress tensor (MST) in vacuum, which is given by:

$$\vec{\mathbf{T}} = \varepsilon \varepsilon_0 \mathbf{E} \otimes \mathbf{E}^* + \mu \mu_0 \mathbf{H} \otimes \mathbf{H}^* - \frac{1}{2} (\varepsilon \varepsilon_0 \mathbf{E} \cdot \mathbf{E}^* + \mu \mu_0 \mathbf{H} \cdot \mathbf{H}^*) \vec{\mathbf{I}}$$

* MERGEFORMAT (8)

Where $\otimes$ denotes a dyadic operation over two tensors, $\vec{\mathbf{I}}$ denotes the unit tensor, $\mathbf{E}$ and $\mathbf{H}$ are the total fields, defined as the sum of the incident and scattered fields, given by:

$$\mathbf{E} = \mathbf{E}_{inc} + \mathbf{E}_{sca}; \quad \mathbf{H} = \mathbf{H}_{inc} + \mathbf{H}_{sca}$$

* MERGEFORMAT (9)

The time-averaged optical force on a dielectric particle could be expressed in terms of the induced multipoles:

$$\begin{aligned} \mathbf{F} = & [\mathbf{F}_p + \mathbf{F}_{Q^e} + \mathbf{F}_{O^e} + \quad] + [\mathbf{F}_m + \mathbf{F}_{Q^m} + \mathbf{F}_{O^m} + \quad] + \\ & + [\mathbf{F}_{pm} + \mathbf{F}_{Q^e Q^m} + \mathbf{F}_{O^e O^m} + \quad] + [\mathbf{F}_{pQ^e} + \mathbf{F}_{Q^e O^e} + \quad] + [\mathbf{F}_{mQ^m} + \mathbf{F}_{Q^m O^m} + \quad] \end{aligned}$$

* MERGEFORMAT (10)

Then, the i-th component of the optical force is given by [11]:

$$\begin{aligned}
F_i = & \frac{1}{2} \text{Re}[p_j \nabla_i E_j^*] + \frac{1}{12} \text{Re}[Q_{jk}^e \nabla_i \nabla_k E_j^*] + \frac{1}{12} \text{Re}[O_{jkl}^e \nabla_i \nabla_l \nabla_k E_j^*] + \\
& + \frac{1}{2} \text{Re}[m_j \nabla_i B_j^*] + \frac{1}{12} \text{Re}[Q_{jk}^m \nabla_i \nabla_k B_j^*] + \frac{1}{12} \text{Re}[O_{jkl}^m \nabla_i \nabla_l \nabla_k B_j^*] - \\
& - \frac{k^4}{12\pi\epsilon_0 c} \text{Re}[\epsilon_{ijk} p_j m_k^*] - \frac{k^6}{2160\pi\epsilon_0 c^2} \text{Re}[\epsilon_{ijk} Q_{lj}^e (Q_{lk}^m)^*] - \frac{k^8}{7560\pi\epsilon_0 c} \text{Re}[\epsilon_{ijk} O_{lnj}^e (O_{lnk}^m)^*] - \\
& - \frac{k^5}{120\pi\epsilon_0} \text{Im}[Q_{ij}^e p_j^*] - \frac{k^7}{1890\pi\epsilon_0} \text{Im}[O_{ijk}^e (Q_{jk}^e)^*] - \\
& - \frac{k^5}{120\pi\epsilon_0 c^2} \text{Im}[Q_{ij}^m m_j^*] - \frac{k^7}{1890\pi\epsilon_0 c} \text{Im}[O_{ijk}^m (Q_{jk}^m)^*]
\end{aligned}$$

Where ϵ_{ijk} is Levi-Civita symbol, $i, j, k, l \in \{1, 2, 3\}$. According to Einstein notation, the summation is performed over all duplicated indices.

To determine the numerical coefficient of the interaction force of 2 octupoles we used expressions for multipole forces in a general form [12]. However, this term turned out to be several orders of magnitude smaller than the other recoil components, so it is not included in the force decomposition given in the main text.

3.2 Pressure force and “HAS destruction” effect

In this section, we examine the effect of the HAS on the light pressure (F_z component of the total force). The results are presented in Figure S3, where the dependence of the pressure force in the center of the beam on the radius of the nanocylinder is shown together with the spatial distributions of optical pressure for specific nanoparticle radii.

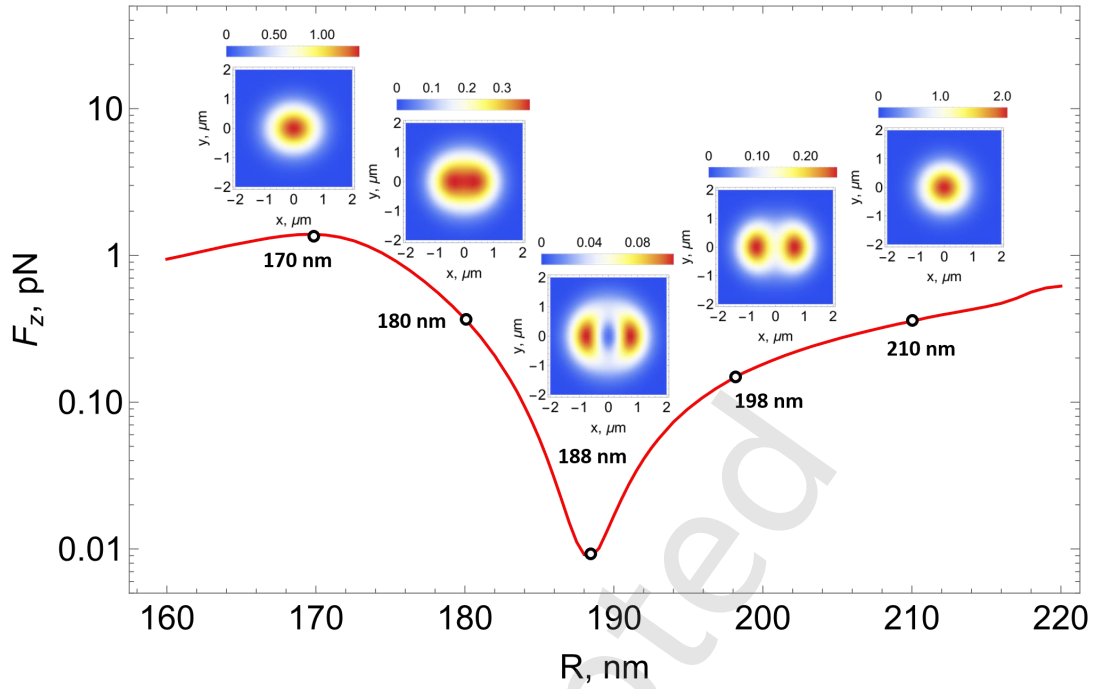


Figure S3. Light pressure, F_z , acting on a nanocylinder located in the centre of the focal plane versus nanocylinder radius. Insets show the spatial distribution of F_z in the focal plane for selected R marked on the red curve.

Notably, in regimes that are far from the hybrid anapole (e.g. $R = 170$ nm and 210 nm), light pressure assumes a radially symmetric Gaussian distribution in the focal plane. As the hybrid anapole regime is approached, the symmetry is gradually broken, the force maximum expands along the x -axis and eventually the distribution decays into 2 lobes in the HAS ($R = 188$ nm). More specifically, the nanocylinder experiences the lowest pressure on the axis of the beam, but as it drifts away from the axis, the pressure starts to increase and the increase is particularly pronounced along the x -direction, which is the consequence of the beam being linearly polarized along the x -axis. At the edge of the beam, however, the pressure falls and, following the exponential decrease of light intensity, drops to the values characteristic of the normal scattering regime. We termed the described effect as “HAS destruction”. This effect is conditioned by a response of multipole moments near the HAS on the high-order field gradients, which will be discussed in details later.

3.3 The effect of octupoles

In the hybrid anapole state (HAS), octupole moments provide a dominant contribution to the scattered field. Figure S4 illustrates the difference between the scattering efficiency computed from dipole and quadrupole terms alone and the total scattering efficiency including the octupole terms.

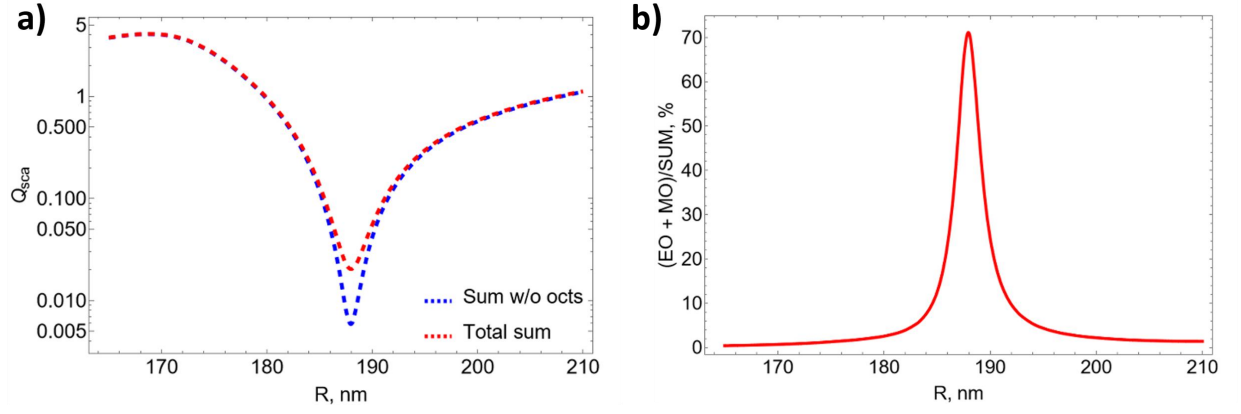


Figure S4. a) A comparison of the scattering efficiency calculated with octupole terms (red dashed curve) and without octupoles (blue dashed curve); b) Octupole (magnetic + electric) contribution to total scattering efficiency versus particle radius. The peak in the region of the HAS ($R = 188$ nm) corresponds to an increase of octupole contribution up to 70 %

For the particles in the scattering regime octupole moments are usually negligible in comparison to dipoles and quadrupoles. However, as seen in Fig. 2(c) in the main text, in the HAS all the dipole and quadrupole components decrease by two times in magnitude, while octupole components remains the same, rendering their contribution to the scattering cross section the most significant. Figure S5(b) shows a significant difference between the force calculated using the multipoles up to octupoles (yellow line), and the truncated expansion without octupoles (blue dotted line). In the scattering regime Figure S5(a) this difference is negligible.

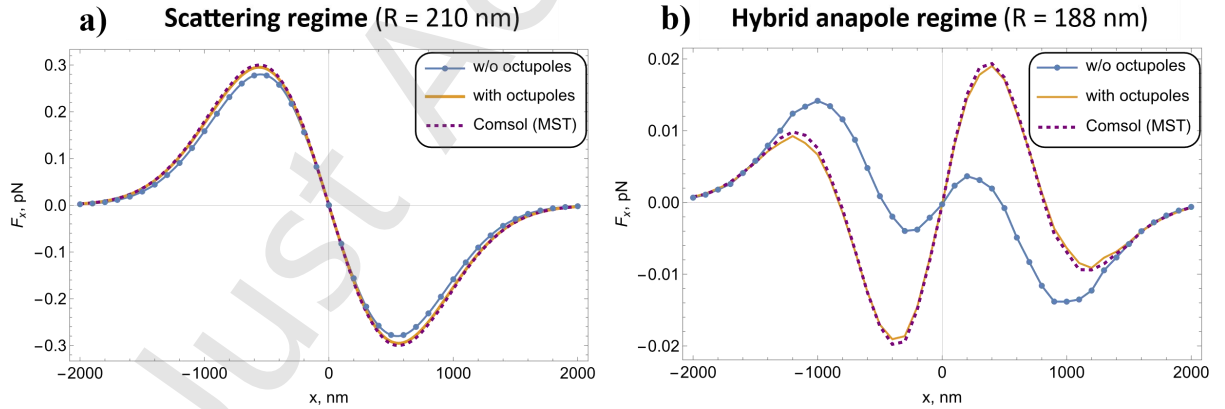


Figure S5. Comparison of total transverse force F_x , calculated using Maxwell stress tensor (pink dashed line) and using multipole decomposition up to octupoles (yellow line) with the force calculated without octupole terms (blue dotted line): a) in scattering regime ($R = 210$ nm), b) in the hybrid anapole state ($R = 188$ nm)

3.4 Multipole decomposition of scattering and forces

Using Eq. we can calculate all the interceptional and recoil components of transverse forces F_x and F_y and compare them in the HAS and in the conventional regime – Figure S6 and Figure S7.

Conventional (H = 200 nm, R = 100 nm)

HAS (H = 538 nm, R = 188 nm)

Interception components

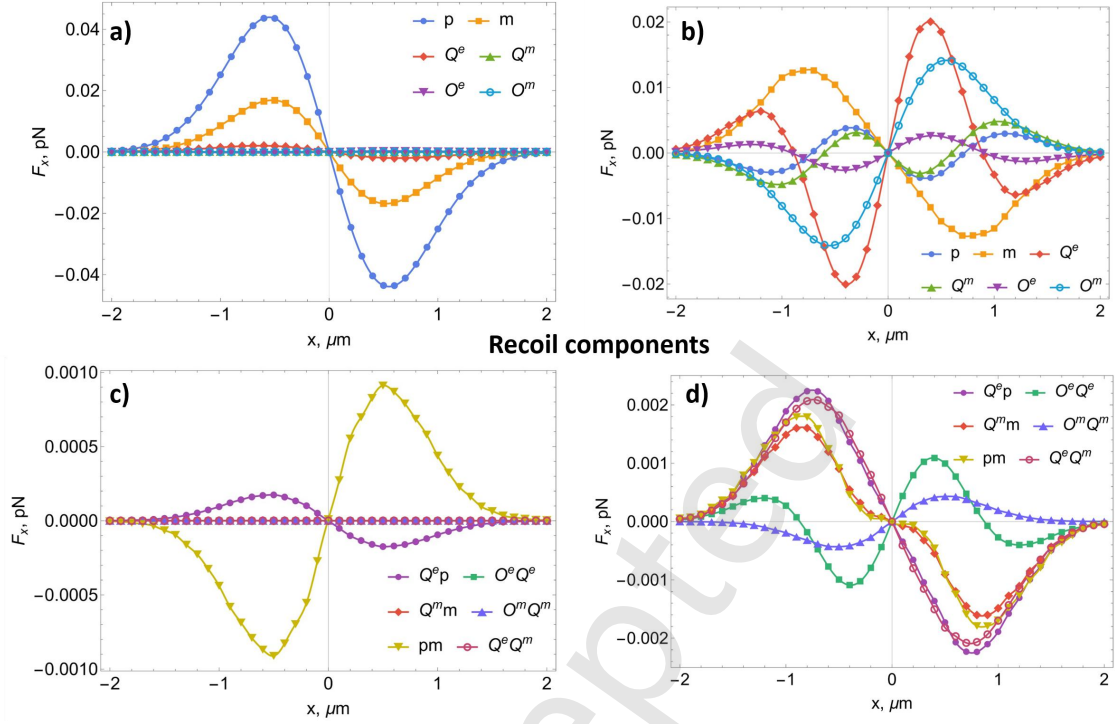


Figure S6. Interceptional and recoil components of the transverse force F_x for the nanoparticle in the conventional scattering regime (a, c) and in the HAS (b, d).

Conventional (H = 200 nm, R = 100 nm)

HAS (H = 538 nm, R = 188 nm)

Interception components

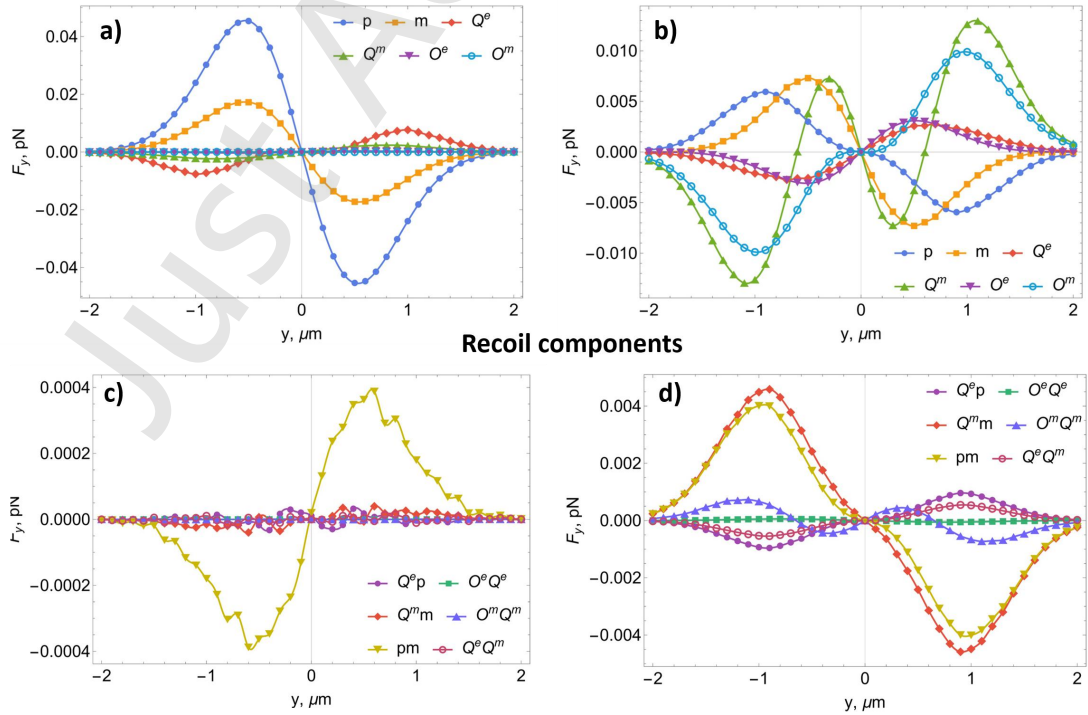


Figure S7. Interceptional and recoil components of the transverse force F_y for the nanoparticle in the conventional scattering regime (a, c) and in the HAS (b, d).

Figures F6 and F7 demonstrate that, in the HAS regime, all interception components up to the octupole have comparable magnitudes. Moreover, nearly all of these components, including the electric dipole, exhibit non-classical behavior with two zero crossings. In contrast, in the conventional regime only the dipole terms contribute significantly, and both dipole forces display the classical gradient profile, unlike their counterparts in the HAS.

Similar graphs are plotted for the F_z force at x- displacement – Figure S8 and Figure S9.

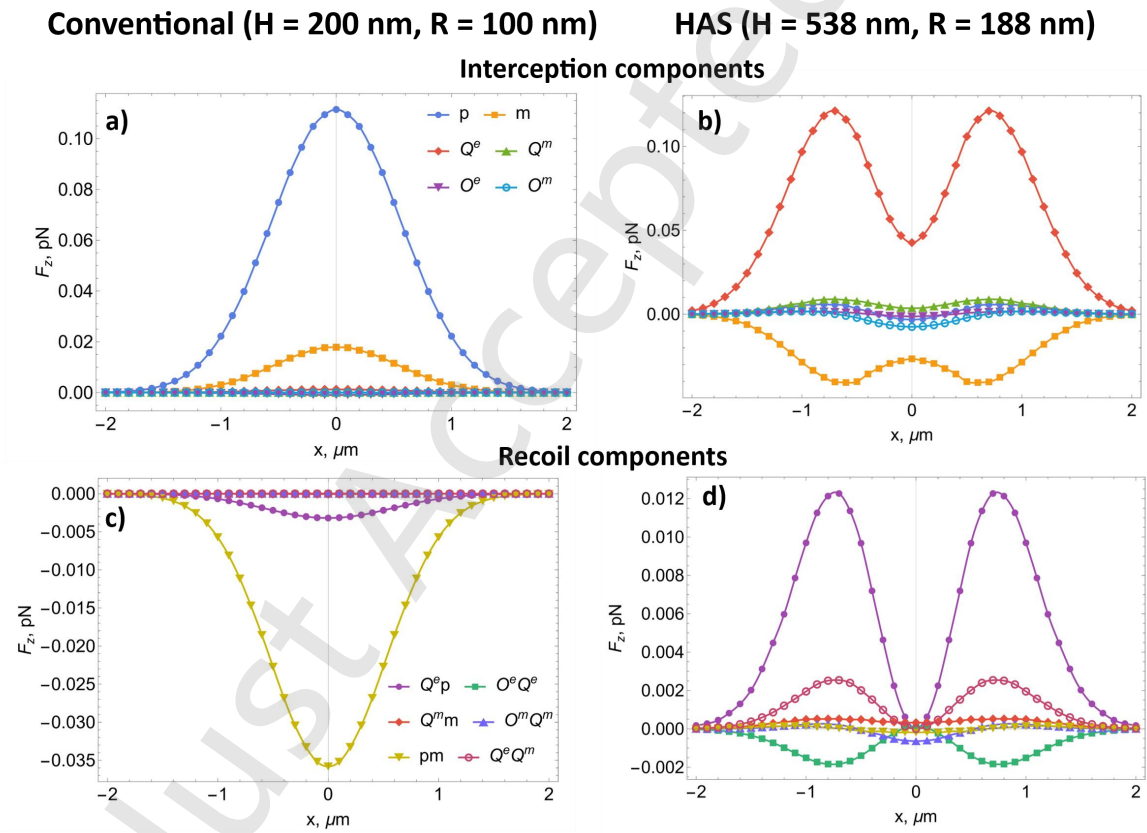


Figure S8. Interceptional and recoil components of the pressure force F_z along the x-axis for the nanoparticle in the conventional scattering regime (a, c) and in the HAS (b, d). The legend is common for each pair of graphs horizontally.

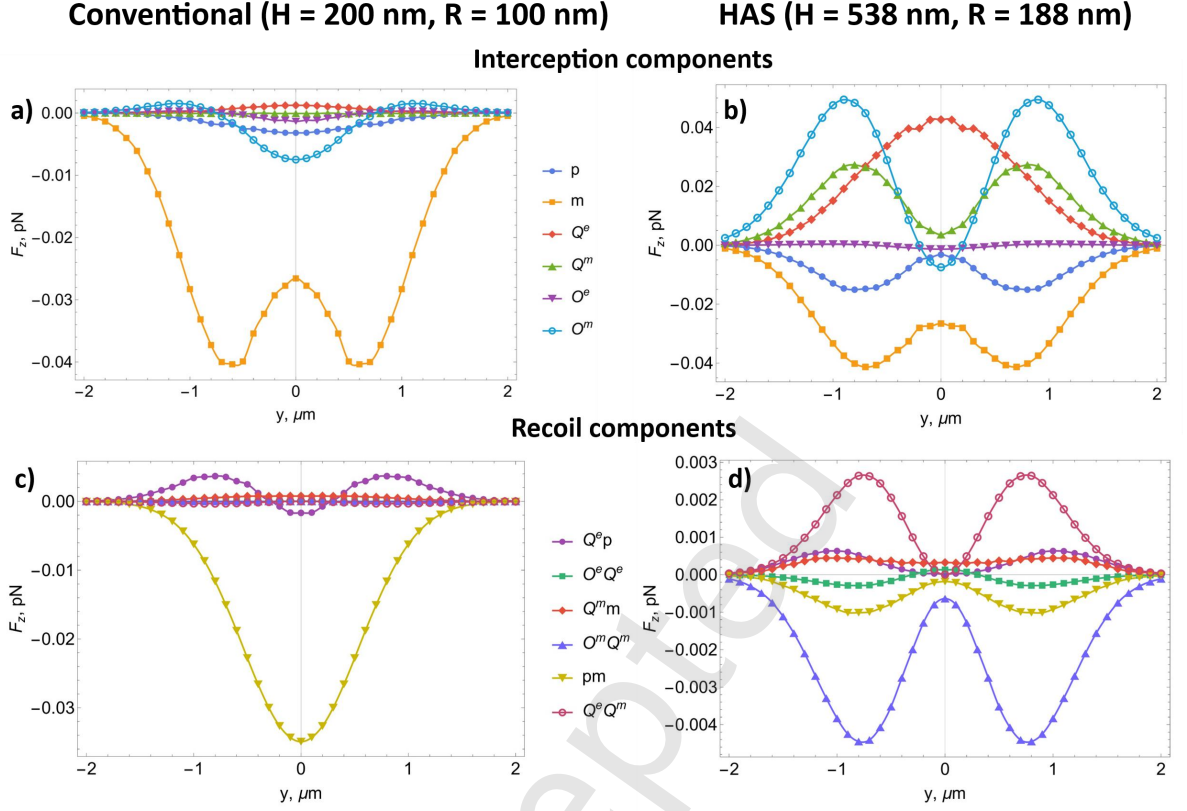


Figure S9. Interceptional and recoil components of the pressure force F_z along the y -axis for the nanoparticle in the conventional scattering regime (a, c) and in the HAS (b, d). The legend is common for each pair of graphs horizontally.

From Figures S8 and S9 we can see that the multipole components of the optical pressure force in the HAS have a dip at the beam axis, which is attributed to the so-called “HAS destruction effect”, observed in Figure S3. This effect manifests itself in both dipole and quadrupole forces, rather than just in octupoles, as one might expect from the anapole conditions. The reason for this is the predominance of the response to higher derivatives of the fields in the HAS, which is discussed in the following section.

3.5 Multipole decomposition of the scattering cross section

For comparison with the multipole decomposition of the optical force discussed above, Figure S10 shows the individual multipolar contributions to the scattering cross section for the conventional and hybrid-anapole regimes as a function of the particle displacement along the x - and y -axes. The same data are presented on linear and logarithmic scales to resolve both the dominant and strongly suppressed components.

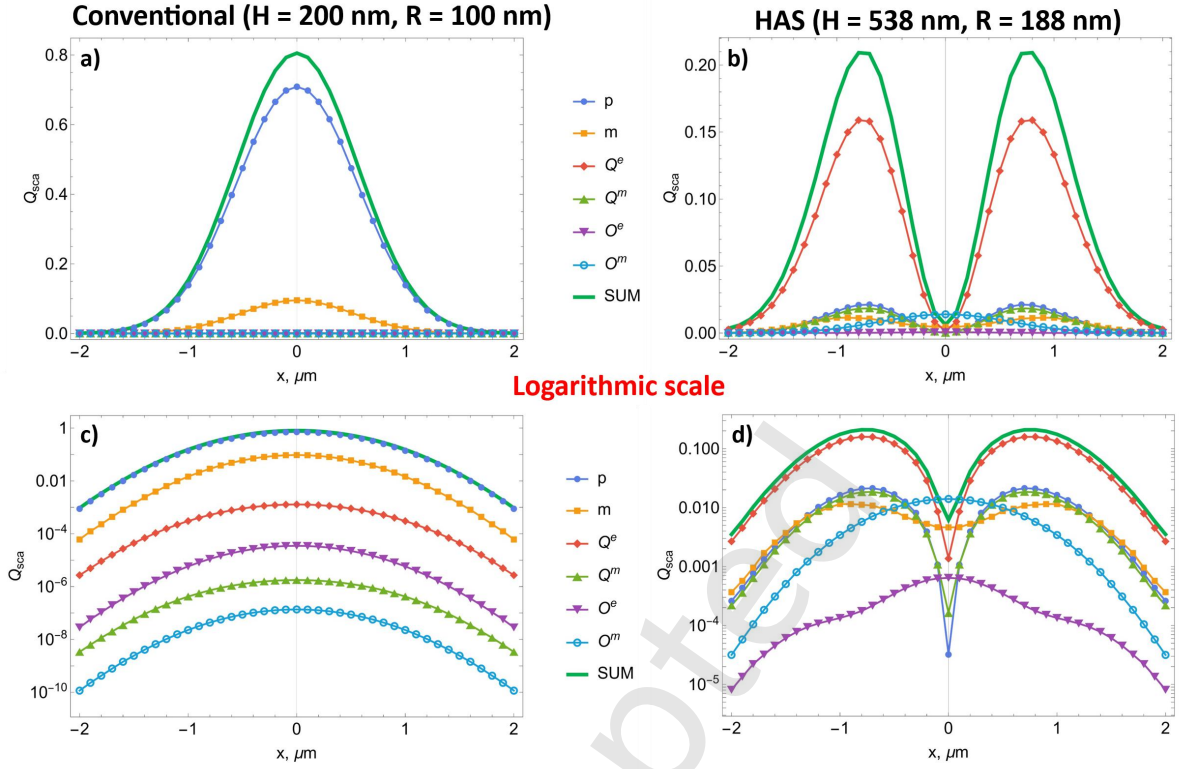


Figure S10. Scattering cross section normalized to geometric section along the x-axis in the conventional scattering regime (a, c) and in the HAS (b, d). The graphs on the lower panels (c,d) are plotted on a logarithmic scale.

The comparison shows that the multipolar hierarchy of the scattering cross section does not necessarily coincide with that of the interception optical force. This difference reflects the fact that the scattering cross section is determined by the radiated power of the induced multipoles, whereas the interception force also depends on their coupling to the corresponding spatial derivatives of the incident field.

4 Nonlocal polarizabilities calculation

4.1 Polarizabilities and scattered field formulation

The main text described the principle of calculating an arbitrary multipole as the sum of responses to an incident electric field $\mathbf{E}$ and its spatial derivatives. The general formula is:

$$M_i[\mathbf{E}(\mathbf{r})] = \alpha_{ij}^M E_j(\mathbf{r}_0) + \sum_{n=1}^N \sum_{k_1 \dots k_n} \frac{1}{n!} \alpha_{ijk_1 \dots k_n}^M \frac{\partial^n E_j}{\partial r_{k_1} \dots \partial r_{k_n}}(\mathbf{r}_0)$$

* MERGEFORMAT (12)

where indices $i = (i_1 \dots i_s)$ denote the set of Cartesian components of the multipole ($s = 1$ – dipole, $s = 2$ – quadrupole, etc.), $N \geq 1$ is the order of the last significant term in the series, $k_1, \dots, k_n \in \{1, 2, 3\}$, $\alpha_{ijk_1 \dots k_n}^M$ is a component of a

polarizability tensor relating the i^{th} component of the multipole M to the spatial derivatives of a j^{th} component of an incident field.

Let us write electric and magnetic dipoles in this formalism, taking into account the symmetry of the system:

$$\begin{aligned} p_i &= \alpha_{ij}^p E_j + \alpha_{ijkl}^p \frac{\partial^2 E_j}{\partial r_k \partial r_l} + \alpha_{ijklmn}^p \frac{\partial^4 E_j}{\partial r_k \partial r_l \partial r_m \partial r_n}; \\ m_i &= \alpha_{ijk}^m \frac{\partial E_j}{\partial r_k} + \alpha_{ijklm}^m \frac{\partial^3 E_j}{\partial r_k \partial r_l \partial r_m}; \end{aligned}$$

* MERGEFORMAT (13)

Where the proportionality coefficients between the multipoles and the derivatives of the incident electric field are the polarizability tensors of the corresponding rank:

$$\begin{aligned} \alpha_{ij}^p &= p_i [\mathbf{e}_j], \quad \alpha_{ijkl}^p = p_i \left[\frac{1}{2!} r_k r_l \mathbf{e}_j \right], \quad \alpha_{ijklmn}^p = p_i \left[\frac{1}{4!} r_k r_l r_m r_n \mathbf{e}_j \right]; \\ \alpha_{ijk}^m &= m_i [r_k \mathbf{e}_j], \quad \alpha_{ijklm}^m = m_i \left[\frac{1}{3!} r_k r_l r_m \mathbf{e}_j \right], \end{aligned}$$

* MERGEFORMAT (14)

The first terms in both expressions in Eq. gives us the conventional approximation, while higher-order terms are usually considered negligible.

In all cases the corresponding polarizabilities can be calculated as the response to a uniform field, a linearly growing field with a unit gradient, and so on. However, none of these fields satisfy Maxwell's equations and thus cannot be directly applied to the particle in a numerical solver.

To address this, we employ the widely used scattered field formulation [13,14]. The main principle of this approach is that the field scattered by the particle (defined as the difference between the total field and the background one) $\mathbf{E}_{sc}(\mathbf{r}) = \mathbf{E}_{tot}(\mathbf{r}) - \mathbf{E}(\mathbf{r})$ can be determined as the radiation from an equivalent current source within the particle:

$$\mathbf{j}_{primary}(\mathbf{r}) = -\frac{i\omega}{4\pi} \Delta\epsilon(\mathbf{r}, \omega) \mathbf{E}(\mathbf{r}, \omega),$$

* MERGEFORMAT (15)

where $\Delta\epsilon = \epsilon(\mathbf{r}, \omega) - \epsilon_{bg}(\mathbf{r}, \omega)$ is the difference between the permittivity of the scatterer and the background media.

The total field, which is used to calculate the multipole moments, is then the sum of the background and scattered fields:

$$\mathbf{E}_{tot}(\mathbf{r}) = \mathbf{E}(\mathbf{r}) + \mathbf{E}_{sc}(\mathbf{r})$$

* MERGEFORMAT (16)

Now, we can substitute any non-physical field into Eq. , and the corresponding currents might be used for further calculations of multipole moments without issue. While this might initially appear to be a trick for solving an otherwise intractable problem, it is not. The optical response to a non-physical field should only be interpreted within the context of an infinite series, which, when summed, represents the response to a physically realizable field. In practical applications, we always work with a finite, truncated series, which provides an approximate solution - this is both sufficient and intended for most use cases.

Numerical simulations of response to power-law were conducted in COMSOL Multiphysics®, the geometry and parameter set were identical to the ones described in the in the main text.

4.2 Convergence of Taylor series

To find out which order of derivatives in the Taylor series is sufficient to approximate the full field, we plotted the shape of the Taylor-approximated field along the x-axis and compared it with the full Gaussian beam given by Eq.

The field along the x-axis is affected only by the 2nd, 4th and 6th derivatives in the Taylor series, so we can break the series on them and write down two approximate expressions:

$$\mathbf{E}_2(\mathbf{r}) = \mathbf{E}(0) + r_m \frac{\partial \mathbf{E}(0)}{\partial r_m} + r_m r_n \frac{\partial^2 \mathbf{E}(0)}{\partial r_n \partial r_m}$$

* MERGEFORMAT (17)

$$\mathbf{E}_4(\mathbf{r}) = \mathbf{E}_2(\mathbf{r}) + r_m r_n r_l r_s \frac{\partial^4 \mathbf{E}(0)}{\partial r_s \partial r_l \partial r_n \partial r_m}; \quad \mathbf{E}_6(\mathbf{r}) = \mathbf{E}_4(\mathbf{r}) + r_m r_n r_l r_s r_j r_k \frac{\partial^6 \mathbf{E}(0)}{\partial r_s \partial r_l \partial r_n \partial r_m \partial r_j \partial r_k}$$

* MERGEFORMAT (18)

Now let us compare fields approximated up to 2nd, up to 4th and up to 6th orders of Taylor series ($\mathbf{E}_2(x)$, $\mathbf{E}_4(x)$ and $\mathbf{E}_6(x)$ respectively) with the exact field $\mathbf{E}_{gauss}(x) = \{E_{gauss,x}(x), 0, E_{gauss,z}\}$. Noteworthy, that $E_{gauss,x} = \text{Re}[E_{gauss,x}]$; $E_{gauss,z} = \text{Im}[E_{gauss,z}]$ and these expressions hold for all the spatial derivatives of these components.

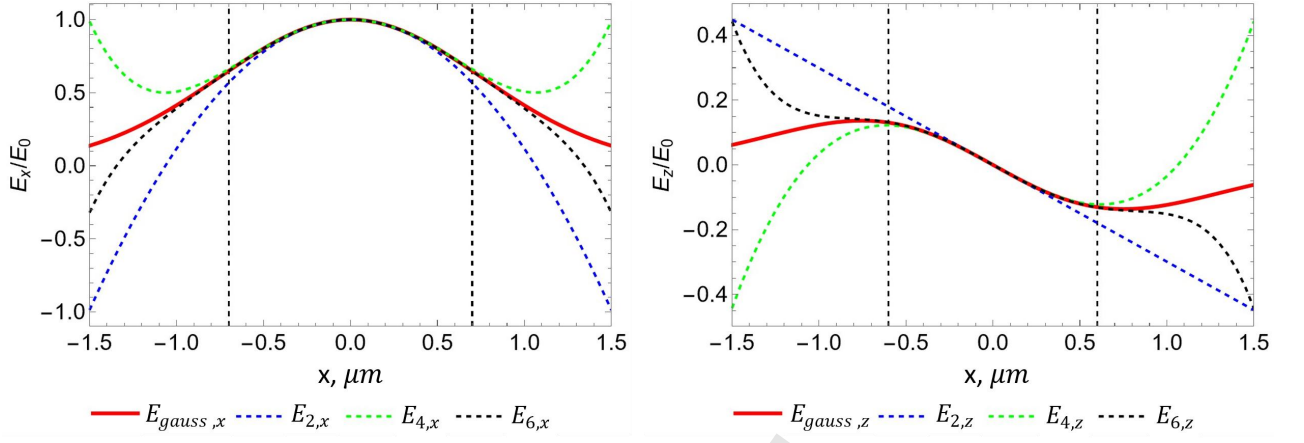


Figure S11. Dependence of the Gaussian beam profile, its 2nd, 4th and 6th derivatives normalized to the maximum on the x-coordinate: a) real part of E_x , b) imaginary part of E_z .

Figure S11 shows that, within the spatial range from -600 to 600 nm, retaining terms up to the fourth order provides an accurate representation of the incident Gaussian field. Thus, for particles with linear dimensions up to 600 nm, considered in our paper, the Taylor expansion to the fourth order of derivatives is sufficient to accurately reproduce the distribution of the incident field inside the particle.

We emphasize that the Taylor expansion is employed only in the reconstruction of the induced multipole moments. The optical-force expressions in Eq. (4) are evaluated using the full incident field and its exact spatial derivatives. Therefore, the truncation with respect to the derivative order enters the force calculation only through the reconstructed multipoles, and convergence of the force follows from convergence of the multipoles entering its multipolar decomposition.

4.3 Conventional local approximation

In the conventional approximation, which is widely used for low-index and small nanoparticles, multipole moments are given by the incident field at the particle's center multiplied by the corresponding polarizability tensors:

$$\mathbf{p} = \alpha_p \mathbf{E}_{inc}; \quad \mathbf{m} = \alpha_m \mathbf{H}_{inc}$$

$$\mathbf{Q}^e = \alpha_{Q^e} \frac{\nabla \mathbf{E}_{inc} + \mathbf{E}_{inc} \nabla}{2}; \quad \mathbf{Q}^m = \alpha_{Q^m} \frac{\nabla \mathbf{H}_{inc} + \mathbf{H}_{inc} \nabla}{2},$$

* MERGEFORMAT (19)

As in the Taylor series method, multipole moments and the electromagnetic field are related through polarizability tensors. However, in the conventional approximation magnetic moments are proportional to the magnetic field and its first gradient, while in Taylor series method all the multipoles are proportional only to the electric field gradients and its gradients. This difference could be overcome by using 3rd Maxwell's equation, which relates the magnetic field with

the rotor to the electric field. Thus, the i^{th} component of the magnetic field can be expressed in terms of the difference of the derivatives of the electric field of the form $(\partial_j E_k - \partial_k E_j)$, and after some symmetrization procedures, magnetic polarizability tensor of 2nd rank (representing response to a magnetic field) can be expressed in terms of electric polarizability tensors of 3rd rank (representing response to the 1st derivative of the electric field).

4.4 Electric dipole force

After the examination of electric dipole in the hybrid anapole state we can explain the shape of the first term in formula, which reflects the contribution of electric dipole to the force. It consists of 3 components proportional to p_x , p_y and p_z . Due to the symmetry of our system, $p_y = 0$. Figure S12 shows the contributions of p_x and p_z to the total electric dipole force in the conventional regime (nanocylinder with $R = 100$ nm, $H = 200$ nm) and in the hybrid anapole state ($R = 188$ nm, $H = 538$ nm):

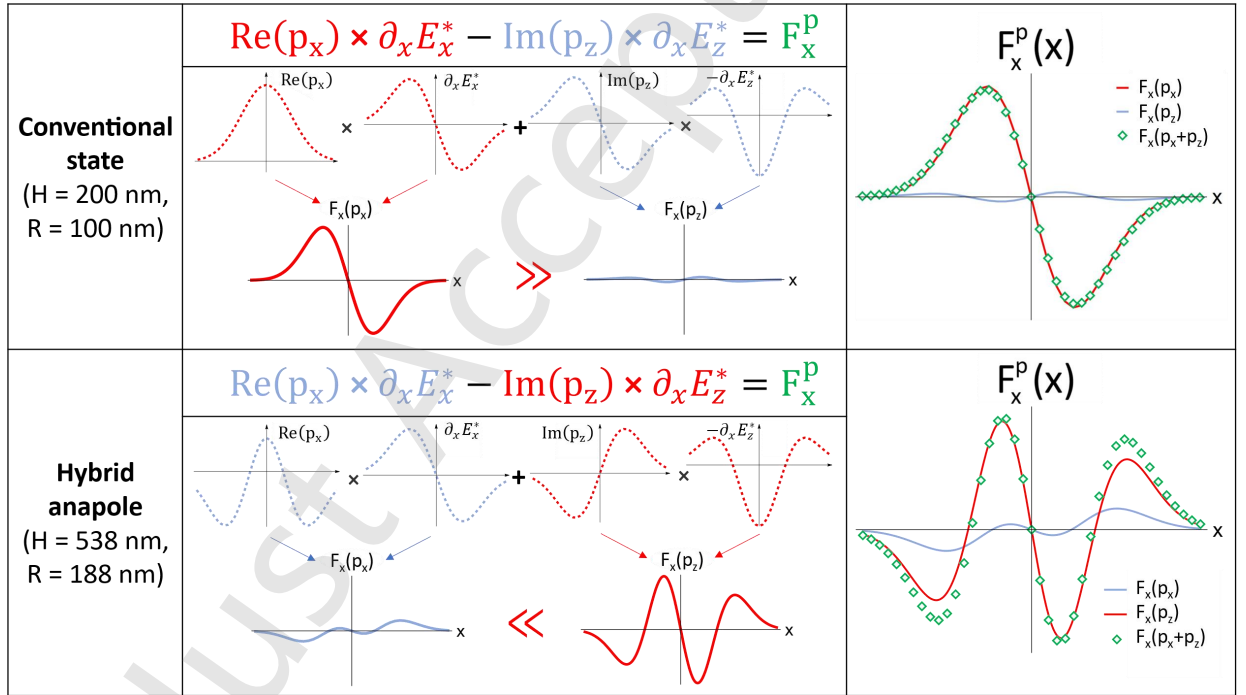


Figure S12. Electric dipole component of the transverse F_x force in the case of conventional trapping (upper panel) and HAS (lower panel). Central panel shows the components of the dipole multiplied by the field derivatives that contribute to the total force. The results are shown below them. Red curves depict the strongest components, light blue curves – the weakest. Right panel compares contributions from p_x , p_z , and the total electric dipole force

Figure S12 reveals a pronounced difference in the contributions of dipole components to the total optical force in different scattering regimes. In the conventional case, the p_x component dominates over p_z , while in the hybrid anapole state, the inverse is true. This example explains another mechanism of non-classical force formation in addition to the nonlocal response: within the HAS, the p_x component decreases, while p_z (which is proportional to the imaginary component of Gaussian beam – E_z) remains significant, thereby becoming the

dominant contributor that forms a force profile with 2 trapping points along the x-axis. In the conventional regime, the contribution of the p_z to the force is negligible compared to p_x so classical trapping is realized. The analysis of other multipoles allowed us to identify the components dominant in the conventional regime (e.g., p_x, Q_{xz}^e, Q_{yz}^m) and in the HAS (e.g., $p_z, Q_{xx}^e, Q_{zz}^e, Q_{zz}^m$).

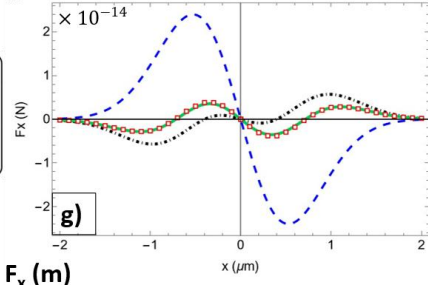
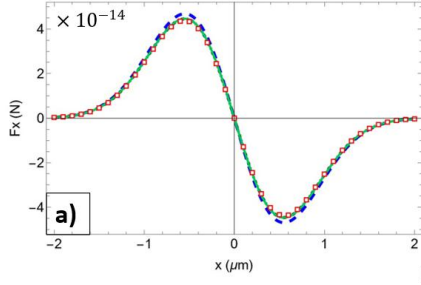
4.5 Interceptional components of force

We now apply the aforementioned Taylor series expansion of the incident field to compute all interceptional components of the force in Eq. , both in the conventional scattering regime and in the HAS – Figure S13.

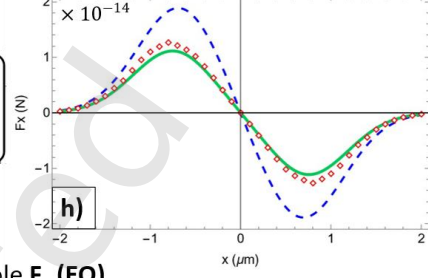
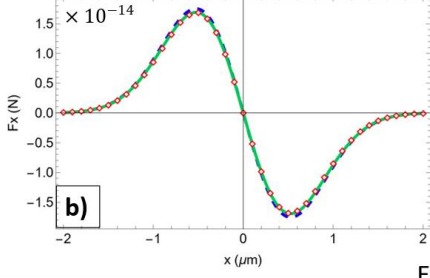
Conventional (H = 200 nm, R = 100 nm)

HAS (H = 538 nm, R = 188 nm)

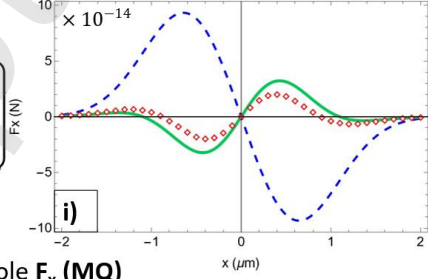
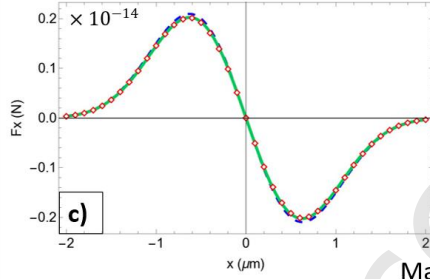
Electric dipole $F_x(p)$



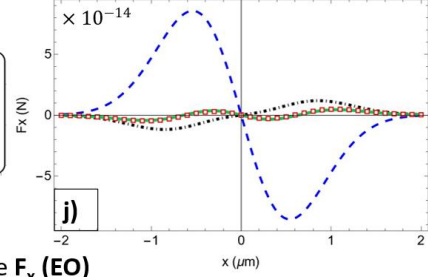
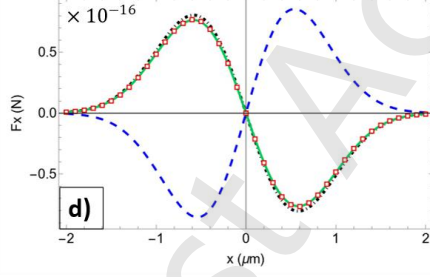
Magnetic dipole $F_x(m)$



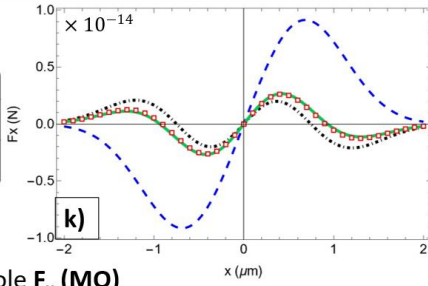
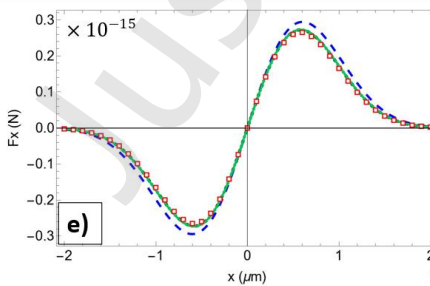
Electric quadrupole $F_x(EQ)$



Magnetic quadrupole $F_x(MQ)$



Electric octupole $F_x(EO)$



Magnetic octupole $F_x(MO)$

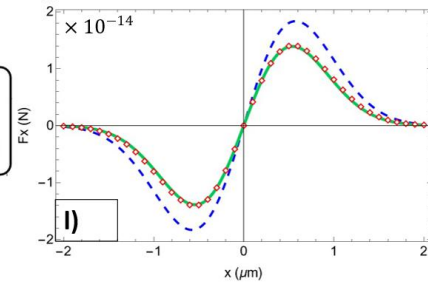
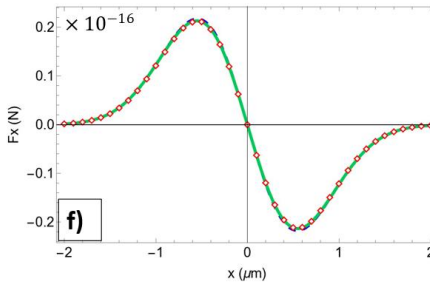


Figure S13. Interceptional components of the transverse F_x force in the case of conventional trapping (panels a-f) and in the HAS (panels g-l). Blue dashed curves – the result of calculation using conventional approximation (i.e., including the response only to the incident field and its first derivatives); black dot-dashed curves – calculation using the Taylor series up to the second derivatives (for multipoles that respond to them), green solid curves – Taylor series up to the highest third or fourth derivatives for corresponding multipoles; and red markers give the full-wave result based on Table 1.

Figure S13 shows that in the conventional regime, it is sufficient to consider the response to the field and its first derivative to obtain a good match between the result and the forces calculated using full-wave simulation. The exception is the magnetic quadrupole, but its contribution is several orders of magnitude smaller than that of the other multipoles, so it is not noticeable when calculating the total force.

In the HAS, the situation is markedly different. To achieve agreement with full-wave simulation, it is necessary to consider the response to derivatives up to the 3rd order for “odd” multipoles (p, MQ, EO) and up to the 4th order for “even” multipoles (m, EQ, MO). This is a new optomechanical property that distinguishes hybrid anapole from conventional scattering regimes.

Acknowledgements. This work was supported by the Russian Science Foundation № 25-22-00266.

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