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A fatigue life approach model of automotive parts based on stress equivalence relation

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Abstract: Fatigue failure under alternating loads represents a significant failure mode for automotive components. This study focuses on predicting the fatigue life of automotive parts subjected to multi-level stress loading. Current nonlinear models often require extensive test data or face challenges in selecting appropriate reference values, which imposes limitations on automotive reliability assessments. By analyzing the process of fatigue damage transformation between two levels of stress, the relationship, i.e., $S_i^2 \times \Delta S_i = S_{i+1}^2 \times \Delta S_{i+1}$, considering the equivalent transformation of adjacent stresses based on the S-N characteristic curve of materials. The equivalent formula for cumulative fatigue damage and the expression for residual fatigue life between adjacent stresses acress two, three, and higher stress levels are derived. Then a fatigue life prediction model based on the stress equivalence relation is proposed. The calculation process of this model relies solely on the fatigue life of the materials subjected to at least two stress levels to determine the S-N characteristic curve. The experimental data utilized include existing test results for two-, three-, four-, and five-level stress loading, as reported by relevant scholars. Specifically, the two-level stress materials analyzed are AL-2024 and 30CrMnSiA, the three-level stress material is LY12CZ, and the four- and five-level stress material is an aluminum alloy. The average and maximum relative errors of the Miner model, Manson model, Subramanyan model, Hashin model, and the proposed model are all calculated and compared. Additionally, the statistical values of cumulative fatigue damage, cumulative fatigue life, and the discrepancies between cumulative fatigue damage and test cumulative fatigue damage predicted by each model are summarized. The results show that the proposed model, which is based on the equivalent transformation of adjacent loads, demonstrates superior overall performance in predicting fatigue life and damage compared to the Miner model, Manson model, Subramanyan model, and Hashin model, thereby offering a more accurate application for fatigue life and damage prediction under multi-level loading conditions.

Keywords: automotive engineering; equivalent transformation; fatigue damage; fatigue life; multi-levels load

1 Introduction

The fatigue failure caused by alternating loads is one of the highly concerned projects in the research and testing of automotive fatigue durability[1]. Miner[2] first proposed a cumulative damage criterion that can explain the fatigue failure mechanism of metal materials. Due to its simplicity, this criterion is still widely used in engineering nowadays. Subsequent researchers have found that when the loading stress sequence changes, there is a significant deviation in the cumulative fatigue damage predicted by the Miner criterion[3-4]. Therefore, relevant scholars have explored the evolution process of cumulative fatigue damage from different perspectives.

In order to consider the influence of stress application sequence, a large number of scholars have established many nonlinear fatigue life prediction models. For example, models based on energy[5-7], damage curve[8-9], material strength degradation[10-11], continuous damage Mechanics[12-13] etc. Among these nonlinear models, damage curve calculations have attracted much attention due to their convenience. For instance, Manson proposed the bilinear damage rule[14-15]. He set the intersection point of two lines as the knee point, and then pro-

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posed an improved double damage curve model. Meanwhile, some researchers have also studied nonlinear models of fatigue accumulation. Marco was the first to propose the nonlinear cumulative damage theory[16]. Subsequently, Manson and Sabramanya further proposed the nonlinear fatigue cumulative damage models based on effective micro crack growth and fixed inflection points based on material S-N characteristics[14,17], respectively. In addition, Hashin, et al. also proposed a power-law rule considering a turning point in the S-N curve to predict residual damage[18]. This method requires setting the fatigue life corresponding to the fatigue limit. Shang proposed a new nonlinear fatigue cumulative damage model based on the evolution law of material ductility during the fatigue damage process[19]. Yao, et al. constructed a fatigue cumulative damage model based on the equivalent principle of strength degradation or stiffness degradation and fatigue damage[20-22]. Djebli proposed a nonlinear fatigue cumulative damage model based on energy parameter application[23]. Mesmacque defined damage stress and proposed a new damage model that follows a sequential pattern during multi-level loading[24], which calculates the ultimate stress of the required material.

Preliminary analysis of the above relevant models indicates that a certain amount of testing data is required for the inflection points, fatigue life corresponding to fatigue limits, and ultimate stress. However, actual engineering applications may not be able to obtain sufficient fatigue loading test data. To this end, this paper establishes a relationship that considers the equivalent transformation of adjacent load actions $S_i^2 \times \Delta S_i = S_{i+1}^2 \times \Delta S_{i+1}$, and proposes a fatigue damage model for adjacent loads. The calculation process of this model relies on the fatigue life of materials with no less than 2 types of stresses to determine the S-N characteristic curve of the material. In addition, this paper also compares the new model with the Miner model, Manson model, Subramanyan model, and Hashin model based on the fatigue test results under two-level to five-level loading stress.

2 Fatigue cumulative damage model

2.1 Miner model

Fatigue of automotive components is a gradual accumulation of damage under alternating loads, and each load cycle will produce a certain amount of damage. Miner was the first to propose the fatigue cumulative damage criterion. The Miner fatigue cumulative damage formula is

$$D = \sum_{i=1}^{m} \frac{n_i}{N_i} \tag{1}$$

where, D is the fatigue cumulative damage variable; n_i is the number of cycles under alternating stress S_i; N_i is the fatigue life under alternating stress S_i.

The Miner's law states that failure occurs when fatigue accumulates damage $D = 1$.

In this formular, the relationship between material stress amplitude and fatigue life curve can be based on the Basquin model[25]. The relationship formula is

$$S = KN^b \tag{2}$$

where b and K can be determined by fitting fatigue test data.

2.2 Nonlinear damage models

A large number of fatigue tests indicate that the characteristics of the impact of load sequence on damage are: Accumulated fatigue damage during fatigue failure $\sum n_i/N_i < 1$ under cyclic loading with high and low order of action; Accumulated fatigue damage during fatigue failure $\sum n_i/N_i > 1$ under cyclic loading with low and high action sequences.

Therefore, to consider the influence of loading order on fatigue cumulative damage, Marco et al. proposed a nonlinear fatigue cumulative damage model:

$$D=\frac{n_m}{N_m}+\left\{\frac{n_{m-1}}{N_{m-1}}+\left\{\frac{n_{m-2}}{N_{m-2}}+\cdots+\left[\frac{n_2}{N_2}+\left(\frac{n_1}{N_1}\right)^{\alpha_1}\right]^{\alpha_2}\cdots\right\}^{\alpha_{m-2}}\right\}^{\alpha_{m-1}} \tag{3}$$

where, $\alpha_1, \alpha_2, \cdots, \alpha_m$ are the coefficient of action; m is the number of levels of loading stress.

Manson determined the coefficient of action based on a large amount of experimental data:

$$\alpha_i = \left(\frac{N_i}{N_{i+1}}\right)^{0.4} \tag{4}$$

Subramanyan gave the coefficient of action based on the benchmark stress amplitude S_{i+1} and fatigue limit S_e as:

$$\alpha_i = \frac{(S_i - S_e)}{(S_{i+1} - S_e)} \tag{5}$$

Hashin used different expressions for the fatigue life N_e at the fatigue limit S_e and the fatigue life N_{i+1} corresponding to the benchmark stress amplitude S_{i+1}, with corresponding action coefficients as:

$$\alpha_i = \frac{\lg(N_i/N_e)}{\lg(N_{i+1}/N_e)} \tag{6}$$

3 Proposed model

Nonlinear models can be well applied in multi-level stress loading. The Manson model determines a response coefficient of 0.4 based on a large amount of experimental data, By default, the coefficients remain constant and there are no related physical quantities associated with them; Although the Subramanyan model and Hashin model are related to the fatigue limit of related materials and their corresponding fatigue life, in practical engineering applications, obtaining the fatigue limit requires more experimental time and higher costs.

To this end, establish an equivalent transformation relationship between adjacent stresses under two-level stress loading as follows:

$$S_1^2 \times \Delta S_1 = S_2^2 \times \Delta S_2 \tag{7}$$

The specific process is as follows: As shown in Figure 1, the fatigue life N_1 corresponding to the first level stress S_1, and the remaining fatigue life corresponding to the fatigue damage d_1 after the first level stress acts on n_1 are

$$N_{1r} = N_1 - d_1 N_1 \tag{8}$$

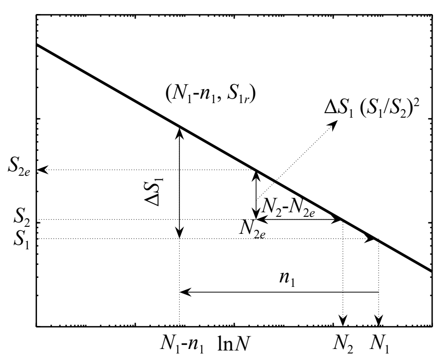

Fig. 1 The transformation process under two-level loading

The equivalent stress corresponding to the *S-N* characteristic curve of N_{1r} in the material is

$$S_{1r} = KN_{1r}^b \tag{9}$$

Define the difference in equivalent stress:

$$\Delta S_1 = S_{1r} - S_1 \tag{10}$$

The equivalent transformation relationship between the first level stress action and the second level stress is proposed as follows:

$$S_1^2 \times \Delta S_1 = S_2^2 \times \Delta S_2 \tag{11}$$

When the first level stress acts on the second level stress, the equivalent stress is

$$S_{2e} = S_2 + \Delta S_1 \frac{S_1^2}{S_2^2} \tag{12}$$

The fatigue life corresponding to the equivalent stress S_{2e} can be further calculated through the *S-N* characteristic curve

$$N_{2e} = e^{b \times \ln(S_{2e}/K)} \tag{13}$$

At this point, the equivalent remaining fatigue damage is

$$d_2 = \frac{N_{2e}}{N_2} \tag{14}$$

That is, the fatigue damage caused by the first level stress of this model is

$$d_{12e} = 1 - \frac{N_{2e}}{N_2} \tag{15}$$

For levels 3 and above, the formula is

$$d_{i(i+1)e} = 1 - \frac{N_{(i+1)e}}{N_{(i+1)}} \tag{16}$$

$$d_{i(i+1)} = d_{i(i+1)e} + \frac{n_{i+1}}{N_{i+1}} \tag{17}$$

Calculate the number of stress levels for $d_{i(i+1)}$ to continue $N_{(i+1)r} = N_{(i+1)} - d_{i(i+1)} N_{(i+1)}$ cycles until loading is completed.

Therefore, the cumulative fatigue damage from level 1 loading to level i is $d_{i(i+1)e}$. If fatigue occurs during level i+1 stress loading, the remaining fatigue life is $1 - d_{i(i+1)e}$.

4 Verification of multi-level loading

4.1 Two-level loading

For two-level loading stress, we adopt fatigue test data of AL-2024[26] and 30CrMnSiA[27]. The loading stress includes high-low loading and low-high loading. The fatigue lifetime under different stress levels is listed in Table 1 and Table 2, respectively. Further, we can calculate the *S-N* correlation coefficient of the material as shown in Table 3.

We calculate the results of fatigue damage of Miner model, Manson model, Subramanyan model, Hashin model and proposed model, respectively. Furthermore, the relative error of fatigue damage and its mean and maximum values are calculated. These results are summarized in Tables 4 and 5, respectively. The result shows that the average and maximum fatigue damage relative errors of the proposed model is better than other models for AL-2024 material, while, the prediction accuracy of

Table 1 Fatigue lifetime under different loading amplitude (AL-2024)

Stress amplitude (MPa)	150	200
Lifetime (time)	430 000	150 000

Table 2 Fatigue lifetime under different loading amplitude (30CrMnSiA material)

Stress amplitude (MPa)	482 (Maximum stress 732, reference stress 250)	586 (Maximum stress 836, reference stress 250)
Lifetime (time)	55 757	7 186

Table 3 Coefficient of *S-N* characteristics of two materials

Materials	AL-2024	30CrMnSiA
K	5 189.39	1 366
b	−0.273	−0.095 4

Table 4 Comparison of experimental results and model-predicted results under two-level loading (AL-2024)

Stress amplitude (MPa)	Experimental result		Miner model		Manson model		Subramanyan model		Hashin model		Proposed model	
	d_1	d_2	d_2	E_r (%)	d_2	E_r (%)	d_2	E_r (%)	d_2	E_r (%)	d_2	E_r (%)
150−200	0.2	0.96	0.80	13.79	0.91	3.97	0.93	2.30	0.90	5.19	0.91	4.43
	0.4	0.89	0.60	22.48	0.75	10.66	0.79	8.06	0.73	12.40	0.80	7.05
	0.6	0.54	0.40	12.28	0.54	0.08	0.58	3.21	0.52	1.92	0.66	10.57
	0.2	0.67	0.80	14.94	0.65	2.05	0.62	6.23	0.68	0.65	0.60	7.88
200−150	0.4	0.24	0.60	56.25	0.45	33.11	0.42	28.11	0.47	36.45	0.33	13.89
	0.6	0.18	0.40	28.21	0.28	13.44	0.26	10.49	0.30	15.45	0.15	3.64
	0.2	0.96	0.80	13.79	0.91	3.97	0.93	2.30	0.90	5.19	0.91	4.43
Mean relative error (%)			24.66		10.55		9.73		12.01		7.91	
Maximum relative error (%)			56.25		33.10		28.11		36.45		13.89	

Table 5 Comparison of experimental results and model-predicted results under two-level loading (30CrMnSiA material)

Stress amplitude (MPa)	Experimental result		Miner model		Manson model		Subramanyan model		Hashin model		Proposed model	
	d_1	d_2	d_2	E_r (%)	d_2	E_r (%)	d_2	E_r (%)	d_2	E_r (%)	d_2	E_r (%)
586−482	0.167	0.662	0.833	20.63	0.546	14.05	0.611	6.177	0.649	1.58	0.722	7.20
	0.208	0.582	0.792	26.58	0.499	10.46	0.563	2.401	0.601	2.38	0.660	9.88
	0.417	0.287	0.583	42.05	0.320	4.66	0.369	11.712	0.400	16.12	0.387	12.64
	0.694	0.125	0.306	22.10	0.149	2.89	0.175	6.127	0.192	8.23	0.130	0.60
482−586	0.223	0.917	0.777	12.28	0.967	4.37	0.942	2.187	0.923	0.54	0.868	4.27
	0.269	0.903	0.731	14.68	0.949	3.94	0.917	1.205	0.894	0.76	0.839	5.46
	0.448	0.750	0.552	16.53	0.838	7.37	0.782	2.665	0.747	0.28	0.715	2.88
	0.628	0.615	0.372	19.55	0.652	2.98	0.586	2.318	0.549	5.34	0.570	3.60
	0.807	0.425	0.193	18.83	0.385	3.22	0.334	7.374	0.307	9.58	0.388	3.01
Mean relative error (%)			21.47		5.99		4.68		4.98		5.67	
Maximum relative error (%)			42.05		14.05		11.71		16.12		12.64	

the four nonlinear models is almost similar, and all of them is better than Miner model.

4.2 Three-level loading

The fatigue life test data of LY12CZ material under different stress amplitudes is listed in Table 6 [28]. The three-level loading sequence includes low-high loading, low-high-low loading, high-low loading, respectively. We calculate the *S-N* coefficient of the material is $K = 1\,023$, $b = 0.112$. Further, the comparisons of five models are listed in Table 7. We can see that the mean and maximum relative errors of Subramanyan model, Hashin model, Manson model and proposed model are close, while the Miner model is relatively larger than them.

Table 6 Fatigue life under different stress amplitudes (LY12CZ material)

Stress level	S_1	S_2	S_3	S_4
Stress amplitude (MPa)	224.20	246.49	359.87	503.18
Lifetime (time)	719 424	312 500	12 098	524

Table 7 Comparison of experimental results and model-predicted results under three-level loading

Loading sequence	Experimental result			Miner model		Manson model		Subramanyan model		Hashin model		Proposed model	
	d_1	d_2	d_3	d_3	E_r (%)	d_3	E_r (%)	d_3	E_r (%)	d_3	E_r (%)	d_3	E_r (%)
S_1, S_3, S_4	0.556	0.248	0.646	0.196	13.52	0.986	23.44	0.939	20.18	0.908	18.05	0.838	13.23
S_1, S_4, S_3	0.556	0.191	0.517	0.253	20.02	0.376	11.16	0.563	3.62	0.620	8.15	0.383	10.61
S_4, S_3, S_2	0.191	0.248	0.159	0.561	93.81	0.037	20.47	0.076	13.95	0.115	7.30	0.028	21.91
Mean relative error (%)				42.45		18.35		12.58		11.17		15.25	
Maximum relative error (%)				93.81		23.44		20.18		18.05		21.91	

4.3 Four- and five-level loading

We adopt the four- and five-level loading fatigue test data of Aluminum alloy material[14]. The fatigue lifetime under different stresses and cycles of loads per level are shown in Table 8. The coefficient $K = 855, b = 0.087\,3$ of S-N curve is obtained by fitting. The loading sequence of loads is shown in Figure 2. The comparisons between the five models are shown in Table 9. We can find that the maximum relative error of Miner model, Manson model, Subramanyan model, Hashin model and proposed model is 26.58%, 8.57%, 34.17%, 32.95% and 9.65%, respectively, and the poor prediction effect of Subramanyan model and Hashin model is reflected in that the cumulative fatigue damage of two models under loading B exceeds 1.5.

Table 8 The applied cycles and failure cycles under different stress amplitudes

Stress level	S_1	S_2	S_3	S_4
Stress amplitude (MPa)	260	275	290	305
Cycles of loads per level	210 000	110 500	60 000	33 750
Lifetime (time)	840 000	442 000	240 000	135 000

5 Comparison of the results of each model

5.1 Comparison of fatigue cumulative life/damage

We summarized and compared the fatigue cumulative damage of each model, as shown in Figure 3. The comparison of the fatigue cumulative life of each model is shown in Figure 4.

As can be seen from Figure 2, a large number of the predicted results of Miner model fall outside the demarcation interval of ±0.2, while most of the other four models fall within the demarcation interval. Of the other four models, the maximum distance of the proposed model is lower than that of the other three models.

As can be seen from Figure 3, several of the predicted results of Miner model fall outside the interval with 3 or 1/3 times of the cumulative fatigue life as the dividing line, and the proposed model falls within the interval, while some predicted results of the other three models fall outside the interval.

5.2 Comparison of statistical eigenvalues

The statistical value and distribution diagram of the difference between the cumulative fatigue damage of the test and the predicted fatigue cumulative damage of each

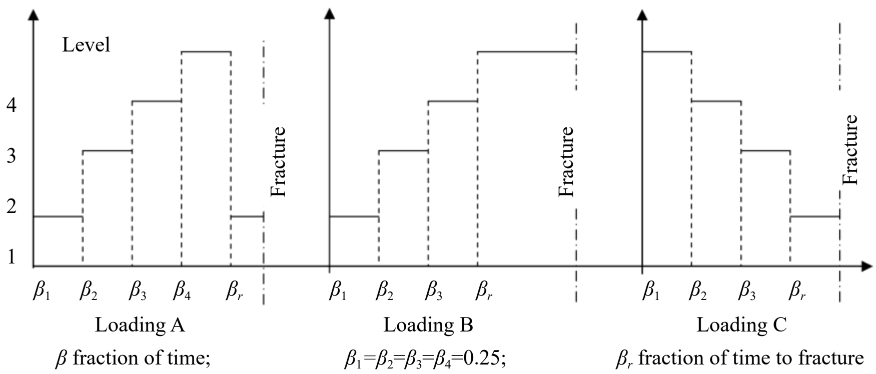

Fig. 2 Loading sequence of levels loading A, B, and C

Table 9 Comparison of experimental results and model-predicted results under four-level loading

Loading sequence	Loading A, S_1-S_2-S_3-S_4-S_1			Loading B, S_1-S_2-S_3-S_4			Loading C, S_4-S_3-S_2-S_1		
Evaluation index	Sum of lifetime (time)	Cumulative fatigue damage	E_r (%)	Sum of lifetime (time)	Cumulative fatigue damage	E_r (%)	Sum of lifetime (time)	Cumulative fatigue damage	E_r (%)
Experimental result	531 000	1.139	-	434 500	1.151	-	236 500	-	-
Miner model	414 250	1.000	12.20	414 250	1.000	13.12	414 250	1.000	26.58
Manson model	522 670	1.129	0.87	447 940	1.250	8.57	266 780	0.824	4.36
Subramanyan model	465 610	1.061	6.84	487 720	1.544	34.17	200 030	0.745	5.70
Hashin model	467 740	1.064	6.61	485 840	1.530	32.95	202 040	0.747	5.40
Proposed model	453 310	1.047	1.73	432 610	1.137	5.22	301 800	0.866	9.65

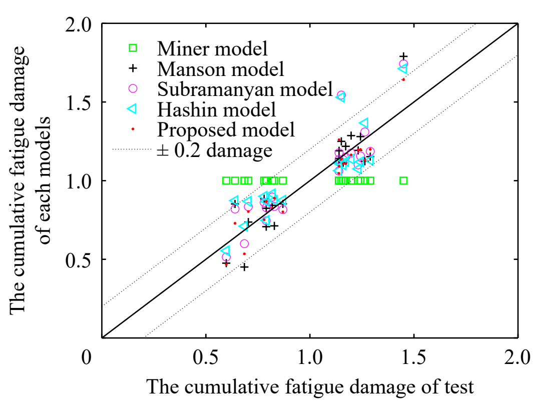

Fig. 3 Comparison of cumulative damage of different models under multi-levels loading

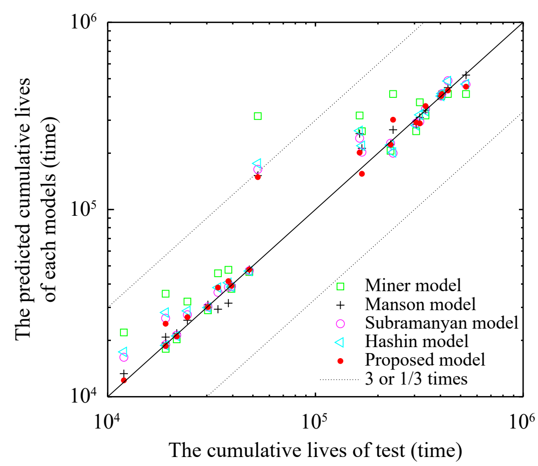

Fig. 4 Comparison of cumulative life of different models under multi-levels loading

model were calculated. The statistical value includes the minimum value, maximum value and standard deviation, as listed in Table 10, and the distribution diagram is shown in Figure 5. As can be seen from Figure 5, the three statistical indicators of the proposed model are better than the other four models, and the statistical values of the three models of Manson, Subramanyan and Hashin are close and better than Miner model.

In summary, by comparing the results of two-level to five-level stress loading tests, we found that the fatigue life/damage prediction of the proposed model is generally better than that of Miner model, Manson model, Subramanyan model and Hashin model.

6 Conclusion

In this paper, a new fatigue cumulative damage model

Table 10 Statistical values of cumulative damage for models prediction under multi-levels loading

	Minimum value	Maximum value	Standard deviation
Miner model	−0.45	0.40	0.25
Manson model	−0.23	0.34	0.12
Subramanyan model	−0.13	0.39	0.13
Hashin model	−0.16	0.38	0.14
Proposed model	−0.13	0.19	0.09

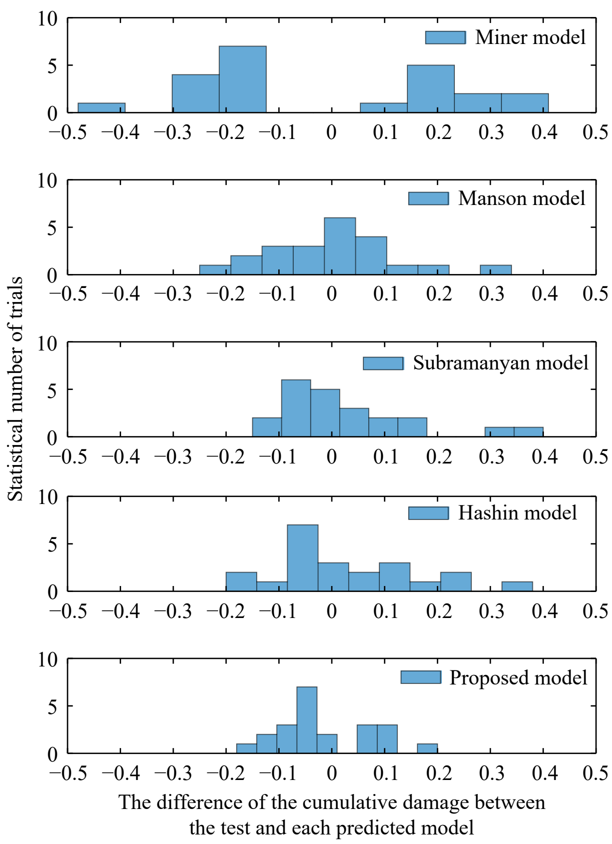

Fig. 5 Comparison of fatigue lives of different models under multi-levels loading

based on the equivalent transformation of adjacent loads $S_i^2 \times \Delta S_i = S_{i+1}^2 \times \Delta S_{i+1}$ is proposed. The calculation process of the model relies on the fatigue life of the material with no less than two level stresses to determine the *S-N* characteristic curve of the material.

The two-level to five-level loading stress test data are applied to validate each model. Regarding the difference of fatigue cumulative damage between the experimental result and each model, Miner model, Manson model, Subramanyan model and Hashin model is 0.25, 0.12, 0.13, 0.14 and 0.09, respectively. The proposed model corresponds to the minimum standard deviation, which also verifies that the model has a high fatigue life/damage prediction accuracy, and has certain engineering application value.

Declaration of competing interest

The authors have no competing interests to declare that are relevant to the content of this article.

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