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Stability Assessment on Safety of Shield Tunneling Based on Game Theory and Extension Cloud

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Abstract: During the construction of the metro shield, the surrounding ground surface and buildings will settle due to the disturbance of sand and gravel during excavation. Therefore, it is essential to evaluate the safety and stability of shield tunneling. Aiming to address the ambiguity, randomness, and significance of each index in the process of evaluating safety and stability during shield tunneling, the study utilizes a method that combines game theory and extension cloud. Monitoring data related to ground subsidence, building settlement, embankment settlement, pipeline settlement, and segment deformation are selected as evaluation indices to develop a comprehensive extension cloud evaluation model for shield tunneling safety. The model is validated using real-world engineering cases. The results show that according to the combined weight results obtained by the game theory, analytic hierarchy process, and entropy weighting method, the safety and stability evaluation indexes of the shield tunnel interval in this case are building settlement, segment deformation, ground settlement, pipeline settlement, and embankment settlement in order of importance. The safety and stability of nineteen monitoring groups are evaluated using the comprehensive extension cloud model, including eight Level III, four Level II, and seven Level I. Among them, monitoring groups at Level II and Level I nearby need to adopt corresponding preventive measures. The comprehensive evaluation model developed in this study, based on game theory and extension cloud, can reflect the uncertain relationships among various indicators. It helps to avoid the one-sidedness of single weighting. Compared with fuzzy comprehensive evaluation, the calculation results are more aligned with objective reality. Based on the aforementioned research, a comprehensive evaluation method is proposed, which combines game theory and extension cloud model. This method offers a novel approach and concept for safety evaluation in shield construction. The research can provide scientific references for the safety and stability evaluation of shield tunneling in sandy and pebble strata in the Chengdu area.

Key words: tunnel engineering; stability assessment; game theory; extension cloud model; shield tunneling; stratum of sandy pebble

1 Introduction

When subway shield construction is carried out in the sand and pebble stratum, the stress state changes caused by the shield tunneling can lead to stratum loss subseg uently, affecing the stability of the tunnel face soil. In severe cases, it will lead to large-scale ground subsidence, signlyicantly impacting the surrounding environment. Scholars both domestically and internationally have conducted a lot of research on the risk analysis and assessment of subway shield construction. ISAKSSON[1] firstly put forward a tunnel construction risk analysis method based on reliability theory; ESKESEN[2] proposed the risk assessment technology for the entire lifecycle of tunnel and underground engineering. In the aspect of subway shield construction, the $R=P\times C$ grading method[3] can evaluate the construction process by analyzing the risk occurrence probability and risk consequences; The extension risk assessment model[4] reduces the subjective impact of the assessment through the qualitative and quantitative analysis of the diversity of assessment indicators and the incompatibility of single-index assessment results; The structural entropy weight method[5] integrates the experience of experts into the evaluation model, reducing the interference of abnormal data on the evaluation results; The introduction of the cloud model[6] into the evaluation model can greatly reduce the subjectivity of evaluation criteria; The combination of cloud

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model and entropy weight method[7] can take into account the fuzziness and randomness of evaluation indexes, overcome the defects of single weighting method and improve the rationality of evaluation. And the matter element theory[8] can solve the uncertainty problem in practical engineering to a certain extent and improve the credibility of the safety evaluation results. It can be seen that whether the evaluation method is comprehensive or not directly affects the rationality and accuracy of the evaluation results. However, when the thing to be evaluated are influenced by multiple factors together, it has been controversial how to solve the assignment of each evaluation index. In view of this, it would be beneficial to combine game theory[9-11], entropy method[12], extension evaluation[13] and cloud model[14] to construct a comprehensive evaluation model to obtain a more comprehensive and reasonable evaluation result. Currently, comprehensive evaluation of various methods has been carried out in slope stability analysis[15-16], culvert safety evaluation[17] and transmission line engineering construction safety evaluation[18], but it is still rare in the subway shield project affected by many factors in the construction process. Based on a shield section project of Chengdu Metro, this paper combines analytic hierarchy process, entropy method and game theory to weight the evaluation indexes of shield tunneling safety and stability. At the same time, the extension cloud model is used to effectively solve the fuzziness and randomness between the evaluation indexes. A comprehensive evaluation method based on game theory and extension cloud model is proposed, which provides a new method and idea for shield construction safety evaluation. The research results offer a theoretical basis and scientific reference for risk early warning and safety prevention in shield construction within water-rich sandy cobble strata in the Chengdu area.

2 Extension cloud principle

2.1 Cloud model

Cloud model is an uncertain mathematical model for qualitative and quantitative transformation of objective things, which is usually represented by cloud digital eigenvalues such as expected Ex, entropy En and hyper entropy He[12]. Among them, expected Ex is the most representative point in the domain space, that is, the best sample. Entropy En is a measure of the uncertainty of qualitative concepts, which is determined by the fuzziness and randomness of qualitative concepts. It not only reflects the discreteness of cloud droplets, but also reflects the value range of cloud droplets recognized by qualitative concepts. Hyper entropy He is the uncertainty to measure entropy, which reflects the thickness of cloud droplets, and its value can be adjusted according to the fuzzy threshold of the index. Expectation Ex and entropy En are calculated as shown in Equations (1), (2) and (3).

$$Ex = (C_{\max} + C_{\min})/2, \tag{1}$$

$$En = (C_{\max} - C_{\min})/6, \tag{2}$$

$$He = s, \tag{3}$$

Where, $C_{\max}$ and $C_{\min}$ are the maximum and minimum boundary values of a grade standard; and s is a constant.

2.2 Extension theory

Extension theory is often used to evaluate the pros and cons and feasibility of the research object, through the introduction of matter element $\boldsymbol{R}$ for qualitative and quantitative analysis[19], where the unit matter element is represented by an ordered triad $\boldsymbol{R} = \{N, c, x\}$. Assuming that N is the evaluation object, c_i is the index of the evaluation object, and x_i is the eigenvalue of c_i, the basic elements are shown in Equation (4).

$$\boldsymbol{R} = \begin{bmatrix} N & c_1 & x_1 \\ & c_2 & x_2 \\ & \vdots & \vdots \\ & c_n & x_n \end{bmatrix}. \tag{4}$$

2.3 Extension cloud model

Extension cloud theory is a method of coupling matter-element model with cloud model and using matter-element theory to analyze cloud model[20-21], and the expression is shown in Equation (5).

$$\boldsymbol{R}_j = (N_j, c_j, x_j) = \begin{bmatrix} N_j & c_1 & (Ex_1, En_1, He_1) \\ & c_2 & (Ex_2, En_2, He_2) \\ & \vdots & \vdots \\ & c_n & (Ex_n, En_n, He_n) \end{bmatrix}, \tag{5}$$

where, $\boldsymbol{R}_j$ is a unit object element; N_j is the jth evaluation category; c_j is the jth characteristic index, namely each monitoring index; $x_j = (Ex_j, En_j, He_j)$ is the value range specified by N_j about feature c_j, that is, the expectation, entropy and hyper entropy corresponding to each monitoring index data.

Assuming that the evaluation category is T, the n-dimensional composite cloud model $\boldsymbol{R}$ consisting of m evaluation categories is obtained from the measured data and in the form of object elements, as shown in Equation (6).

$$\boldsymbol{R} = (T, c_i, x_i) = \begin{bmatrix} & T_1 & T_2 & \cdots & T_m \\ c_1 & \mu_1(x_{11}) & \mu_2(x_{12}) & \cdots & \mu_m(x_{1m}) \\ c_2 & \mu_1(x_{21}) & \mu_2(x_{22}) & \cdots & \mu_m(x_{2m}) \\ \vdots & \vdots & \vdots & & \vdots \\ c_n & \mu_1(x_{n1}) & \mu_2(x_{n2}) & \cdots & \mu_m(x_{nm}) \end{bmatrix}, \quad (6)$$

Where $T_j (j=1,2,\cdots,m)$ is the jth evaluation category, which is usually three levels of danger, warning and safety in the safety evaluation of shield tunneling.

The correlation degree between the sample to be evaluated and each index of the evaluation category is calculated based on the composite cloud model. In other words, Matlab is used to generate a normal random number En' calculated with En as the mean and He as the standard deviation. The deterministic value in the sample to be evaluated is x_i, and its cloud entropy is (x_i, μ_i). The correlation calculation is shown in Equation (7).

$$\mu_i = \exp\left[-\frac{(x_i - Ex)^2}{2(En'_i)^2}\right]. \quad (7)$$

According to the Equation (7), the correlation degree between the sample to be evaluated and the evaluation index category is calculated, and the comprehensive evaluation matrix is formed, which can be the extension cloud matrix $\boldsymbol{Q}$. As shown in Equation (8).

$$\boldsymbol{Q} = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1m} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2m} \\ \vdots & \vdots & & \vdots \\ \mu_{n1} & \mu_{n2} & \cdots & \mu_{nm} \end{bmatrix}, \quad (8)$$

Where, μ_i is the correlation between the standard normal cloud of the sample to be evaluated c_i and the standard normal cloud of the evaluation index category j; n is the number of evaluation indexes; and m is the number of evaluation categories.

2.4 Determination of risk level

To perform risk level determination, a comprehensive determination matrix $\boldsymbol{B}$ is calculated by multiplying the indicator weight vector $\boldsymbol{W}$ with the extension cloud matrix $\boldsymbol{Q}$. Each sample has a corresponding degree of certainty for each level. The same level of certainty is accumulated and the evaluation level of the sample is determined according to the principle of maximum certainty. The comprehensive determination matrix is shown in Equation (9).

$$\boldsymbol{B} = \boldsymbol{W} \cdot \boldsymbol{Q}. \quad (9)$$

3 Game theory weighting method

3.1 Game theory

Game theory, also known as "game theory"[22-23], is a mathematical model that uses mathematical language to express the dependence between the characteristics and quantities of things to be evaluated. When a thing to be evaluated is influenced by multiple factors and the degree of influence of each factor is different, a single subjective assignment will make the evaluation result biased by subjective factors, while a single objective assignment cannot reflect the comprehensiveness of the evaluation result. The combination of empowerment based on game theory can just solve the empowerment problem objectively, rationally and scientifically. Assuming that there are n evaluation indicators for the things to be evaluated, m weighting methods can be used to determine the weight of each indicator, and the resultin weight vector $\boldsymbol{W}_k$, as shown in Equation (10).

$$\boldsymbol{W}_k = (W_{k1}, W_{k2}, \cdots, W_{kn}), k = 1,2,\cdots,m. \quad (10)$$

The weight set $\boldsymbol{W}$ of the indicators to be evaluated can be determined according to the weight vector $\boldsymbol{W}_k$, as shown in Equation (11).

$$\boldsymbol{W} = \sum_{k=1}^{m} \alpha_k \boldsymbol{W}_k^{\mathrm{T}}, \ \alpha_k > 0, \quad (11)$$

Where, $\boldsymbol{W}_k$ is the basic weight vector set; α_k is the linear combination weight coefficient.

The α_k in Equation (11) is solved optimally to minimize the departure of the weight set from each weight vector, and a countermeasure model is constructed from it, as shown in Equation (12).

$$\min \| \sum_{k=1}^{m} \alpha_k \boldsymbol{W}_k^{\mathrm{T}} - \boldsymbol{W}_k \| , \ k = 1,2,\cdots,m. \quad (12)$$

The linear equations of the first-order derivative conditions for the optimization of the countermeasure model are shown in Equation (13), and thus the weight coefficient vector α_k of the linear combination can be obtained, namely (α_1, α_2, $\cdots$, α_m).

$$\sum_{k=1}^{m} \alpha_k \boldsymbol{W}_k \boldsymbol{W}_k^{\mathrm{T}} = \boldsymbol{W}_k \boldsymbol{W}_k^{\mathrm{T}}, \ k = 1,2,\cdots,m. \quad (13)$$

The linear combination weight coefficients α_k are normalized according to Equation (14) and the combination weight vector $\boldsymbol{W}^*$ is determined according to Equation (15).

$$\alpha_k^* = \alpha_k / \sum_{k=1}^{m} \alpha_k. \quad (14)$$

$$\boldsymbol{W}^* = \sum_{k=1}^{m} \alpha_k^* \cdot \boldsymbol{W}_k^{\mathrm{T}}. \quad (15)$$

3.2 Objective empowerment

The entropy method[24] is an objective assignment method that can reflect the degree of change of indicators. By using the inherent information of evaluation indicators to discern the utility value of indicators, the influence of subjective factors on the evaluation results is avoided to a certain extent. Assuming that there are m samples to be tested forming the evaluation object set $\{A_i\}(i = 1,2,\cdots,m)$ and n indicator data forming the indicator set $\{X_j\}(j = 1,2,\cdots,n)$. Among them, x_{ij} denotes the original value of the jth indicator of the ith sample to be measured. After the standardization process, the standard matrix $\boldsymbol{Y}$ consisting of the weight y_{ij} of different quantities in different indicators is obtained, as shown in Equation (16). The calculation of the proportion y_{ij} of the value j in indicator i is shown in Equation (17), and the entropy e of any indicator is calculated according to Equation (18).

$$\boldsymbol{Y} = (y_{ij})_{m\times n}, i = 1,2,\cdots,m; \ j = 1,2,\cdots,n, \quad (16)$$

$$y_{ij} = \frac{x_{ij}}{\sum_{j=1}^{m} x_{ij}}, i = 1,2,\cdots,m; \ j = 1,2,\cdots,n, \quad (17)$$

$$e_j = -k \sum_{j=1}^{m} y_{ij} \ln y_{ij}, \ j = 1,2,\cdots,n, k \geqslant 0, e_j \geqslant 0. \quad (18)$$

The k in Equation (18) is related to the sample number m of the system and is taken as a constant. When the degree of order is 0, its entropy value is maximum, i.e., $e = 1$, When m samples are in completely disordered distribution, $y_{ij} = 1/m$, and the utility value of entropy to comprehensive evaluation is zero. Therefore, the coefficient of variation g_j for the jth indicator is determined by the difference between the entropy value of that indicator and 1, as shown in Equation (19). The higher the value coefficient, the greater its importance. The weight w_j of the jth indicator is shown in Equation 20.

$$g_j = 1 - e_j, \ j = 1,2,\cdots,n, \quad (19)$$

$$w_j = \frac{g_j}{\sum_{i=1}^{n} g_j}, \ j = 1,2,\cdots,n. \quad (20)$$

3.3 Subjective empowerment

The subjective weight mainly uses the analytic hierarchy process to analyze the importance of each factor u_{ij} in the evaluation index factor $\{U_1, U_2, \cdots, U_m\}$. The value can be determined according to the '1-9' scale method[25], and the judgment matrix is established, as shown in Table 1. The judgment matrix is solved to obtain the maximum eigenvalue $\lambda_{\max}$ of the matrix and the corresponding eigenvector $\boldsymbol{A} = [a_1, a_2, \cdots, a_m]$, and the weight of each index is determined by normalization. The calculation of the maximum eigenvalue is shown in Equation (21).

$$\lambda_{\max} = \frac{1}{m} \sum_{i=1}^{m} \frac{(\boldsymbol{UA})_i}{a_i}. \quad (21)$$

Table 1 Judgment matrix of evaluation index factor

Judgment matrix	U_1	U_2	$\cdots$	U_m
U_1	u_{11}	u_{12}	$\cdots$	u_{1m}
U_2	$1/u_{12}$	u_{22}	$\cdots$	u_{2m}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
U_m	$1/u_{1m}$	$1/u_{2m}$	$\cdots$	u_{mm}

The consistency of the judgment matrix directly reflects the objective order of the evaluation object, which can be calculated by Equation (22). R_{C} is the consistency ratio of judgment matrix, I_{C} is the consistency index, which can be calculated by Equation (23). I_{R} is a random consistency index, and its value can be determined by Table 2.

$$R_{\mathrm{C}} = \frac{I_{\mathrm{C}}}{R_{\mathrm{C}}}, \quad (22)$$

$$I_{\mathrm{C}} = \frac{1}{m-1}(\lambda_{\max} - m). \quad (23)$$

The consistency test shows that the greater the I_C, the worse the consistency of the judgment matrix. When $I_C = 0$, the judgment matrix is completely consistent; when $R_C < 0.1$, the judgment matrix has relatively satisfactory consistency; in other cases, the judgment matrix needs to be adjusted.

Table 2 Consistency index of judgment matrix

Order	1	2	3	4	5	6	7	8	9	10	11	12
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.52	1.54

Table 3 Main parameters of soil in shield tunnel section

Name of stratum	Natural density ρ (g/cm^3)	Angle of internal friction φ (°)	Modulus of deformation E_0 (MPa)	Poisson's ratio μ	Permeability coefficient k (m/d)	Bearing capacity f_k(kPa)	Self-stability
Compact pebble soil	2.20	38	50	0.25	28	700	差

4 Case analysis

4.1 Project overview

The shield section of Fengxihe Station to City No. 5 Hospital Station on Chengdu Metro Line 17 is 1 610.186 m long for the left tunnel and 1 611.485 m long for the right tunnel, with one shield machine for each of the left and right lines. The shield interval is arranged in a north-south direction along Fengxi Avenue, with heavy traffic flow along the line, many buildings (structures) on both sides, and Fengxi River on the west side of the line. The longitudinal slope of the interval tunnel is 10.063‰, and the minimum plane curve radius is 450 m. The minimum burial depth of the top of the interval tunnel is about 9.5 m, and the maximum burial depth is about 20 m. The interval tunnel mainly crosses the dense pebble soil layer with poor self-stability, and the shield construction will bring serious impact to the surrounding environment. The project location and line alignment are shown in Figure 1, the soil parameters are shown in Table 3, and the shield interval pebbles are shown in Figure 1.

Fig. 1 Pebbles in shield section

4.2 Combination weight calculation based on game theory

4.2.1 Selection of evaluation indicators

In order to accurately evaluate the safety and stability in the process of shield tunneling, combined with the situation of surrounding buildings, according to the monitoring and measurement management method of Chengdu rail transit construction project and the requirements of relevant specifications[26-28], five monitoring items including ground settlement, building settlement, river embankment settlement, pipeline settlement and segment settlement are selected as the evaluation indexes. The maximum deformation values in the monitoring points of ground settlement, building settlement, river embankment settlement, pipeline settlement and pipe sheet settlement on the cross section of shield excavation face in 19 consecutive effective working days during the tunneling of 120 m linear segment of shield interval are selected for statistical analysis. The measured data are shown in Table 4.

4.2.2 Calculation of evaluation index weight

The analytic hierarchy process is used to calculate the subjective weight of the selected evaluation indexes, and the judgment matrix is constructed, as shown in Table 5.

The evaluation index data in Table 4 were assigned entropy values according to Equations (16) to (20), respectively, and the combination weights of each evaluation index were calculated as shown in Equation (15), and the calculation results are shown in Table 7.

Table 4 Monitoring data of straight shield tunnel section

Number	Ground settlement c_1(mm)	Building settlement c_2(mm)	Riverbank settlement c_3(mm)	Pipelines settlement c_4(mm)	Pipe sheet settlement c_5(mm)
1	−5.89	−1.85	−1.84	−3.86	−4.53
2	−9.59	−2.67	−1.42	−5.21	−3.97
3	−8.27	−1.92	−1.24	−6.11	−5.78
4	−9.70	−1.72	−0.99	−9.08	−3.91
5	−11.51	−1.98	−1.22	−5.34	−8.32
6	−14.07	−2.42	−1.26	−6.74	−9.05
7	−13.71	−2.63	−1.74	−8.25	−8.15
8	−16.90	−3.71	−1.78	−5.30	−8.39
9	−18.36	−4.00	−2.19	−6.96	−7.82
10	−13.48	−5.48	−2.09	−9.81	−7.23
11	−13.67	−5.33	−2.43	−9.75	−11.66
12	−20.06	−6.07	−2.47	−20.06	−14.80
13	−24.80	−6.57	−2.00	−24.80	−17.53
14	−23.77	−6.36	−2.43	−23.77	−18.79
15	−24.10	−6.43	−2.31	−24.10	−19.31
16	−23.59	−6.29	−2.27	−23.59	−18.25
17	−19.12	−6.30	−2.59	−18.95	−18.22
18	−19.01	−6.23	−2.93	−18.29	−15.32
19	−18.57	−6.49	−2.85	−18.57	−13.28

Table 5 Judgment matrix of shield stability evaluation index

Judgment matrix	c_1	c_2	c_3	c_4	c_5
Ground settlemen	1	1	7	5	1
Building settlement	1	1	5	3	2
Riverbank settlement	1/7	1/5	1	1/3	1/5
Pipelines settlement	1/5	1/3	3	1	1/3
Pipe sheet settlement	1	1/2	5	3	1

Table 6 Results of consistency test

Eigenvector	λ_{max}	I_C	I_R	R_C
$[0.319, 0.310, 0.045, 0.091, 0.235]^T$	5.142	0.036	1.120	0.032

Table 7 Entropy weighting of evaluation index

Evaluation index	c_1	c_2	c_3	c_4	c_5
Entropy e_j	0.978 8	0.966 2	0.986 0	0.943 2	0.961 2
Diversity factor g_j	0.021 2	0.033 8	0.014 0	0.056 8	0.038 8
Entropy weight w_j	0.129 0	0.205 3	0.084 8	0.345 1	0.235 7

According to the calculated subjective weight vector and objective weight vector, the countermeasure model is constructed according to the Equations (10) to (15), and the comprehensive weight vector is calculated. Among them, the linear combination coefficient α_1 of subjective weight vector is 0.650 8, and the linear combination coefficient α_2 of objective weight vector is 0.471 5. After normalization, α_1^* is 0.579 9, α_2^* is 0.420 1. The calculation results of comprehensive weight are shown in Table 8.

Table 8 Weighting calculation of evaluation index

Weighting items	c_1	c_2	c_3	c_4	c_5
Subjective weights ω'	0.319	0.310	0.045	0.091	0.235
Objective weights ω''	0.129	0.206	0.085	0.346	0.236
Combined weights ω	0.239	0.266	0.062	0.198	0.235

4.3 Construction of extension cloud model

4.3.1 Criteria for judging safety and stability of shield tunneling

According to the requirements of "Chengdu rail transit construction project monitoring and measurement management method" and relevant specifications[26-28], the safety and stability of shield tunneling is divided into three grades: danger (level I), warning (level II) and safety (level III). The safety and stability criterion of shield tunneling is shown in Table 9.

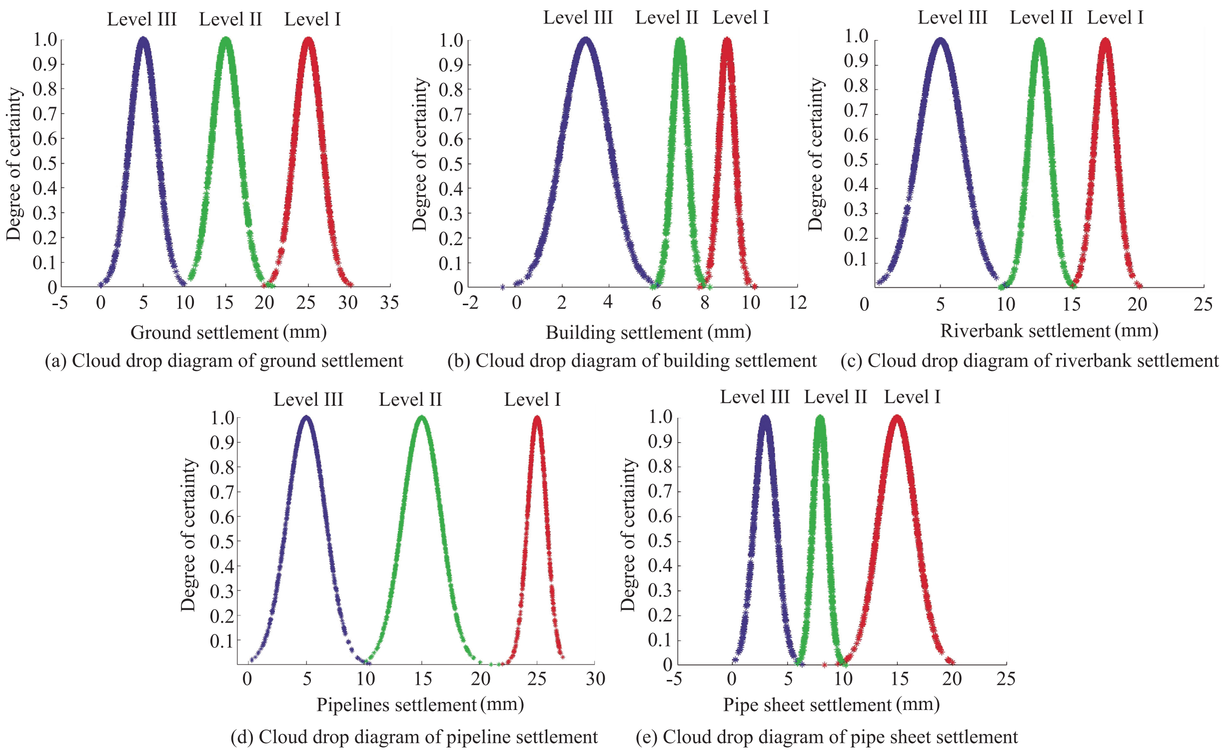
Table 9 Criteria for safety and stability of shield tunneling

Evaluating indicator	Danger (level I)	Warning (level II)	Safety (level III)
Ground settlement (mm)	(30,20)	(20,10)	(10,0)
Building settlement (mm)	(10,8)	(8,6)	(6,0)
Riverbank settlement (mm)	(20,15)	(15,10)	(10,0)
Pipelines settlement (mm)	(30,20)	(20,10)	(10,0)
Pipe sheet settlement (mm)	(20,10)	(10,6)	(6,0)

4.3.2 Extension Cloud Comprehensive Evaluation Model

According to the safety and stability criterion of shield tunneling and Equations (1) to (3), the expected Ex and entropy En of shield tunneling evaluation index are calculated. The digital characteristic hyper entropy He value of cloud droplets was calculated by reverse cloud generator. The ground subsidence was 0.005 9, the building subsidence was 0.012 9, the embankment subsidence was 0.004 6, the pipeline subsidence was 0.011 7, and the segment subsidence was 0.009 3. The calculated *He* was adjusted repeatedly according to the fuzzy threshold of each index, and finally the *He* of all evaluation indexes was determined to be 0.001. The extension cloud model $\boldsymbol{R}_0$ is constructed according to matter element theory, as shown in Equation (24). Matlab is used to calculate the cloud drop diagram of each evaluation index, as shown in Figure 2. The constructed extension cloud matrix is shown in Table 10.

$$\boldsymbol{R}_0 = \begin{array}{c} \\ \end{array}\left[\begin{array}{cccc} & v_1 & v_2 & v_3 \\ c_1 & (25.0,1.67,0.001) & (15.0,1.67,0.001) & (5.0,1.67,0.001) \\ c_2 & (9.0,0.33,0.001) & (7.0,0.33,0.001) & (3.0,1.00,0.001) \\ c_3 & (17.5,0.83,0.001) & (12.5,0.83,0.001) & (5.0,1.67,0.001) \\ c_4 & (25.0,1.67,0.001) & (15.0,1.67,0.001) & (5.0,1.67,0.001) \\ c_5 & (15.0,1.67,0.001) & (8.0,0.67,0.001) & (3.0,1.00,0.001) \end{array}\right]. \tag{24}$$



(a) Cloud drop diagram of ground settlement (b) Cloud drop diagram of building settlement (c) Cloud drop diagram of riverbank settlement

(d) Cloud drop diagram of pipeline settlement (e) Cloud drop diagram of pipe sheet settlement

Fig. 2 Cloud drop of each index in every safety level

The expected value *Ex* in the extension cloud model represents the point value of the classification level of each evaluation index for shield tunneling safety evaluation. Entropy *En* reflects the randomness and fuzziness of data samples in the process of shield tunneling safety evaluation. And the hyper entropy *He* is a description of the discrete degree of data samples, which reveals the degree of correlation among ground settlement, building settlement, river bank settlement, pipeline settlement and pipe sheet settlement during shield tunneling. As shown in Figure 2, the cloud droplets are all normally distributed, and the clouds corresponding to the two adjacent levels of each evaluation index have different degrees of intersection. The intersection area of cloud layers also reflects the fuzziness and uncertainty of the cloud model, in which the intersection area of pipe settlement is slightly larger than the intersection area of other indexes.

Table 10 Evaluation index extension cloud matrix

Number	c_1	c_2	c_3	c_4	c_5
1	0.563 3	0.519 7	0.166 9	0.792 2	0.311 2
2	0.000 0	0.949 3	0.100 5	0.992 1	0.624 6
3	0.000 4	0.558 0	0.079 1	0.801 4	0.021 1
4	0.000 0	0.447 9	0.055 7	0.050 6	0.660 3
5	0.112 4	0.597 9	0.077 3	0.979 5	0.892 5
6	0.856 5	0.844 0	0.081 5	0.580 7	0.293 3
7	0.742 4	0.932 5	0.148 7	0.150 1	0.975 3
8	0.523 4	0.779 9	0.156 4	0.984 0	0.844 1
9	0.132 1	0.593 6	0.242 6	0.502 2	0.964 6
10	0.660 8	0.042 7	0.218 6	0.015 7	0.517 2
11	0.728 8	0.066 2	0.306 4	0.017 5	0.135 9
12	0.012 6	0.016 1	0.317 4	0.012 6	0.992 9
13	0.992 9	0.426 4	0.199 0	0.992 8	0.318 2
14	0.762 6	0.156 3	0.305 9	0.762 3	0.076 3
15	0.865 0	0.212 2	0.273 5	0.864 7	0.035 8
16	0.699 9	0.111 1	0.262 2	0.700 5	0.150 2
17	0.047 7	0.085 8	0.353 0	0.061 0	0.155 9
18	0.055 9	0.055 6	0.463 2	0.144 4	0.981 8
19	0.102 0	0.329 7	0.436 3	0.101 5	0.588 0

4.3.3 Evaluation results

The comprehensive extension cloud evaluation matrix is calculated according to Equation (9), as shown in Table 11, and the determinations of the same grades are accumulated to obtain the determinations of each monitoring point under each grade, and the safety grade of shield tunneling for each monitoring point is determined according to the principle of maximum determination. Taking the first group of data as an example, according to the safety level boundary values shown in Table 9, it can be seen that the ground settlement is −5.89, which is safe (level III); the building settlement is −1.85, which is safe (level III); the river bank settlement is −1.84, which is safe (level III); the pipeline settlement is −3.86, which is safe (level III); and the pipe sheet settlement is −4.53, which is safe (level III). In the first group of data, each indicator corresponds to safety (level III), and the determinacy corresponding to each indicator in Table 11 is summed to obtain a comprehensive determinacy of 0.531 0. Since there are no determinacy values for hazard (level I) and warning (level II) in the first group of data, the safety level of the first group of data is level III according to the principle of maximum determinacy. By this method, the comprehensive determination of each group of data is calculated, and the evaluation results are shown in table 12. The safety level of field monitoring results in the table takes the highest level of safety level of each evaluation index as the safety level of monitoring points.

Table 11 Comprehensive extension cloud evaluation matrix

Number	c_1	c_2	c_3	c_4	c_5
1	0.134 6	0.138 2	0.010 3	0.156 8	0.073 1
2	0.000 0	0.252 5	0.006 2	0.196 4	0.146 8
3	0.000 1	0.148 4	0.004 9	0.158 7	0.005 0
4	0.000 0	0.119 2	0.003 5	0.010 0	0.155 2
5	0.026 9	0.159 0	0.004 8	0.193 9	0.209 7
6	0.204 7	0.224 5	0.005 1	0.115 0	0.068 9
7	0.177 4	0.248 1	0.009 2	0.029 7	0.229 2
8	0.125 1	0.207 4	0.009 7	0.194 8	0.198 4
9	0.031 6	0.157 9	0.015 0	0.099 4	0.226 7
10	0.157 9	0.011 4	0.013 6	0.003 1	0.121 5
11	0.174 2	0.017 6	0.019 0	0.003 5	0.031 9
12	0.003 0	0.004 3	0.019 7	0.002 5	0.233 3
13	0.237 3	0.113 4	0.012 3	0.196 6	0.074 8
14	0.182 3	0.041 6	0.019 0	0.150 9	0.017 9
15	0.206 7	0.056 4	0.017 0	0.171 2	0.008 4
16	0.167 3	0.029 6	0.016 3	0.138 7	0.035 3
17	0.011 4	0.022 8	0.021 9	0.012 1	0.036 6
18	0.013 4	0.014 8	0.028 7	0.028 6	0.230 7
19	0.024 4	0.087 7	0.027 1	0.020 1	0.138 2

4.4 Analysis of evaluation results

4.4.1 Result analysis of extension cloud comprehensive evaluation

From the results of the comprehensive extension cloud evaluation, it can be seen that the safety levels of monitoring points 1 to 4 (safety), 7 and 10 (warning), 12 to 16 and 18 to 19 (danger) are consistent with the discriminatory levels based on the field monitoring results. Comparing the field monitoring data of monitoring

Table 12 Comprehensive extension cloud evaluation results

Number	Danger (level I)	Warning (level II)	Safety (level III)	Extension cloud evaluation results	Field monitoring results
1	0.000 0	0.000 0	0.513 0	Ⅲ	Ⅲ
2	0.000 0	0.000 0	0.602 0	Ⅲ	Ⅲ
3	0.000 0	0.000 0	0.317 1	Ⅲ	Ⅲ
4	0.000 0	0.000 0	0.287 8	Ⅲ	Ⅲ
5	0.000 0	0.236 6	0.357 8	Ⅲ	Ⅱ
6	0.000 0	0.273 6	0.344 5	Ⅲ	Ⅱ
7	0.000 0	0.406 6	0.287 0	Ⅱ	Ⅱ
8	0.000 0	0.323 5	0.412 0	Ⅲ	Ⅱ
9	0.000 0	0.258 2	0.272 4	Ⅲ	Ⅱ
10	0.000 0	0.279 5	0.028 0	Ⅱ	Ⅱ
11	0.031 9	0.174 2	0.040 1	Ⅱ	Ⅰ
12	0.238 8	0.004 3	0.019 7	Ⅰ	Ⅰ
13	0.508 6	0.113 4	0.012 3	Ⅰ	Ⅰ
14	0.351 1	0.041 6	0.019 0	Ⅰ	Ⅰ
15	0.386 4	0.056 4	0.017 0	Ⅰ	Ⅰ
16	0.341 3	0.029 6	0.016 3	Ⅰ	Ⅰ
17	0.036 6	0.046 3	0.021 9	Ⅱ	Ⅰ
18	0.230 7	0.056 7	0.028 7	Ⅰ	Ⅰ
19	0.138 2	0.132 2	0.027 1	Ⅰ	Ⅰ

points 7 and 10, show that the settlement of the pipe sheet is within the warning value and close to the lower limit of the control value. The safety status of the whole construction needs to be closely monitored due to the increase of the settlement of the pipe sheet, and corresponding technical measures are taken to reduce and prevent the safety risk caused by shield excavation. From the monitoring data of monitoring points 12 to 16 and 18 to 19 show that the ground settlement, pipeline settlement and pipe sheet settlement are close to the upper limit of the control value. The shield construction situation is severe, and it is necessary to pay close attention to the shield tunneling status, tunneling parameters and various monitoring data, and to reduce the safety risk by adjusting the shield construction parameters to avoid construction safety problems caused by shield tunneling. For monitoring points 5 to 6, 8 to 9, 11 and 17 with different evaluation results, the grade of field monitoring results was determined by the safety grade of pipe sheet settlement due to the high safety grade of pipe sheet settlement compared to the safety grade of other indicators. Although pipe sheet settlement has a significant impact on the safety of shield construction, there are many factors that cause pipe sheet settlement and not all of them will lead to safety problems. The monitoring data of other evaluation indexes are all within the safe and controllable range, so the results of the comprehensive evaluation of extension cloud are lower

4.4.2 Comparison with the results of fuzzy comprehensive evaluation

The monitoring data were analyzed using the fuzzy comprehensive evaluation[29-30] based on the combined weights, and the evaluation results and comparison results are shown in Table 13.

According to the comparison results shown in Table 13, the comprehensive extension cloud evaluation results are compared with the fuzzy comprehensive evaluation results. The safety levels of monitoring points 1 to 6, 8, 9, and 12 to 16 are the same, and the safety levels of monitoring points 7, 10, 11, and 17 to 19 are higher than the fuzzy comprehensive evaluation results, and the safety levels of monitoring points 18 and 19 are higher than two levels. It can be seen that the introduction of game theory into the combination of hierarchical analysis and entropy method of assigning weights, calculating the linear combination of subjective and objective weights coefficients, reduces the influence of human factors, and obtains a more objective, reasonable and scientific assignments. The introduction of object element theory into the cloud model effectively solves the fuzziness and randomness among the evaluation indexes, reflects the uncertainty relationship among the indexes qualitatively and quantitatively, and improves the accuracy of the comprehensive evaluation results. Compared with the fuzzy comprehensive evaluation, the game theory-based extension cloud evaluation model reflects the inherent correlation among monitoring items more effectively and is a comprehensive reflection of each monitoring data, which is more in line with the actual situation. It can be seen that the comprehensive evaluation model of shield tunneling safety and stability based on game theory and extension cloud has application and promotion value, and the evaluation results are more favorable to the control of shield construction safety and stability.

Table 13 Fuzzy comprehensive evaluation results

Number	Danger (level I)	Warning (level II)	Safety (level III)	Fuzzy comprehensive evaluation	Extension cloud evaluation results	Field monitoring results
1	0.000 0	0.000 0	0.513 2	Ⅲ	Ⅲ	Ⅲ
2	0.000 0	0.000 0	0.602 0	Ⅲ	Ⅲ	Ⅲ
3	0.000 0	0.000 0	0.317 1	Ⅲ	Ⅲ	Ⅲ
4	0.000 0	0.000 0	0.287 8	Ⅲ	Ⅲ	Ⅲ
5	0.000 0	0.236 6	0.357 8	Ⅲ	Ⅲ	Ⅱ
6	0.000 0	0.273 6	0.344 5	Ⅲ	Ⅲ	Ⅱ
7	0.000 0	0.406 6	0.287 0	Ⅲ	Ⅱ	Ⅱ
8	0.000 0	0.323 5	0.412 0	Ⅲ	Ⅲ	Ⅱ
9	0.000 0	0.258 2	0.272 4	Ⅲ	Ⅲ	Ⅱ
10	0.000 0	0.279 5	0.028 0	Ⅲ	Ⅱ	Ⅱ
11	0.031 9	0.174 2	0.040 1	Ⅲ	Ⅱ	Ⅰ
12	0.238 8	0.004 3	0.019 7	Ⅰ	Ⅰ	Ⅰ
13	0.508 6	0.113 4	0.012 3	Ⅰ	Ⅰ	Ⅰ
14	0.351 1	0.041 6	0.019 0	Ⅰ	Ⅰ	Ⅰ
15	0.386 4	0.056 4	0.0170	Ⅰ	Ⅰ	Ⅰ
16	0.341 3	0.029 6	0.016 3	Ⅰ	Ⅰ	Ⅰ
17	0.036 6	0.046 3	0.021 9	Ⅲ	Ⅱ	Ⅰ
18	0.230 7	0.056 7	0.028 7	Ⅲ	Ⅰ	Ⅰ
19	0.138 2	0.132 2	0.027 1	Ⅲ	Ⅰ	Ⅰ

5 Conclusions

(1) Aiming at the problems of single reference index, one-sided evaluation results and inability to reflect the uncertainty relationship between each index in the safety and stability evaluation of shield tunneling, a comprehensive evaluation model of extension cloud suitable for water-rich sandy cobble stratum in Chengdu area is proposed by combining game theory with extension cloud model. Among them, the combination weight of hierarchical analysis and entropy theory is used to weight each evaluation index, which avoids the subjective one-sidedness of single weighting, and the calculation results are in line with objective reality.

(2) Taking the construction of the shield interval of Chengdu subway as an example, the safety and stability of 19 monitoring points were evaluated using the extension cloud integrated model based on five evaluation indexes, such as ground settlement, building settlement, river bank settlement, pipeline settlement and pipe sheet settlement, and compared with the evaluation results of on-site monitoring, among which, the evaluation results of each monitoring point in level III state were basically the same. For the monitoring points in the early warning state, the evaluation results of the extension cloud integrated model are one grade higher than the evaluation results of on-site monitoring. And the extension cloud model has greater superiority in evaluating the uncertainty relationship between indicators, so the extension cloud integrated evaluation results are more in line with the actual situation.

(3) The comprehensive evaluation model based on game theory and evaluation cloud takes into account the fuzziness, randomness and actual distribution laws in the information of the shield tunneling safety and stability evaluation indexes, avoiding the lack of information caused by normalizing the data, and at the same time, the combined weighting of the original data makes the weight of each index more scientific and reasonable, which can provide scientific reference for the evaluation of shield tunneling safety and stability under the conditions of water-rich sand and pebble strata in Chengdu.

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Declaration of competing interest

The authors have no competing interests to declare that are relevant to the content of this article.

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